

HEAT TRANSFER PHENOMENON OF HEATED CYLINDER AT VARIOUS
LOCATIONS IN A SQUARE CAVITY

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I hereby declare that the work in this project is my own except for quotations and summaries which have been duly acknowledged. The project has not been accepted for any degree and is not concurrently submitted for award of other degree.

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ABSTRACT

In this thesis, the theory of lattice Boltzmann method is been described in first chapter. The lattice Boltzmann equation method has been found to be useful in many application involving interfacial dynamics and complex boundaries. First, the introduction of this report is described the objective of the project. This project objective is to study the plume behavior of heated cylinder at various locations in square cavity. Next, the problem statement is explained in further detailed. The problems solve using the lattice Boltzmann method theory and some flow simulation. The background of the project is relating to the lattice Boltzmann method equation that is involving the Navier-Stoke equation, the governing equation and Bhatnagar-Gross-Krook (BGK) approximation. Then, the literature will explain and described further detail about the lattice Boltzmann method. The methodology is the simulation of the isothermal and thermal of the lattice Boltzmann. The isothermal include the Poiseuille and Couette flow. The thermal include the Porous Couette flow. The isothermal and thermal of lattice Boltzmann equation have been derived from the Boltzmann equation by discretization in both time and phase space. The result of heated cylinder at various locations in square cavity at different Rayleigh number that has been done compute that when the Rayleigh number is increase the flow will become distorted and the plume will emerge in the enclosure. This is because of the buoyancy induced and convection become more predominant than conduction. The isotherms move upward and larger plumes exist on the top of the inner square, which gives rise to the stronger thermal gradient on the top of the enclosure. Therefore, the flow strongly imposes on the above of the enclosure, which also cause the form of a thinner thermal boundary layer in this area and develops the heat transfer.

ABSTRAK

Tesis ini menerangkan teori kaedah kekisi Boltzmann dalam bahagian satu. Persamaan kekisi Boltzmann telah ditemui amat berguna kerana membabitkan dinamik antara muka dan sempadan kompleks. Pertama, pendahuluan menerangkan tentang objektif projek ini. Objektif projek ini adalah untuk mengkaji sifat pemanasan silinder dalam ruang segi empat. Seterusnya, permasalahan projek ini diterangkan dengan lebih terperinci. Masalah The problem is diselesaikan dengan kaedah kekisi Boltzmann dan simulasi aliran. Latar belakang projek ini berkaitan dengan kaedah kekisi Boltzmann yang membabitkan persamaan Navier-Stoke dan penghampiran Bhatnagar-Gross-Krook (BGK). Kemudian, penulisan akan menerangkan lebih lanjut tentang kaedah kekisi Boltzmann. Dalam simulasi Isothermal dan pemanasan kekisi Boltzmann. Dalam Isothermal terdapat aliran Poiseuille dan Couette. Dalam pemanasan terdapat aliran Porous Couette. Persamaan Thermal dan Isothermal Boltzmann diterbitkan daripada persamaan kekisi Boltzmann melalui diskrit masa dan fasa. Hasil pemanasan silinder dalam ruang segi empat pada nombor Rayleigh yang berlainan menunjukkan apabila nombor Rayleigh meningkat, aliran akan menjadi bengkok dan bentuk seperti pelepah akan terbentuk dalam ruang tersebut. Ini adalah kerana apungan berlaku dan konveksi akan menjadi lebih dominan daripada konduksi. Isotherm akan bergerak ke atas dan pelepah yang lebih besar akan terbentuk di atas ruang segi empat, yang mana akan meningkatkan kecerunan suhu di atas ruang segi empat tersebut. Oleh itu, aliran yang kuat terjadi di atas ruang segi empat tersebut, seterusnya menjadikan lapisan sempadan suhu menjadi nipis di kawasan ini dan menyebabkan pemindahan haba berlaku.

TABLE OF CONTENTS

	Page
SUPERVISOR’S DECLARATION	ii
STUDENT’S DECLARATION	iii
ACKNOWLEDGEMENTS	iv
ABSTRACT	v
ABSTRAK	vi
TABLE OF CONTENTS	vii
LIST OF TABLES	ix
LIST OF FIGURES	x
LIST OF SYMBOLS	xii
LIST OF ABBREVIATIONS	xiii
CHAPTER 1 INTRODUCTION	1
1.1 Objective	1
1.2 Problem Statement	1
1.3 Project Background	2
1.4 Project Flow Chart	7
CHAPTER 2 LITERATURE REVIEW	9
2.1 Navier Stoke Equation	9
2.2 Computational fluid dynamics (CFD)	10
2.3 Lattice Boltzmann Method (LBM)	12
2.4 Boltzmann Bhatnagar Gross Krook (BGK)	15
2.5 Poiseuille Flow	19
2.6 Couette Flow	20
2.7 Porous Couette Flow	21
2.8 Natural Convection in Square Cavity	22

CHAPTER 3	METHODOLOGY	24
3.1	Original LBM Algorithm	24
3.2	Simulation of Isothermal Lattice Boltzmann	25
	3.2.1 Poiseuille Flow	26
	3.2.2 Couette Flow	26
3.3	Simulation of Thermal Lattice Boltzmann	27
	3.3.1 Porous Couette Flow	28
CHAPTER 4	RESULT AND DISCUSSION	30
4.1	Grid Dependence Study	31
4.2	Heated Cylinders at Different Rayleigh Number	34
4.3	Heated Cylinders at Bottom of Enclosure	38
4.4	Heated Cylinders at Top of Enclosure	41
4.5	Relationship of Nusselt Number and Rayleigh Number	43
4.6	Horizontal and Vertical Velocity at Different Rayleigh Number	44
CHAPTER 5	CONCLUSION AND RECOMMENDATIONS	49
5.2	Conclusion	49
5.2	Recommendation	50
REFERENCES		51
APPENDIX		55
A	Main Programme Source Code	55

LIST OF TABLES

Table No.	Title	Page
4.1	Grid dependence study of $Ra=10^5$	32
4.2	Grid dependence study of $Ra=10^6$	33

