INVESTIGATING BENDING STRENGTH OF SPUR GEAR : A REVIEW

Ismail Ali B Hj Abdul Aziz[1,a], Daing Mohamad Nafiz Bin Daing Idris[1,b], Wafiuddin B Mohd Ghazali[1]

[1]Faculty of Mechanical Engineering, Universiti Malaysia Pahang,
26600 Pekan, Pahang, Malaysia,
[a]smaelali82@gmail.com, [b]daingnafiz@ump.edu.my

ABSTRACT

Having a lightweight and efficient gear system could improve engine efficiency that will reduce emission and enhanced fuel economy in order to reduce the global carbon footprint. Gear tends to play a very vital role in all industries. As other mechanical systems, it is subject to design parameter, installation errors, method of manufacture and load uncertainties arising from randomness. Before engineers could design an efficient and safe gear, it is importance to understand on how gear can fail is much needed. This article reviews the methodology used to investigate bending strength of spur gear; Finite Element Method (FEM), Numerical Calculation and Investigational Techniques were usually carried out in order to understand the bending strength of thin-rimmed spur gear. Works and experiment from the literature were studied and their findings were extracted to understand the methods used. The most common method used to investigate bending strength is the numerical calculation. This method was used with several types of established equations and standards to predict gear failures. Next stage is to simulate the gear using Finite Element Method (FEM) in order to get the analysis of gear strength and later to be used for data verification. The most important stage is to put the gears to physical experiment or testing facilities to determine and validate all data from the numerical calculation and FEM method.

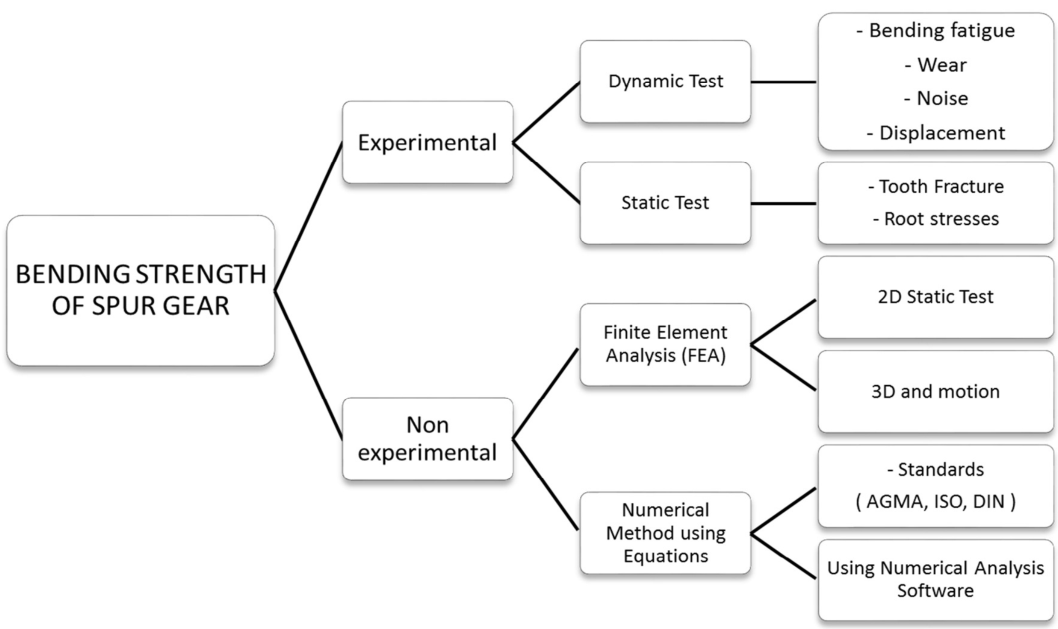
Keywords: Thin-Rimmed Spur Gear, Bending Strength, Finite Element Method (FEM), Numerical Calculation

INTRODUCTION

Nowadays, there are huge demands to have lightweight and efficient gear in industry especially in aerospace and transportation applications[1]. Since the creation of gearing system to transmit water from river thousand years ago, the importance of gearing system become more crucial. With growing demands for sustainable environment, it is an utmost importance for industries to reduce the global carbon footprint and conserve natural resources[2]. Aircraft manufacturers and automakers are showing more interest in developing innovative solutions that reduce emissions with improved fuel economy by reducing weight. In gearing applications such as gearbox for vehicles and aircrafts, thin-rimmed gears can help to reduce the weight of gearbox. Therefore, the usage of thin-rimmed gear became more important and often used in application where lightweight and compact designs were demand.

