Influence of Slip Velocity and Aligned Magnetohydrodynamics on Convective Boundary Layer Flow of Jeffrey Fluid with Convective Boundary Condition Across Stretching Sheet

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Abstract-The solutions on the flow and heat transfer of a Jeffrey fluid on a stretching sheet with effect of aligned Magnetohydrodynamics (MHD) is presented in this paper. The governing non-linear equations are first transformed into ordinary differential equations using appropriate transformation before solving using BVP4C. In order to verified the correctness of the present computation, the comparison on the limiting case with the previous publication has been made and the results shows an excellent agreement.

Keywords- Jeffrey fluid; slip velocity; Magnetohydrodynamics; heat transfer; convective boundary conditions

1. INTRODUCTION

Much attention has been given to the flow over a stretching sheet due to the application in engineering fields. For instance, the features of a moving surface that is constant are owned by material manufactured by extrusion process and heat-treated materials that travels between a feed roll and windup roll or on conveyor belts.

Besides, the flow over a stretching sheet also important in the process of constructing papers, linoleum, polymeric sheets, roofing shingles, insulating materials, and fine-fiber. Some of important part in nuclear power plants, gas turbines and numerous propulsion devices for aircraft, missiles, satellites and space vehicles are involve by convective heat transfer. The primary connection between the medium boosts the starting of heat transfer between the sheet and the adjacent fluid. There is a decrease of the sheet temperature in the streams direction when the fluid temperature is lower as compare to the sheet. Then, a stream wise increase of the latter will be observed if the fluid temperature is greater than the sheet temperature. The advancement of knowledge in heat transfer where temperature variation appear are essential for the design on thermal processing station for moving sheets. [1-6]. Final product quality is highly affected by the cooling rate and thus, heat transfer is substantial. Merkin and Kumaran [7], have been investigated on the problem of unstable Magnetohydrodynamic boundary layer flow over a shrinking. Ariel [8], has apply Homotopy perturbation approach in the study on boundary layer flow of a viscous fluid through a stretching sheet while the mixed convection flow of an incompressible viscous fluid toward a vertical permeable stretching sheet in a porous medium has been considered by Mukhopadhyay [4]. In 2010, the group of researchers studied the influence of thermal radiation and chemical reaction on the Magnetohydrodynamic flow across an unsteady permeable stretching surface (see Hayat et al. [9]). The derivation on the exact solutions in solving the problem of two-dimensional Magnetohydrodynamic flow across a stretching sheet with partial slip has been completed by Fang et al.[10]. Previously the same researcher do the investigation on boundary layer flow over a stretching sheet with power-law velocity (see Fang [11]). The field of metal-working processes and modern metallurgical enhances the understanding of the work involving effect Magnetohydrodynamic (MHD) flow of an electrically conducting fluid. Various examples of such fields include the process of fusing metals in an electrical furnace through a magnetic field and the process of cooling of the first wall inside a nuclear reactor containment vessel where the hot plasma is isolated from the wall through a magnetic field [12]. A significant role of the use of magnetic field can be envision in the control of heat transfer and momentum in the boundary layer flow of differing fluids across a stretching sheet [13]. The streamlines are made more steep by magnetic field and resulting on the thinner in boundary layer [14]. Motivating to the above study, the analyzing of the Jeffrey fluid, MHD and heat transfer across a sheet based on the effect of convective boundary conditions are the focus this investigation. BVP4C method is applied to attain the numerical solutions, where the presentation of the results is done in the tabular and graphical form.

2. MATHEMATICAL MODEL

The geometrical physical represent the moving Jeffrey fluid influenced by aligned MHD convective boundary layer across a stretching sheet at a slip velocity $u_w(x) > 0$, with uniform ambient temperature T_∞ due to a permeable stretching surface corresponding with the plane $y = 0$, where the flow being confined to $y > 0$, is illustrated in Figure 1.

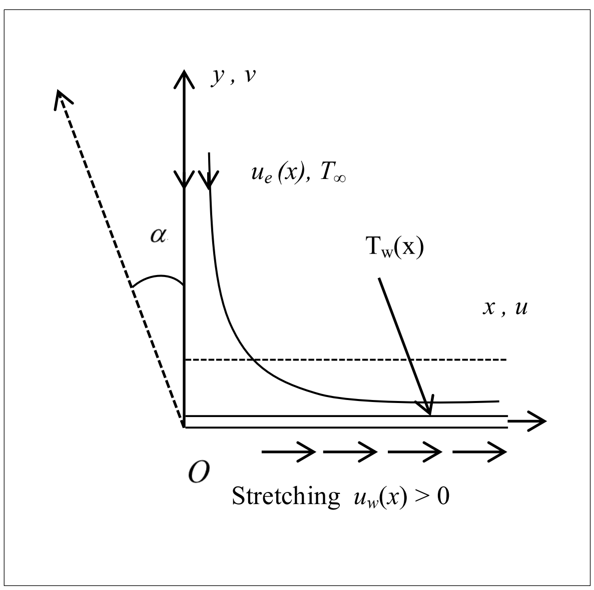

Fig. 1 The geometry of physical flow

Introducing two equal and opposite forces along the x-axis so that the surface is stretched with the velocity $u_w(x)$. It is also assumed that the velocity outside the boundary layer (potential flow) is $u_e(x)$. Under these assumptions, the boundary layer equations governing the flow and temperature fields are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\frac{v}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial y^2}+\lambda_2\left(u\frac{\partial^3 u}{\partial x\partial y}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right)\right]-\frac{\sigma B_0^2}{\rho}\sin^2(\alpha_1) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty) \tag{3}$$