LIST OF FIGURES

Figure No.	Title	Page
1.1	Lattice Boltzmann method	5
1.2	Project flow chart	8
2.1	Poiseuille flow	20
2.2	Couette flow	21
2.3	Porous Couette flow	22
2.4	Heated cylinder in square cavity	23
3.1	LBM algorithm flow chart	25
3.2	Poiseuille flow graph	26
3.3	Velocity profiles across the normalized channel width at different times	27
3.4	Porous Couette flow graph at different Reynolds number	29
3.5	Porous Couette flow graph at different Prandtl number	29
4.1	Geometry and boundary condition of heated cylinder in enclosure	30
4.2	The dimension of mesh grid of cylinder and enclosure	31
4.3	Graph of Nusselt number against grid for $Ra=10^5$	32
4.4	Graph of Nusselt number against grid for $Ra=10^6$	33
4.5	Comparison result of Isotherm for Rayleigh number $Ra= 10^3$ and Nor Azwadi C.S., A. R. M. Rosdzimin	34
4.6	Comparison result of Isotherm for Rayleigh number $Ra= 10^4$ and Nor Azwadi C.S., A. R. M. Rosdzimin	34
4.7	Comparison of result Isotherm for Rayleigh number $Ra= 10^5$ and Nor Azwadi C.S., A. R. M. Rosdzimin	35
4.8	Comparison of result Isotherm for Rayleigh number $Ra= 10^6$ and Nor	

Azwadi, C.S., A. R. M. Rosdzimin	35
4.9 Stream function for low Rayleigh number $Ra=10^3$ and $Ra=10^4$	36
4.10 Stream function for high Rayleigh number $Ra=10^5$ and $Ra=10^6$	36
4.11 Isotherm for Rayleigh number $Ra=10^4$ when cylinder at bottom of enclosure	38
4.12 Isotherm for Rayleigh number $Ra=10^5$ when cylinder at bottom of enclosure	39
4.13 Streamline for Rayleigh number $Ra=10^4$ when cylinder at bottom of enclosure	39
4.14 Streamline for Rayleigh number $Ra=10^5$ when cylinder at bottom of enclosure	40
4.15 Isotherm for Rayleigh number $Ra=10^4$ when cylinder at top of enclosure	41
4.16 Isotherm for Rayleigh number $Ra=10^5$ when cylinder at top of enclosure	41
4.17 Streamline for Rayleigh number $Ra=10^4$ when cylinder at top of enclosure	42
4.18 Streamline for Rayleigh number $Ra=10^5$ when cylinder at top of enclosure	42
4.19 Graph of Nusselt number against Rayleigh number	43
4.20 Horizontal velocity profile at the mid height of the cavity	45
4.21 Vertical velocity profile at the mid height of the cavity	45
4.22 Horizontal velocity profile at the mid height of the cavity of Nor Azwadi C.S.	46
4.23 Vertical velocity profile at the mid height of the cavity of Nor Azwadi C.S.	47

LIST OF SYMBOLS

f	Density distribution function
c	Microscopic velocity
Ω	Collision integral
f^{eq}	Equilibrium distribution function
f_i	Initial density distribution function
τ	Relaxation time
u	Velocity
P	Pressure
ν	Kinematic shear viscosity
χ	Thermal diffusivity

LIST OF ABBREVIATIONS

BGK	Bhatnagar-Gross-Krook
CFD	Computational fluid dynamic
ELB	Entropic lattice Boltzmann
LBE	Lattice Boltzmann equation
LBGK	Lattice Bhatnagar-Gross-Krook
LBM	Lattice Boltzmann method
LGA	Lattice gas automata

CHAPTER 1

INTRODUCTION

1.1 OBJECTIVE

This project objective is to study the plume behavior of heated cylinder at various locations in square cavity. This project also attempts to deal with the analysis of an investigation of the natural convection of heat transfer in a square enclosure containing solid cylinder. The effects of the cylinder position on the heat transfer and the flow structures inside the cavity are to be studied and highlighted.

1.2 PROBLEM STATEMENT

This project is to study the heat transfer phenomenon of heated cylinder at various locations in square cavity. The project scope is to analysis heat transfer limit to natural convection only. The problem will be tested at $Ra = 10^5$ and 10^6 . This study will include the natural convection interactions in a heated cavity with an inner body. A specifically developed numerical model, based on the lattice Boltzmann method (LBM), is used for the solutions of the governing equations. Natural convection in heated enclosures, housing inner bodies has received significant attention because of its interest and importance in industrial applications. Some applications are solar collectors, fire research, electronic cooling, aeronautics, chemical apparatus, building constructions and nuclear engineering. This will contributes to the development of the LBM. Effects of the cylinder position on the heat transfer and the flow structures inside the cavity are to be studied and highlighted.

1.3 PROJECT BACKGROUND

The Lattice Boltzmann Method (LBM) is another approach to finite difference, finite element and finite volume method for solving Navier-Stoke Equation. Lattice Boltzmann approach has found current achievement in the host of fluid dynamical study including in porous media, magnetohydrodynamics, immiscible fluid and turbulence. The numerical and experimental study of natural convection of heat transfer in the partitioned enclosure has received significant interest in the recent year due to the useful engineering application. The application that related to this project is the solar collectors, thermal insulation, cooling of the electronic component and designing building. In nearly all of the earlier studies on natural convection in a square cavity containing partitions or solid bodies, with or without heat generation, the influence of radiation is ignored. There have been not many studies on the both heat-transfer problem involving convection and radiation. On the other hand, it is well recognized that when natural convection in air is involved, the heat transfers by convection and radiation are usually of the equal order of magnitude. In this project the objective is to study the plume behavior of heated cylinder in the square cavity at the various locations using the Lattice Boltzmann Method. The analysis of the heat transfer will be limited to natural convection only. A complete parametric study is made for different Rayleigh numbers. The problem will be tested at $Ra = 10^5$ and 10^6 .

