FINITE DIFFERENCE OF THERMAL LATTICE BOLTZMANN
SCHEME FOR THE SIMULATION OF NATURAL
CONVECTION HEAT TRANSFER

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I hereby declare that I have checked this project and in my opinion, this project is adequate in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering

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I hereby declare that the work in this project is my own except for quotations and summaries which have been duly acknowledged. The project has not been accepted for any degree and is not concurrently submitted for award of other degree.

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ACKNOWLEDGEMENT

In the name of Allah s.w.t, the most Gracious, the Ever Merciful Praise is to Allah, Lord of the Universe and Peace and Prayers be upon His final prophet and Messenger Muhammad s.a.w.

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ABSTRACT

In this thesis, a method of lattice Boltzmann is introduced. Lattice Boltzmann method (LBM) is a class of computational fluid dynamics (CFD) methods for fluid simulation. Objective of this thesis is to develop finite difference lattice Boltzmann scheme for the natural convection heat transfer. Unlike conventional CFD methods, the lattice Boltzmann method is based on microscopic models and macroscopic kinetic equation. The lattice Boltzmann equation (LBE) method has been found to be particularly useful in application involving interfacial dynamics and complex boundaries. First, the general concept of the lattice Boltzmann method is introduced to understand concept of Navier-Stokes equation. The isothermal and thermal lattices Boltzmann equation has been directly derived from the Boltzmann equation by discretization in both time and phase space. Following from this concept, a few simple isothermal flow simulations which are Poiseulle flow and Couette flow were done to show the effectiveness of this method. Beside, numerical result of the simulations of Porous Couette flow and natural convection in a square cavity are presented in order to validate these new thermal models. Lastly, the discretization procedure of Lattice Boltzmann Equation (LBE) is demonstrated with finite difference technique. The temporal discretization is obtained by using second order Runge-Kutta (modified) Euler method from derivation of governing equation. The discussion and conclusion will be presented in chapter five.
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LIST OF SYMBOLS

\( R \)  \hspace{1cm} \text{Gas constant}
\( t \)  \hspace{1cm} \text{Time}
\( T \)  \hspace{1cm} \text{Temperature}
\( T_C \)  \hspace{1cm} \text{Cold temperature}
\( T_H \)  \hspace{1cm} \text{Hot temperature}
\( u \)  \hspace{1cm} \text{Horizontal velocity}
\( u \)  \hspace{1cm} \text{Velocity vector}
\( U \)  \hspace{1cm} \text{Horizontal velocity of top plate}
\( v \)  \hspace{1cm} \text{Vertical velocity}
\( V \)  \hspace{1cm} \text{Volume}
\( x \)  \hspace{1cm} \text{Space vector}
\( P \)  \hspace{1cm} \text{Pressure}
\( \tau_{f,g} \)  \hspace{1cm} \text{Time relaxation}
\( \nu \)  \hspace{1cm} \text{Shear viscosity}
\( \beta \)  \hspace{1cm} \text{Thermal expansion coefficient}
\( \varepsilon \)  \hspace{1cm} \text{Internal energy}
\( \rho \)  \hspace{1cm} \text{Density}
\( T_{\infty} \)  \hspace{1cm} \text{Infinity temperature}
\( T_f \)  \hspace{1cm} \text{Film temperature}
\( T_s \)  \hspace{1cm} \text{Surface temperature}
\( A \)  \hspace{1cm} \text{Area of contact A}
\[ \eta \quad \text{Proportionally constant} \]
\[ \chi \quad \text{Thermal diffusivity} \]
\[ \Omega \quad \text{Collision operator} \]
\[ T_m \quad \text{Average temperature} \]
\[ g \quad \text{Acceleration due to gravity} \]
\[ c \quad \text{Microscopic velocity} \]
\[ f^{eq} \quad \text{Equilibrium distribution function} \]
\[ f \quad \text{Distribution function} \]
\[ Pr \quad \text{Prandtl number} \]
\[ Ra \quad \text{Rayleigh number} \]
\[ Re \quad \text{Reynolds number} \]
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<td>Bhatnagar Gross krook</td>
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CHAPTER 1