with boundary conditions

$$u=cx+g\frac{\partial u}{\partial y}\ ,\ v=0\ ,\ -k\frac{\partial T}{\partial y}=h_f\left(T_w-T\right)\ \text{(CBC)}\ ,\ T=T_w(x)\ \text{(CWT) at }\ y=0$$

$$u\to u_e(x)=ax\ ,\ \frac{\partial u}{\partial y}\to 0\ ,\ T\to T_\infty\ \text{ as }\ y\to\infty, \tag{4}$$

where u and v are the velocity components in the x and y directions, T is the fluid temperature, $v=\dfrac{\mu}{\rho}$ is the kinematic viscosity, λ_1 is the ratio of relaxation and retardation time and λ_2 is the relaxation time. It is expected a uniform magnetic field of strength B_0 is imposed along y-axis. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field is assumed to be negligible, σ is the electrical conductivity, ρ is the density, α_1 is inclined angle (aligned angle), c_p is the specific heat at constant pressure, T_∞ is free stream temperature, α is the thermal diffusivity, a , c (>0) are constant, g is the gravitational acceleration acts in the downward direction, $k=g\sqrt{\dfrac{c(1+\lambda_1)}{v}}$ is thermal conductivity, h_f is heat transfer coefficient, and T_f is heat the surface of the sheet.

The following transformations has been proposed in order to reduce the complexity of equations.

$$\eta=y\sqrt{\frac{c(1+\lambda_1)}{v}}\ ,\ u=cxf'(\eta)\ ,\ v=-\sqrt{\frac{cv}{1+\lambda_1}}f(\eta)\ ,\ \theta=\frac{T-T_\infty}{T_w-T_\infty}\ \text{(CBC)} \tag{5}$$

After applying (5) the equation (1-3) reduced to

$$f'''+ff''-(f')^2+\beta\left((f'')^2-ff^{(4)}\right)-Mf'\sin^2(\alpha_1)+\delta^2=0, \tag{6}$$

$$\theta''+\Pr f\theta'+\Pr Q\theta=0, \tag{7}$$

with respected to boundary condition

$$f'(0)=1+kf''(0)\ ,\ f(0)=0\ ,\ \theta'(0)=-\gamma(1-\theta(0))\ ,$$

$$f''(\infty)=0\ ,\ \theta(\infty)=0\ ,\ f'(\infty)=\delta\ , \tag{8}$$

For the case of constant wall temperature, one of the boundary condition at $\eta = 0$ is

$\theta(0) = 1$ (CWT)

where $\beta = \lambda_2 c$ is the Deborah number, $M = \dfrac{\sigma\beta_0^2}{c\rho}$ is magnetic parameter, $\delta = \dfrac{a}{c}$ is stretching parameter, $\Pr = \dfrac{v}{\alpha}$ is prandtl number, $Q = \dfrac{Q_0}{c\rho c_p}$ is heat generating, $\gamma = \dfrac{h_f}{k\sqrt{\dfrac{c(1+\lambda_1)}{v}}}$ is the conjugate parameter. It is important to mentioned here, the case of CWT has been taken into account as the case to verified the accuracy of the computation.

3. RESULTS AND DISCUSSION

The numerical scheme was performed on various parameters like fluid parameter, δ, stretching parameter, and prandtl number $\Pr$. A comparative study has been made between the previous published results with the current solutions as shown in Table 1. The results of $-\theta'(0)$ for the case of CWT from M.Turkyilmazoglu and Pop [15] has been used for this purpose. It is notice that, the strong agreement has been found.

Table 1 comparison between the previously result for boundary condition and present solution for the CWT case at $M = 0$, $Q = 0$, $k = 0$.

$-\theta'(0)$			
Pr	δ	**M.Turkyilmazoglu, I.Pop [15]**	**Present**
0.5	0.5	0.49039	0.48965
	1	0.56419	0.56335
1	0.5	0.71544	0.71470
	1	0.79788	0.79701
2	0.5	1.0392	1.0385
	1	1.1284	1.1275
5	0.5	1.6886	1.6881
	1	1.7841	1.7834
10	0.5	2.4244	2.4244
	1	2.5231	2.5228

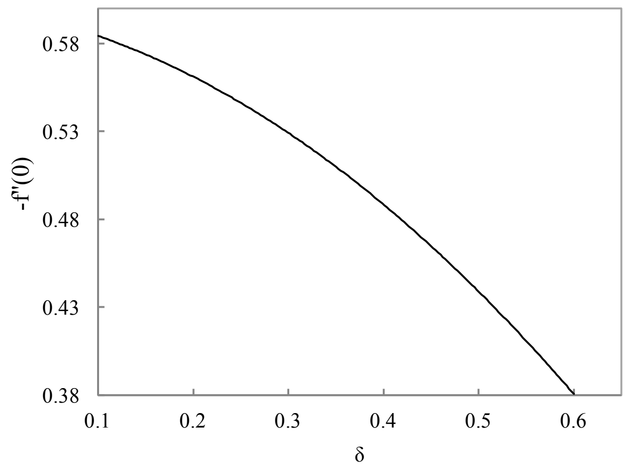

Fig.2 values of $-f''(0)$ for some value of δ when $\beta = 0.5$, $\alpha_1 = \frac{\pi}{6}$, $M = 1$, $\Pr = 0.7$, $Q = 0.005$, $k = 0.5$ and $\gamma = 0.005$

Figure 2 shows the relationship between the results on the values of $-f''(0)$ and δ. It is clear captured that, the increasing on the values of stretching parameter led to decrease on the valued of $-f''(0)$.

4. CONCLUSION

The problems of convective boundary flow of Jeffery fluid with effect of slip velocity and aligned MHD under convective boundary conditions have been studied. The governing boundary layer equations are first going through the transformation processes in order to reduce the complexity of equation. The final equations in the form of ordinary differential equation are the solved using BVP4C method.

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