The mathematical relationship governing fluid flow is the famous continuity equation. The Navier-Stoke Equation is given by:

$$\nabla \cdot \mathbf{u} = 0 \quad (1.1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} \quad (1.2)$$

\mathbf{u} = velocity

P = pressure

ν = kinematic shear viscosity

Source: J Ryong Lee, Man Yeong Ha and S. Balachandar (2007)

As well known, the Navier-Stoke equation is nonlinear partial differential equations. It is too difficult and there is no analytical answer to them except for a small amount of particular cases. The information about physical process of fluid dynamics is often given by real dimension. The study analysis involving full scale tools can be used to guess how indistinguishable copies of the tools would act upon under the same state. On the other hand, in nearly all cases the investigations are costly and frequently unattainable.

At the present time, the fresh development in the computing power of microprocessor, numerical and solution of flow problems can be brought to the desktop. The employ of computer is necessary to come to a decision of the fluid motion of a few problems. The Computational Fluid Dynamic (CFD) has developed to turn out to be significant tool in solving the Navier-Stoke equation, continuity equation or the equations that are achieve from them. CFD is the science for determination the numerical answer to the governing equation during space or time to attain numerical details of the entire flow field of consideration. To accurately replicate fluid flow on the computer, it is essential to work out the Navier-Stoke equation with infinite exactness.

Lattice Boltzmann method is an additional technique to finite difference, finite element, and finite volume process for solving the Navier-Stoke equations. Lattice Boltzmann develops since the expansion of the lattice gas automata and takes over a few appearances from its pioneer, the lattice gas technique. The significant development to improve the computational competence has been made to Lattice Boltzmann method. The continuous Boltzmann equation is express as in Eq. (1.3)

$$f(x + c\Delta t, t + \Delta t) - f(x, t) = \Omega(f) \quad (1.3)$$

f = density distribution function

c = microscopic velocity

$\Omega(f)$ = collision integral

Source: Nor Azwadi C.S. (2007)

The development is the completion of Bhatnagar-Gross-Krook (BGK) estimate that is the single relaxation approximation. The primary pace in the lattice Boltzmann method is to follow the progression of single particle distribution. This will involve the probable quantity of molecules in an assured amount at assured moment complete since huge number of particles in a structure that travel liberally with no collisions for extended space judge against to their sizes. Following the distribution functions are achieved, the hydrodynamic equation can be attained. The most important purpose of LBM advance is to construct a connection or relation involving the microscopic and macroscopic dynamics, slightly than to deal with macroscopic dynamic straightforwardly. The goal is to attain macroscopic equation since microscopic dynamics by signify of statistic.

The collision integral equation is express as in Eq. (1.4)

$$\Omega(f) = \frac{1}{\tau} (f^{eq} - f) \quad (1.4)$$

τ = relaxation parameter

f^{eq} = equilibrium distribution function

Source: Nor Azwadi C.S. (2007)

The combination of the continuous Boltzmann equation and collision integral equation will give the Lattice Boltzmann BGK equation. The Lattice Boltzmann BGK is express as in Eq. (1.5)

$$f(x + c\Delta t, t + \Delta t) - f(x, t) = -\frac{f - f^{eq}}{\tau} \quad (1.5)$$

f_i = density distribution function

τ = relaxation parameter

f^{eq} = equilibrium distribution function

Source: Junya Onishi, Yu Chen and Hirotsada Ohashi (2001)

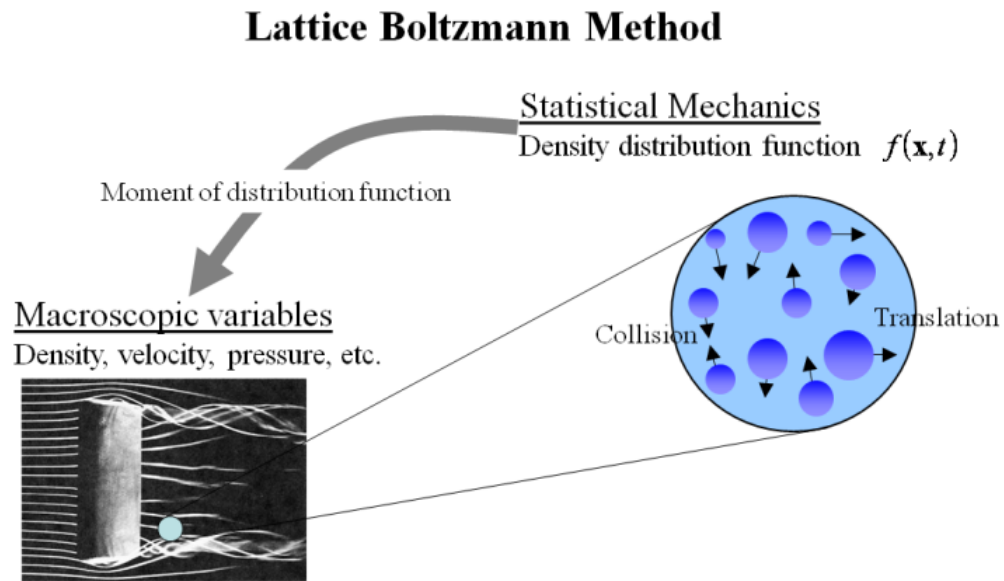


Figure 1.1: Lattice Boltzmann Method

Source: Nor Azwadi C.S. (2007)

The newest thermal lattice Boltzmann method goes to three sorts: the passive scalar approach, multispeed approach and thermal energy distribution model. The figure 1.1 shows the lattice Boltzmann method and the relationship between the macroscopic and microscopic variables. The multispeed technique employ the equivalent purpose in determine the macroscopic velocity, pressure and temperature. To preserve the kinetic energy in the collision on every one lattice point, this model necessary extra dissimilarity of velocity than isothermal form. The equilibrium distribution function commonly include elevated order velocity structure but this form on the other hand has severe numerical instability and not competent. The passive scalar model has enhanced numerical constancy than the multispeed form. The flow ground and temperature of the inactive scalar model distinguish by two distribution functions. Macroscopic function is advection by flow speed but does not manipulate the flow ground. The isothermal and thermal lattice Boltzmann equation (LBE) is resulting from the Boltzmann equation by discretization in together time and stage space. The origins straightforwardly link the LBE to the Boltzmann equation. Consequently, the LBE can be constructing on well-

known origin of the Boltzmann equation and the effect of Boltzmann equation can be prolonged to the LBE. To verify the newest developed lattice arrangement, the numerical simulations of the porous plate Couette flow complexity and the natural convection in a square or cubic cavity have to be figure.