INTRODUCTION

1.1 NAVIER-STROKES EQUATION

The Navier-stokes equation can derive as the motion of fluid substances that is substances which can flow. These equations arise from applying Newton's second law to fluid motion, together with the assumption that the fluid stress is the sum of a diffusing viscous term (proportional to the gradient of velocity), plus a pressure term. The derivation of the Navier–Stokes equations begins with the conservation of mass, momentum, and energy being written for an arbitrary control volume. These equations describe how the velocity, pressure, temperature, and density of a moving fluid are related. The mathematical relationship governing fluid flow is the famous continuity equation and Navier-stokes equation is given by

\[ \nabla \cdot \mathbf{u} = 0 \]  \hspace{1cm} (1.1)

\[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \left( \frac{2\tau - 1}{6} \right) \nabla^2 \mathbf{u} \]  \hspace{1cm} (1.2)

The Navier-Stroke equation is nonlinear partial differential equations in almost every real situation and so complex that there is currently no analytical solution to them except for a small number of special cases. The Navier–Stokes equations dictate not position but rather velocity. A solution of the Navier–Stokes equations is called a velocity
field or flow field, which is a description of the velocity of the fluid at a given point in space and time. Once the velocity field is solved for, other quantities of interest (such as flow rate or drag force) may be found. (C.S. Nor Azwadi, 2007)

Nowadays, the use of a computer is necessary to determine the fluid motion of a particular problem because fluid related problems arising in science and engineering are extremely complex by nature.

1.2 COMPUTATIONAL FLUID DYNAMICS

Computational Fluid Dynamic (CFD) is based on the fundamental governing equation of fluid dynamics: the continuity, momentum, and energy equations. The most fundamental consideration in CFD is how one treats a continuous fluid in a discretized fashion on a computer. One method is to discretize the spatial domain into small cells to form a volume mesh or grid, and then apply a suitable algorithm to solve the Navier-Stokes equation or an equation derived from them. To exactly simulate fluid flow in a computer it would be necessary to solve Navier-Stokes equation with infinite accuracy. In reality, numerical researchers must choose a method to discretize the problem. Some of the general numerical methods used in computational fluid dynamics are described here.

1.3 LATTICE BOLTZMANN METHOD (LBM)

The Lattice Boltzmann method (LBM) is a recently developed method for simulating fluid flows and modeling physics in fluids. Unlike the traditional CFD methods, which solve the conservation equations of macroscopic properties (i.e., mass, momentum, and energy) numerically, LBM models the fluid consisting of fictive particles, and such particles perform consecutive propagation and collision processes over a discrete lattice mesh. Due to its particulate nature and local dynamics, LBM has several advantages over other conventional CFD methods, especially in dealing with complex boundaries, incorporating of microscopic interactions, and parallelization of the algorithm. It is also
known as an alternative approach to the well-known finite difference, finite element and finite volume technique for solving the Navier-Stokes equations. Lattice Boltzmann methods (LBM) is a class of computational fluid dynamics (CFD) methods for fluid simulation. Instead of solving the Navier–Stokes equations, the discrete Boltzmann equation is solved to simulate the flow of a Newtonian fluid with collision models such as Bhatnagar-Gross-Krook (BGK). LB scheme is a scheme evolved from the improvement of lattice gas automata and inherits some features from its precursor, the Lattice Gas Automata (LGA).

The LBM recognizes that Boltzmann’s transport equation is a computational tool that can be solved on the lattice. The collision term of this equation can be simplified using the Bhatnagar-Gross-Krook (BGK) approximation where the distribution function relaxes to a local equilibrium with a constant relaxation time. The main motivation for the transition from LGA to LBM was the desire to remove the statistical noise by replacing the Boolean particle number in a lattice direction with its ensemble average, the so-called density distribution function. Accompanying this replacement, the discrete collision rule is also replaced by a continuous function known as the collision operator. In the LBM development, an important simplification is to approximate the collision operator with the Bhatnagar-Gross-Krook (BGK) relaxation term. This lattice BGK (LBGK) model makes simulations more efficient and allows flexibility of the transport coefficients. (Xiaoyi He and Li-Shi Luo, 1997)
Although LBM approach treats gases and liquids as systems consisting of individual particles, the primary goal is to build the connection between the microscopic and macroscopic dynamics, rather than to deal with macroscopic dynamics directly. In other words, the goal is to derive macroscopic equations from microscopic dynamics by means of statistic, rather than to solve macroscopic equation.