Gears can fail in many different ways, and except for an increase noise level and vibration, there is no early indication of difficulty until total failure occurs[3]. The general types of failure modes (in decreasing order of frequency) include fatigue[4], fracture[5], wear[6], and stress rupture. Gear tooth failures occur in two distinct regions, the tooth flank and the root fillet[7]. For thin-rimmed gears, bending stress can be different because of rim and web thickness factor [8-10].

The importance of understanding on how gear can fail is much needed before engineers could design efficient and safe gears. Figure 1 shows approaches done via various research in understanding gear bending strength. Gear testing can help designers to understand various kinds of gear failures, as well as estimating the gear life in service. [11]. The aid to the designer is the ability to recognize the exact type of initial failure and come out with method to avoid those failures. There are several numerical standards used to design gear as published by German Institute for Standardization (DIN), American Gear Manufacturers Association (AGMA) and International Organization for Standardization (ISO) with complex equations and calculations. In addition, Finite Elements Method (FEM) used to predict gear failure, estimates gears life in service and being used to validate data from experimental test on bending strength. On top of numerical calculation and FEM, experimental gear testing is the best approach to find out gear bending strength. Gear design calculations and experimental tests are necessary to develop and confirm a hypothesis for the probable cause of failure

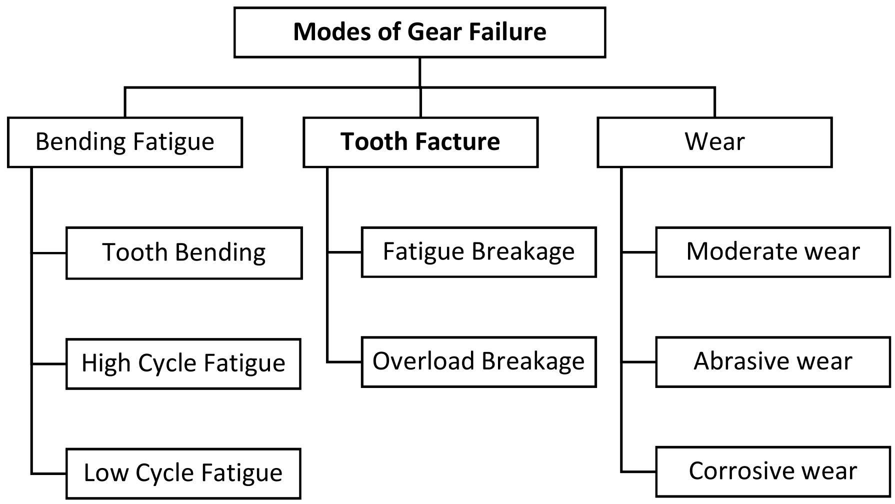


[12].

Figure 1: Approach in understanding Gear Bending Strength

Gear Failures

A pair of gear tooth in action generally subjected to cyclic stress failure that includes tooth-bending fatigue, tooth-bending impact, and tooth wear[13]. Gear tooth failures occur in two distinct regions, the tooth flank and the root fillet. Gear can fail in



several modes as shown in Figure 2.