The macroscopic equation for isothermal equation is express as in Eq. (1.6)

$$\begin{aligned}\nabla \bullet u &= 0 \\ \frac{\partial u}{\partial t} + u \nabla \bullet u &= -\nabla P + \left(\frac{2\tau - 1}{6} \right) \nabla^2 u\end{aligned}\tag{1.6}$$

$$\nu = \frac{2\tau - 1}{6}\tag{1.7}$$

u = velocity

P = pressure

ν = kinematic shear viscosity

τ = relaxation time

Source: Nor Azwadi C.S. (2007)

The numerical answer of the porous Couette flow problem for a great variety of the Rayleigh numbers is representing that the form is suitable and numerically steady for the computational of elevated Rayleigh. The computations of natural convection in a cavity predictable the flow element for dissimilar Rayleigh number. The models utilize shorter imitation time and can be relate successfully in engineering function.

The macroscopic equation for thermal express as in Eq. (1.8) and (1.9)

$$\begin{aligned}\nabla \bullet u &= 0 \\ \frac{\partial u}{\partial t} + u \nabla \bullet u &= -\nabla P + \left(\frac{2\tau_f - 1}{6} \right) \nabla^2 u\end{aligned}\tag{1.8}$$

$$\frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = \left(\tau_g - \frac{1}{2} \right) \nabla^2 T \quad (1.9)$$

$$\tau_f = 3\nu + \frac{1}{2} \quad (1.10)$$

$$\tau_g = \chi + \frac{1}{2} \quad (1.11)$$

u = velocity

P = pressure

ν = kinematic shear viscosity

τ = relaxation time

Source: H.N. Dixit and V. Babu (2006)

1.4 PROJECT FLOW CHART

The figure 1.2 shows the project flow chart which is basically referred to the theory of lattice Boltzmann method. The theory of lattice Boltzmann contained the governing equation, basic principle of lattice Boltzmann, Collide Function of BGK, Equilibrium Distribution Function, Time Relaxation, Discretization of Microscopic Velocity and the Derivation of Navier Stoke Equation. After the theory of lattice Boltzmann has been studied, the isothermal fluid flow is simulated. The isothermal fluid flows have two basic flows which is the flow in pipe or the Poiseuille flow and the Couette flow. The extension of lattice Boltzmann model is the thermal lattice Boltzmann theory and the Porous Couette flow is simulated. The final part of the flow chart is to do the main project that is to study the Heated Cylinder Geometry and Boundary Condition Analysis