**Figure 1.1**: Historically stages in the development of lattice Boltzmann model

Source: Wolf Gladrow, 2000

### 1.4 CLASSICAL CFD VERSUS LATTICE BOLTZMANN METHODS

The conventional simulation of fluid flow and other physical processes generally starts from non linear partial differential equation (PDEs). These PDEs are discretized by finite differences, finite element finite volume or spectral methods. The resulting
algebraic equations of ordinary differential equation are solved by standard numerical methods. In LBM, the starting point is a discrete microscopic model which by construction conservation equation of mass and momentum for Navier-Stokes equation. The derivation of the corresponding macroscopic equation requires multi-scale analysis. (Wolf Gladrow, 2000)

![Diagram](diagram.png)

**Figure 1.2**: Classical CFD versus LBM

### 1.5 PROJECT OBJECTIVE

To develop finite difference Lattice Boltzmann Scheme for the Natural Convection Heat Transfer

### 1.6 PROJECT SCOPES

The first project scope is to analysis heat transfer limit to natural convection only. The second project scope is the problem will be test at Rayleigh number, \( Ra = 10^3 \) to \( 10^5 \). This limitation due to Lattice Boltzmann Method (LBM) can perform well at low Rayleigh number and at high Rayleigh number LBM having a problem. This selection high Rayleigh number is to show that this scheme can simulate problem at high Rayleigh number. For the last project scope is to simulate natural convection in a
square cavity. Detail characteristic numerical value of the flow will be carrying out; isotherm line, stream line and average of Nusselt number where they occur will be compared.

1.7 PROJECT BACKGROUND

The lattice Boltzmann Method (LBM) is an alternative approach to the well-known finite difference, finite element and finite volume techniques for solving the Navier-Strokes equations. LB scheme is a scheme evolved from the improvement of lattice gas automata (LGA) and inherits some features from its precursor, the LGA. The implementation of the Bhatnagar-Gross-Krook (BGK) approximation is a improvement to enhance the computational efficiency has been made for the LB method. The algorithm is simple and can also easily modify to allow for the application of other, more complex simulation component. In mathematics, finite difference methods are numerical methods for approximating the solution to difference equation using finite difference equation to approximate derivatives. Finite difference lattice Boltzmann method is obtained using second order Runge-Kutta (modified) Euler Method.

1.8 THESIS OUTLINE

The aim of this thesis is to study the methods of the lattice Boltzmann equations in order by using finite difference method. These subjects, newly emerged in 1980’s utilize the statistical mechanics of simple discrete models to simulate complex physical systems. The theory of lattice Boltzmann method in 4-discrete velocity and 9-discrete velocity are reviewed in detail.

In the Chapter two, the concept of a distribution function is considered and the derivation and the theory of the classical Boltzmann equation are discussed briefly. Then the theory of lattice Boltzmann method and its coefficients from the Boltzmann equation (Chapman-Enskog expansion) are also presented.

For the four discrete velocities is found in the isothermal model and for the 9-discrete velocity, happen in thermal model that will discuss in chapter three. By using 4-
type of discrete velocity, we can apply to develop Porous Couette flow problem for the thermal fluids problem. Using 9-discrete velocity model can apply in Poiseulle flow and Couette flow

In Chapter four, the simulation by using Finite difference method was applied among the first approaches applied to the numerical solution of differential equations. This method is directly applied to the differential form of the governing equations. The finite difference is a second order equation upwind scheme.

Finally in chapter five, conclusions and discussion on future studies are presented and also some of recommendations will elaborate in order to improve the project on future.
CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

For the literature review will consist the theory of Lattice Boltzmann Model (LBM) that a content are governing equation, basic principle of LBM, collide function Bhatnagar-Gross-Krook (BGK), equilibrium distribution function, time relaxation, discretization of microscopic velocity and lastly is a derivation of Navier-Stokes equation. For the derivation of Navier-Stokes equation was already state in Chapter 1. Beside, derivation of two type of boundary condition which are Bounce-back and also periodic also presented. For this chapter also, we state briefing about Isothermal Lattice Boltzmann Model which are Poiseulle Flow and Couette Flow and Thermal Lattice Boltzmann Model which is Porous Couette Flow.

LBM is a computational fluid dynamics (CFD) method or alternative method to simulate the fluid problems especially to simulate the complex fluid flow problems including single and multiphase flow in complex geometries. The main objective to achieve is to create a connection between the microscopic and macroscopic dynamics. From the Navier-Stokes equation, we already know the density, velocity, pressure and etc. So, this is called macroscopic scale. But, for the Boltzmann equation, we only have the particles that moves to one place to another and called microscopic scale.
2.2 GOVERNING EQUATION

From this equation, we can the relation between microscopic and macroscopic dynamics. The Boltzmann equation given is shown below:

\[
f(x + c\Delta t, t + \Delta t) - f(x, t) = \Omega(f)
\]  

Where  
- \( f \) = density distribution function  
- \( c \) = microscopic velocity  
- \( \Omega \) = collision integral