Figure 2: Modes of Gear Failure

A fracture of a tooth is the cause to be most concerned about as damaged gear can cause the entire operation of a machine to come to a standstill and worst can lead to the destruction of the machine. Tooth breakage caused by an overload, which exceeds the tensile in a short-cycle break, has a finer stringy appearance but still shows evidence of being pulled apart abruptly [13]. In thin-rimmed case, the rim of a gear usually fails between two adjacent teeth. Cracks propagate through the rim and into the web. Sometimes cracks appear in the web near the rim and web junction without disturbing the rim itself. Gear tooth failure from bending fatigue generally appears from a crack initiation at the root section of the gear tooth[4]. There are other causes lead to bending fatigue failures, too much load and cyclic loading that stress at the gear tooth beyond the endurance limits of the material may be one of major concern.

Gear tooth failure from bending fatigue mostly resulted from a crack patenting in the root section of the gear tooth. Based on gears rotational motion and the amount of load they carry, it's subjected to fatigue[14]. Failures that occur under applied load less than 10, 000 cycles considered as low cycle fatigue. Whereas high cycle fatigue defines as a failure that occurred after more than 10,000 cycles of applied load. The stress levels involved in low cycle fatigue are normally higher than the yield strength of the material.

As the nature of gearing system to have contact on each other, it tends wear out. The most common wear failure is moderate wear. This happen for a very huge number of cycles and might involves lack of lubricant or fine dirt in the system. Abrasive wear is caused due to the presence of metal particles from the gears and bearings, weld spatter, scale, rust, sand and dirt in the lubricant of the gears [13]. Meanwhile corrosive wear occurs due to corrosion action on the teeth of the gears. This type of wear regime is visually looked at as spotty dots appearing scattered on the tooth of the gear.

FINITE ELEMENT METHOD (FEM)

Before actual experiment being conducted on a specimen, usually researchers will conduct simulations of the specimen. Works that use the finite element method usually will help the researchers to validate the data that will be obtained from the experiment.

Most recent study by Vincent Savaria [2] used FE model based on the well-known Crossland criterion; calibrated with representative axial and torsion laboratory specimens. The geometry used was a simplified 3D FE model including only one tooth and being compared on Von Mises strain maps obtained by digital image correlation and the FE model as shown in Figure 3. The aim was to predict the effect of material properties on bending fatigue strength of hardened aeronautical gears. Results from FEA test later used to validate data obtained by experiment on single tooth bending fatigue rig (STBF).

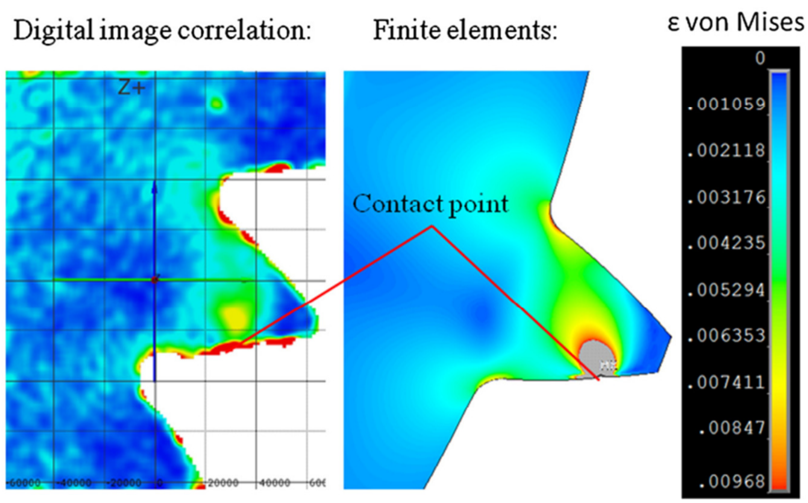


Figure 3: Comparison of Von Mises strain maps obtained by digital image correlation and the FE model [2].

Toni Jabbour et. al [15] also verified their data by finite element calculations. Results from the experiment to find a method for the calculation of the root and contact stresses for metal, spur and helical gear were presented and verified. Geometry of the two gears as well as the assembly was drawn using SolidWorks. Another gear associated to enable the simultaneous rotation of the two gears with consistent gear ratio. By varying the contact position of the tooth, it allowed to perform finite element calculations in accordance with the kinematics of the pair of gears in contact. In order to choose the most effective mesh refinement, the maximum contact pressure values, obtained from several analyses were compared because the precision of the contact pressure value depends on the number of elements.