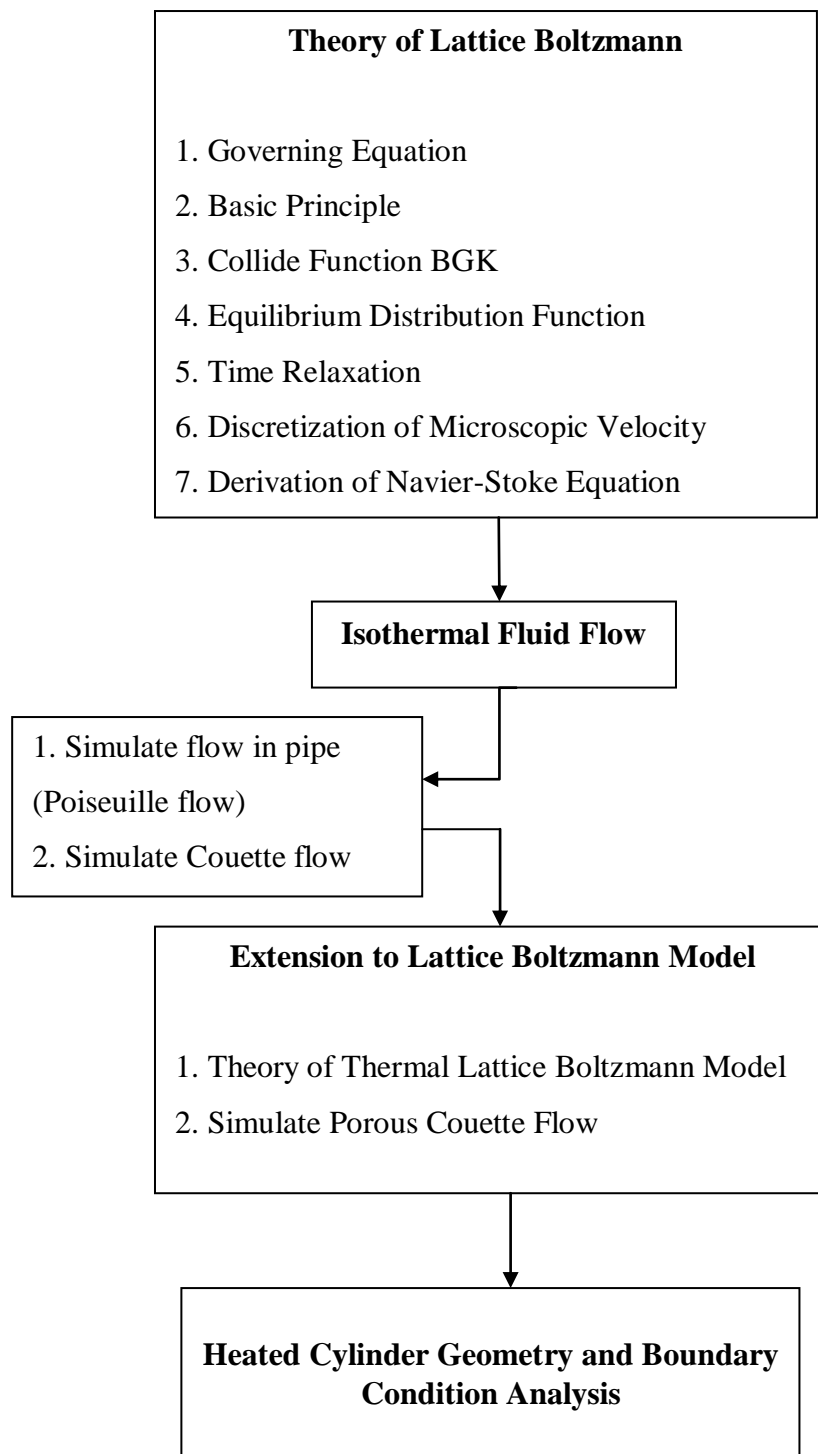


Figure 1.2: Project flow chart

CHAPTER 2

LITERATURE REVIEW

2.1 NAVIER STOKES EQUATION

In the last few years we have seen a quick growth of latest numerical methods for the result of partial differential equations, especially Navier-Stokes equations. The history of the Navier–Stokes equations is it named after Claude-Louis Navier and George Gabriel Stokes. Navier-Stokes equations explain the motion of fluid material that is material which can flow. These equations obtained from relate Newton's second law to fluid movement and collectively with the statement that the fluid pressure is the sum of a spread viscous expression, plus a pressure expression (Batchelor, G.K., 1967). They are one of the mainly practical sets of equations because they explain the physics of a large number of phenomena of academic and economic attention. The application is the weather, ocean currents, water stream in a pipe, flow about an airfoil and movement of stars within a galaxy. These equations in together complete and shorten outline are employed in the design of airplane and vehicle, the learning of blood stream, the devise of power post and the investigation of the effect of pollution.

In a purely mathematical sense, the Navier–Stokes equations are in the great attention. On the other hand, mathematicians have not yet confirm that in three dimensions answers always subsist or that if they do subsist they do not include any infinities, singularities or discontinuities (Batchelor, G.K., 1967). These are known the Navier–Stokes continuation and smoothness troubles. The Clay Mathematics Institute has known this one of the seven mainly significant open questions in mathematics. The Navier–Stokes equations are differential equations which do not explicitly create a relation between the variables of concern example like velocity and pressure. They

establish relations among the rates of change. The Navier–Stokes equations for simple case of an ideal fluid can affirm that acceleration is proportional to the gradient of pressure. It also states not position but rather velocity (Frisch, U., Hasslacher, B. and Pomeau, Y. 1986). A result of the Navier–Stokes equations is called a velocity field or flow field, which is a description of the velocity of the fluid at a given point in space and time. Once the velocity field is resolved for, other amount of concern such as flow rate or drag force may be establish. This is dissimilar from what one normally sees in classical mechanics, where answers are typically trajectories of position of a particle or deflection of a continuum. Studying velocity as an alternative of position makes more sense for a fluid but for visualization reasons one can compute a variety of trajectories.

The Navier-Stoke equation is nonlinear partial differential equations. The information about physical process of fluid dynamics is frequently given by genuine measurement (Nor Azwadi C.S, 2007). The experimental study involving full scale equipment can be used to expect how identical copies of the equipment would perform under the same state. Yet, in nearly all cases the tests are expensive and always impossible.

$$\nabla \cdot \mathbf{u} = 0 \quad (2.1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} \quad (2.2)$$

\mathbf{u} = velocity

P = pressure

ν = kinematic shear viscosity

Source: J Ryong Lee, Man Yeong Ha and S. Balachandar (2007)

2.2 COMPUTATIONAL FLUID DYNAMICS (CFD)

Computational fluid dynamics (CFD) is one of the undergrowth of fluid mechanics that uses numerical scheme and algorithms to work out and study problems that engage fluid flows. Computers are used to carry out the millions of calculations

needed to simulate the relations of fluids and gases with the not easy surfaces used in engineering. Still with high-speed supercomputers barely inexact solutions can be attained in many cases. Continuing study, on the other hand, may give way software that give better accuracy and speed of difficult simulation situations such as transonic or turbulent flows (Acheson, D. J., 1990). Software is frequently carried out by a wind tunnel with the final justification coming in flight analysis. The basic of CFD problem is the Navier-Stokes equations, which describe whichever single-phase fluid flow. Navier-Stokes equations can be simplified by eliminate terms explaining viscosity to give in the Euler equations (Batchelor, G.K., 1967). Advance simplification, by eliminate terms explaining vorticity give in the complete possible equations. Finally, these equations can be linearized to give in the linearized possible equations.

In these days, CFD has developed from a mathematical attention to become significant instrument in solving Navier-Stoke equation and the continuity equation. It is the science of determining numerical answer of the governing equation of fluid flow during proceeds the solution through space or time to achieve a numerical explanation of the whole flow field of attention (Acheson, D. J., 1990). For the fact, numerical researcher must select a method to discretise the difficulty. The settings up of the numerical simulation initiate with built a computational grid. The flow variables are calculated at the node point of this grid in some approach and at in-between points. The spacing between grid points has to be very well sufficient to achieve a high enough degree of precision. There are some benefits but to remain the number of grid point small since of additional grid point indicate more computer memory needed and a greater time is desired to carry out each iteration of the calculation (Nor Azwadi C.S, 2007). The uncomplicated computational grid rectangular lattice by unchanging spacing between node points in every dimension. There are series of way that use unstructured grids where the density of the node point is not constant and is higher in the area where the precision is wanted. Unstructured meshes often end up being connected in a triangular or tetrahedral style since these form fill space well and they needed least number of vertices. Several way employ adaptive meshes where the node point are generated and devastated as flow featured shift though the computational domain. This will remain number of nodes to a least but still providing the wanted dimension for the certain flow elements.

2.3 LATTICE BOLTZMANN METHOD (LBM)

The Boltzmann equation (LBE) is also known as the Boltzmann transport equation. It is derived by Ludwig Boltzmann. LBE explains the statistical distribution of one particle in a fluid. In addition, it is one of the mainly significant equations of non-equilibrium statistical mechanics. The region of statistical mechanics that deals with systems far from thermodynamic equilibrium for example when there is an applied temperature slope or electric field. LBE is employed to study how a fluid transports physical amount such as heat and charge and to obtain transport properties for example electrical conductivity, hall conductivity, viscosity, and thermal conductivity.

The primary successful lattice gas automata (LGA) for hydrodynamics in two sizes. Several latest LGA form have been presented each year and applied to a variety of problems and between them hydrodynamics in three dimensions, flow through porous media (J. Bernsdorf, G. Brenner and F. Durst, 2000), immiscible fluids(N.S. Martys and H. Chen, 1996), several fluids and external forces, turbulences (G. Amati, S. Succi and R. Piva, 1999), magnetohydrodynamics (S. Chen, H. Chen, D. Martinez and W.H. Matthaeus, 1991) and diffusion-reaction equations. The LBE is a specially discretized figure of the continuous Boltzmann equation. The evidence is accurate and straight and in particular creates no use of the LGA. Therefore set up of the LBE on a solid theoretical basis of the Boltzmann equation. The quarrel also has instant practical cost. The arbitrary mesh grids can be put into practice with LBE methods and the Reynolds number simple to get to in hydrodynamic simulations by LBE methods can be considerably improved. Some of the imperfections in a few presented LBE models turn out to be obvious from other derivation to recommend development with a sound theoretical basis.

The lattice Boltzmann equation (LBE) has been well-known to be a triumphant computational instrument for a wide range of difficult physical systems that are problematic for conventional way. Even though the great concern in the LBE, it has yet to be located on an accurate theoretical basis. This is because be short of a main concern perceptive of the LBE has imperfect its function. For example, calculations based on the

LBE have not been very doing well in utilize arbitrary mesh grids and it has not set up probable to suitably reproduce thermohydrodynamic systems by means of LBE technique. The continuous Boltzmann equation is express as in Eq. (2.3)

$$f(x+c\Delta t, t+\Delta t) - f(x, t) = \Omega(f) \quad (2.3)$$

f = density distribution function

c = microscopic velocity

$\Omega(f)$ = collision integral

Source: Nor Azwadi C.S. (2007)

Lattice Boltzmann methods (LBM) is one of computational fluid dynamics (CFD) system for fluid simulation. As an option of work out the Navier–Stokes equations, the discrete Boltzmann equation is work out to simulate the flow of a Newtonian fluid with collision outline such as Bhatnagar-Gross-Krook (BGK). Simulating the contact of a limited sum of particles the viscous flow manners come into view automatically from the essential particle streaming and collision procedure (P.L. Bhatnagar, E.P. Gross, M. Krook, 1954). LBM is a moderately latest simulation technique for complicated fluid structure and has concerned thoughtfulness of researchers in computational physics. It is not the alike like the earlier period CFD practice, which determine the conservation equations of macroscopic properties numerically, LBM models the fluid consisting of fictive particles, and such particles carry out consecutive spread and collision processes over a discrete lattice mesh. For the reason that of its particulate nature and local dynamics, LBM has many returns over the CFD practice. This is mostly in dealing with not easy boundaries; integrate of microscopic connections, and parallelization of the algorithm.

Lattice Boltzmann method is truly begun from the lattice gas automata (LGA) scheme. It can be deliberate as a basic fabricated molecular dynamics model in which gap, time, and particle velocities are all separate (He X. and Luo L. 1997). Every lattice node is associated to its neighbors by 6 lattice velocities. There can be whichever not any or one particle at a lattice node propagated in a lattice course. As soon as the time

phase is over, every one of the particle will pass through to the neighboring node in its course. This propagating practice is called the propagation or streaming step. When there are further than one particles arriving at the similar node from dissimilar path, they collide and alter their path according to a set of collision policy. The suitable collision policy be supposed to preserve mass, momentum, and energy sooner than and after the collision but it was establish that LGA occurrence from a quantity of native imperfection that lack of Galilean invariance, statistical noise and exponential complexity for three-dimensional lattices (S. Succi, 2001). The most important basis for the shift from LGA to LBM was the desirable to take away the statistical noise by replace with the Boolean particle number in a lattice path with its collection common. So it is named density distribution function. The discrete collision regulation is also replace with by a continuous function identified as the collision operator (Dieter Wolf-Gladrow, 1995). Today for the LBM improvement, significant simplification is to estimate the collision operator with the Bhatnagar-Gross-Krook (BGK) relaxation expression. This lattice BGK (LBGK) model makes simulations well-ordered and agrees to flexibility of the transfer coefficients. In addition, it has been revealed that the LBM technique can also be considered as a special discretized delineate of the continuous Boltzmann equation. More than a Chapman-Enskog investigation, one can develop the governing continuity and Navier-Stokes equations from the LBM algorithm (Sydney Chapman and Thomas George Cowling, 1990). In addition, the pressure field is also straightforwardly easy to get to from the density distributions and consequently there is no further Poisson equation to be answer as in old CFD technique.

Above and beyond that, reproduce multiphase flows has forever been a trial to conventional CFD while of the propagating and deformable boundary. The boundaries among dissimilar stage that are liquid or vapor element start from the faithful links among fluid molecules. Because of that it is not easy to put into practice such microscopic connections into the macroscopic Navier–Stokes equation. On the other hand the particulate kinetics provides a relatively simple and stable way to incorporate the underlying microscopic connections by altering the collision operator (S. Succi, 2001). More than a few of LBM multiphase models have been developed and phase divider are produce automatically from the particle dynamics and no unique managing is necessary to manage the line as in traditional CFD technique. The purpose of

multiphase LBM models can be found in a range of complex fluid systems, as well as interface instability, bubble or droplet dynamics, wetting on solid surfaces, interfacial slip, and droplet electrohydrodynamic deformations (S. Chen, H. Chen, D. Martinez and W.H. Matthaeus, 1991). Next to the rising popularities of LBM in simulating difficult fluid method, this advance has some limitations. The elevated Mach number flows within aerodynamics are still not simple intended for LBM, and a steady thermo-hydrodynamic scheme is not current but with Navier–Stokes based CFD, LBM technique have been efficiently attached to thermal-specific answer to allocate heat transfer simulation capability (Jian Guo Zhou, 2004). In the multiphase models, the boundary thickness is commonly enormous and the density fraction across the boundary is little at what time assess with genuine fluids. In addition the broad applications and fast development of this technique through the past twenty years have well-known it's probable in computational physics. The LBM show hopeful outcomes in the area of high Knudsen number that is obvious by the ratio of the mean free path involving molecules to a geometric length scale of flows.