In order to reduce bending stress in external spur gears by redesign the standard cutting tools, Niels L. Pederson [16] simulated Finite Element Modelling of external spur gear with Poisson's ratio of $v = 0.3$. The load and supports as well as the geometry were earlier specified. Results from FEA simulated by F.Cura et. al [17] views that crack propagation path and crack initiation point were strongly influenced by centrifugal load. Spur gears with different web and thin-rimmed were modelled before boundary conditions consist in displacement loads to simulate the presence of the whole gear.

ANALYTICAL CALCULATIONS METHOD

Analytical Calculations were one of the most important stages in designing a gear. Through this method, design and analysis of the spur gears to resist bending failure of the teeth as well as pitting failure of tooth surfaces can be implemented.

The Lewis Bending Equation

Wilfred Lewis[18] in 1892 introduced the bending equation that still remain the most common or basic standard for gear design. Lewis calculated stress in the gear base using a cantilever beam under an applied bending moment[19]. A rectangular cantilever beam of cross-sectional dimensions F and t, having a length l and a load W^t, uniformly distributed across the face width F. The section modulus I/C is $Ft^2/6$ nd therefore the bending stress is

$$\sigma = \frac{M}{I/c} = \frac{6W^t l}{Ft^2} \quad (1)$$

This basic formula being completed by including factor y that is called the Lewis form factor so we have

$$\sigma = \frac{W^t P}{FY} \quad (2)$$

where P is diametrical pitch. The used of this equation for Y means that only the bending of the tooth is considered and that the compression due to the radial component of the force is neglected.

AGMA Stress Equations

Based on Equation (2), ANSI/AGMA 2001-D04 and 2101-D04 contains dynamic factor K_v. Two fundamental stress equations are used in the AGMA methodology, one for bending stress and another for pitting resistance (contact stress). The fundamental equations are

$$\sigma = W^t K_o K_v K_s \frac{1}{bm_t} \frac{K_H K_B}{Y_J} \quad \text{(SI Unit)} \quad (3)$$

where

W^t is the tangential load, (N)	K_H is the load –distribution factor
K_o is the overload factor	K_B is the rim-thickness factor
K_v is the dynamic factor	b is the face width of the narrower member, (mm)
K_s is the size factor	
m_t is the transverse module	Y_J is the geometry factor for bending strength

Aida & Terauchi – On the Bending Stress of a Spur Gear

In 1962, Aida & Terauchi [20] arrived at the conclusion that was rational to take the sharing force into consideration for the calculation of the bending stress of a gear tooth. They have calculated the bending stress of a standard gear tooth by defining the thickness and the position of the weakest section and the worst loaded point.

Therefore, the subsequent formula has been presented for the bending stress of a gear tooth:

$$\sigma_t = \left(1 + 0.08\,\frac{S}{\rho}\right)\left(0.66\,\sigma N_b + 0.40\,\sqrt{\sigma {N_b}^2 + 36\,\tau N^2} + 1.15\,\sigma\,N_c\right) \tag{4}$$

the nominal stress σN_B, σN_c and τN can be calculated by means of the following equations;

$$\sigma N_b = \frac{6P\,sin\theta\,(l_a + l_d)}{S^2 b} \tag{5-1}$$

$$\sigma N_c = -\frac{P\,cos\theta}{Sb} - \frac{6P\,cos\theta\,y}{S^2 b} \tag{5-2}$$

$$\tau N = \frac{P\,sin\theta}{Sb} \tag{5-3}$$

Many researchers since then have performed analytical studies to investigate gear tooth bending stress [21-24]. Chen et. al [15] came out with a method for the calculation of root and contact stresses for metal, spur and helical gears. They verified their finding by finite elements calculations. Finding shows the spur gears, the location of the point of contact that leads to the critical tooth-root stress depends on the contact ratio of the pair of gears that rises with the number of teeth.