2.4 BOLTZMANN BHATNAGAR GROSS KROOK (BGK)

Recently the lattice Boltzmann equation method has broadly studied in a vision point of its appropriate and has been concern to a variety of problems. The lattice Boltzmann method that are possesses a velocity-independent pressure, is named the lattice Boltzmann BGK method, since it utilize the single relaxation time approximation earliest pioneer by Bhatnagar, Gross and Krook in 1954 is really make simpler the collision operator of the Boltzmann equation. The lattice method was planned as an addition of lattice describe as gas automata which was establish to include different problem according to the wide learning. Similar to the derivation of the lattice gas automata, the derivation of the lattice Boltzmann method is too based on a necessity that it gives a Navier–Stokes equation at a maximum of small Knudsen number (S. Succi, 2001). In detail, the lattice Boltzmann method in regular use was considered to provide an incompressible Navier–Stokes equation. The derivation of it is complex and, hence, it is not obvious the density and the velocity of the fluid are describe as adequate how to expand the lattice Boltzmann method so that, for example, it decrease to the Navier–Stokes equation at an edge of small Knudsen number while the equation thus resulting

is the Boltzmann equation itself. For this reason the current derivation method gives us an easy and flexible formula to build the lattice Boltzmann method, and makes it simple to build Boltzmann method in a variety of method. The lattice Boltzmann model with BGK collision operator, BGK is the standard in LB fluid models. In the past, the achievement of the LB method is to a huge level founded on BGK. This model is mainly frequently employed to explain the incompressible Navier-Stokes equations. In that case, it employs a quasi-compressible approach, in which the fluid is fixed into adopting somewhat compressible manners to answer the pressure equation (P.L. Bhatnagar, E.P. Gross, M. Krook, 1954). The BGK method can also be employ to create compressible flows at low Mach-number, other than together the bulk viscosity and the velocity of sound are lattice constants which cannot be attuned. The BGK model has been entire in numerous attempt, such as those planned on to improve numerical steadiness or precision for exact problems, or to characterize extra physical happening. It is on the other hand observed that at the same time as these added extras expand a little component of BGK; they also set up fresh problems. BGK keep on consequently the model of choice in a lot of conditions, for the reason that of its simplicity of completion as well as its dependability.

The BGK collision operator carries out on the off-equilibrium component of every particle populations on a lattice node, multiply every one of them with the linked relaxation consideration (S. Succi, 2001). An addition of this, clarify by the idea of a General Collision Matrix, is to carry out with a linear operator on a vector consisting of all off-equilibrium particle populations. In the case that this operator is diagonalizable, its eigen values attain a nice clarification as relaxation parameters acting on modes of the particle populations (Schroeder and Daniel V., 2000). A few modes are physically having a significant consequence, and tackling with them convert the physics of the model. On the other hand some of them are not and they can be cooperating with to extend the numerical constancy of the model. Theoretically speaking whichever model with a diagonalizable General Collision Matrix can be examined as a Multiple-Relaxation-Time model. This is actually a model in which the relaxation considerations vary since a mode toward one other. Within the regularized model, improved accurateness and steadiness are attained by take away higher order; non-hydrodynamic situation initial the particle populations. This model is foundation on the inspection that

the hydrodynamic boundary of the BGK model is not reliant on the facts of the particle populations, although just on the value of the initial three moments. The plan is to work out these three moments at each time step and on each node. The common BGK collision is executed. Executing one after the other regularization and a BGK collision has the outcome of a diagonalizable linear operation on the off-equilibrium piece of the particle populations. This model can for that reason be qualified to the family of Multiple-Relaxation-Time models. The significance of the relaxation parameters is on the other hand resolute from a physical quarrel, base on the Chapman-Enskog extension of BGK, and not on a numerical constancy investigation.

The entropic lattice Boltzmann (ELB) form is linked to the BGK one. The collision operator is once more an easy relaxation in the direction of stability. The main dissimilarity is the evaluation of the equilibrium distribution function and a limited alteration of the relaxation time. The stability is no more in use as a discretization of the continuous Maxwell-Boltzmann equilibrium distribution on the other hand slightly as an extremum of the discretized entropy beneath the conservations limit of the system (S. Succi, 2001). The relaxation time is adjusted close by in order to stay away from an entropy diminish throughout collision. This final process is publicized to stay away from the distribution functions from adopting negative values, which make sure unqualified numerical steadiness. Unluckily, it also has a elevated computational cost in view of the fact that one has to answer an implicit equation at every lattice node for every time step. Analytical result of the two-dimensional triangular and square lattice Boltzmann BGK models have been achieved for the plane Poiseuille flow and the plane Couette flow (Nor Azwadi C.S, 2007). The analytical results are written in conditions of the attribute velocity of the flow, the single relaxation time, and the lattice spacing. The analytic explanations are the correct illustration of these two flows with no at all approximation. By means of the analytical answer, it is revealed that in Poiseuille flow the bounce-back boundary state introduces an inaccuracy of initial order in the lattice spacing. In lattice gas automata it is used to simulate Poiseuille flow and also measured for the triangular lattice Boltzmann BGK model. A methodical answer is achieved and used to explain that the boundary state introduces an inaccuracy of next order in the lattice spacing.

The significant alteration to develop the effectiveness has been completed to Lattice Boltzmann method. The development is the completion of Bhatnagar-Gross-Krook (BGK) approximation that is the single relaxation approximation. The initial step in the lattice Boltzmann method is to follow the development of single particle distribution. This will involve the possible number of molecules in a assured volume at definite time completed from huge number of particles in a structure that move without restraint and devoid of collisions for long space judge against to their sizes. Following the distribution functions are achieved, the hydrodynamic equation can be achieved. The major purpose of LBM approach is to construct a link or relationship among the microscopic and macroscopic dynamics, somewhat than to deal with macroscopic dynamic straightforwardly. The goal is to obtain macroscopic equation from microscopic dynamics by mean of statistic.