Li Shunting, since 2002 [9, 25-27] have come out with several studies on bending strength of thin-rimmed spur gears. One of his recent studies on analyses of thin-rimmed spur gear with inclined webs founds that the web position and the web angle of the thin-rimmed gears have a significant effect on the tooth contact stresses, the root bending stresses, and the joint stresses. His findings actually aligned with calculation from Miyachika [28], F Cura [17] and Opalic [29] that also take consideration on effect of rim thickness and web arrangements on tooth bending strength of thin-rimmed spur gears.

EXPERIMENTAL TECHNIQUES

Reviews also revealed several set up to study gear strength been made by researchers. Most of researchers focusing the leading causes of failure appeared to be tooth-bending fatigue, tooth-bending impact and abrasive tooth wear.

Most recent study done by Pawar, P. B et al [30] prepared hardness test of composite samples and conducted experiment using on Automatic Optical Brinell Hardness Tester with range 250 kg to 3000 kg. A year before, Nizar Ahamed et al [31] develop a methodology which was tough for fault detection of gears under fluctuating load and speed settings. A multiple-pulse independently rescaled-time synchronous averaging (MIR-TSA) technique in conjunction with conventional time synchronous averaging has been projected. It has been observed that the proposed method improves the fault detection under fluctuating speed conditions. A silimar setup was used by Nick Bretl et al [32] to investigate tooth bending strength of case hardened gears in the range of high cycle fatigue. A pulsator test rig shown in Figure 4 was consistently carried out the bending test.

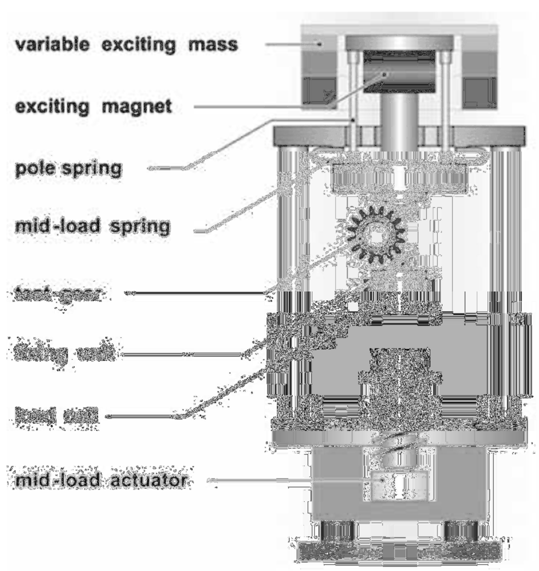


Figure 4: Illusration of a Pulsator Test Rig [32]

David B. Stringer et al [33] set up a new high-speed test capability for determining the high-cycle bending-fatigue characteristics of gear teeth. These test various in load circumstances and cycle-rates. The cycle-rate varied from 50 to 1000 Hz. Li, Shuting [34] assembled a ‘‘power-circulating form’’ test rig for vibration tests of the thin-walled gears at the speed range 500–3000 rpm and then strain phase method was presented to identify the resonance mode shapes of the thin-walled gears when they are running in a complete resonance state. M. J Handschuh et al. [35] established a test setup with equipment for the measurement of root stresses. They used an open-architecture gearbox with drive and load capacity equipped with motor and brake to operate gears under high-load and low-speed conditions. Later, a study by Qi Zhang [36] used a setup of The FZG test rig that was a back to back gear test rig of the closed power loop type.

CONCLUSIONS

From the reviewed articles, it could be concluded that there are several stages had been done in order to investigate gear bending strength. Researchers used the basic Lewis Bending Equation and AGMA Standards with modification to get first view of gear tooth strength. In fact, many researchers formulate their own calculations to predict gear strength. Second stage is to simulate gears using Finite Element Method (FEM) in order to have the analysis of gear strength and later to be used for data verification. The most important stage is to put the gears to experiment or testing facilities to determine and validate all data from numerical calculation and FEM method.

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