The collision integral equation is express as in Eq. (2.4)

$$\Omega(f) = \frac{1}{\tau} (f^{eq} - f) \quad (2.4)$$

τ = relaxation parameter

f^{eq} = equilibrium distribution function

Source: Nor Azwadi C.S. (2007)

The substitution of the continuous Boltzmann equation and collision integral equation will give the Lattice Boltzmann BGK equation. The Lattice Boltzmann BGK is express as in Eq. (2.5)

$$f(x + c\Delta t, t + \Delta t) - f(x, t) = -\frac{f - f^{eq}}{\tau} \quad (2.5)$$

f_i = density distribution function

τ = relaxation parameter

f^{eq} = equilibrium distribution function

Source: Junya Onishi, Yu Chen and Hirotsada Ohashi (2001)

If the lattice Boltzmann model is used on a triangular lattice, for a channel flow, a triangular lattice is created. There are two kinds of particles on each node of a node that are relaxed particles and propagated particles. To acquire the steady-state analytical answer in the LBGK simulations, the boundary state is supposed to be properly selected for the simulation. If we give the analytical solution on the boundary, we will be capable to achieve the analytical answer in this area also. Specification of the analytical solution on the boundary does not offer a boundary state of common reason. In addition, the analytical answer will provide several assistance in improved boundary conditions of common function for the model (S. Succi, 2001). The square lattice Boltzmann BGK model is confirmed to be advanced vigorous than the triangular model in numerical simulations. It is essential and attractive to discover analytical solutions for it. The square lattice Boltzmann BGK model employs three kinds of particles. On behalf of the Couette flow, the above boundary is a propagating one. The analytical solution specified now will provide assistance in raising an appropriate boundary condition for propagating boundaries.

2.5 POISEUILLE FLOW

For the Poiseuille flow, numerical simulation of the Poiseuille flow driven by a pressure incline was made to explore the legitimacy of the isothermal lattice Boltzmann model (S. P. Suter, R. Skalak, 1993). For this flow the pressure incline is set among the inlet and outlet finish of the channel. The density required to be put at somewhat dissimilar rate between the two ends. The velocity is does not to be set to any rate. Apply bounce back boundary conditions at the top and bottom walls. Allowing the systems expand, it is shown that it reaches a steady condition consequent to the parabolic answer of the channel flow (Nor Azwadi C.S, 2007). Two forms of measurement were in employ in the simulation that are the velocity, u and the other element is pressure all along the channel. Each measurement is in use following the stable state is attained. As a result of the simulation, not just the velocity profile is parabolic shape on the other hand also the pressure distribution is linear along the channel length. The figure of 2.1 shows the Poiseuille condition in pipe and how the fluid flows inside it.

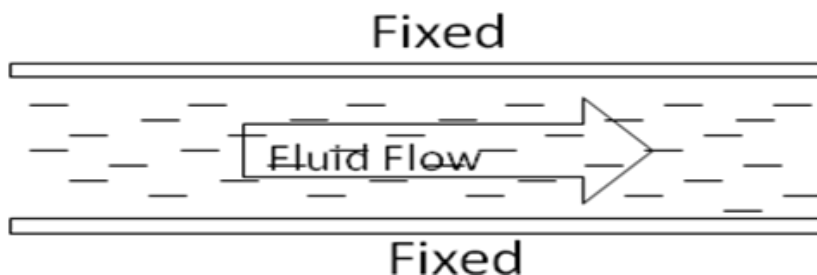


Figure 2.1: Poiseuille Flow

Source: A. R. M. Rosdzimin (2008)

2.6 COUETTE FLOW

The Couette flow is the laminar flow of a viscous fluid in the room among two parallel plates, one of which is propagating relative to the other. The flow is strength by good value of viscous drag force performing on the fluid and the apply pressure slope equivalent to the plates. This type of flow is named in respect of Maurice Marie Alfred Couette, a Professor of Physics at the French university of Angers in the late 19th century (B.R. Munson, D.F. Young, and T.H. Okiishi, 2002). It is regularly affect in physics and engineering lessons to show shear-driven fluid propagation. The easiest conceptual configurations find out two unlimited, equivalent plates separated by a gap. The top plate, infer with a steady velocity u_0 in its own plane. The numerical research involving the time development of the Couette flow is revealed in which the top plate move with steady velocity at the similar moment as the underneath plate is stable. The condition of Couette flow is shown in figure 2.2. The near the beginning condition keep in touch to null velocity all over the place not including on the top boundary where the velocity is $u = (U, 0)$ (Nor Azwadi C.S, 2007). Horizontal component of the velocity is maintain, at the equivalent time as the lower is at rest. No pressure slope incorporated for this case. The periodic boundary state is utilized in the horizontal way. Steady state case answer is very well identified and be in contact to the velocity increasing linearly in view of the fact that zero at the lower to U at the top plate.

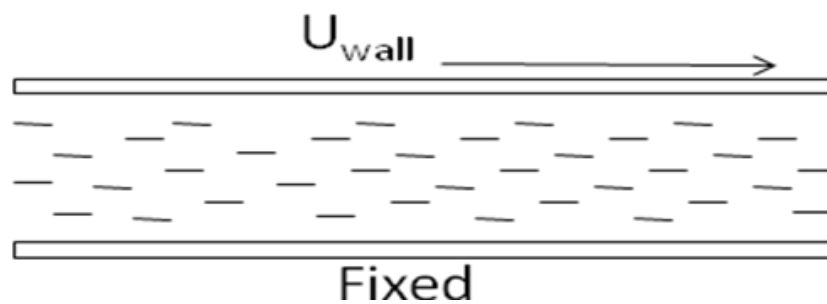


Figure 2.2: Couette Flow

Source: A. R. M. Rosdzimin (2008)

For additional frequent Couette flow state happen at the same time as a pressure slope is forced in a way alike to the plates. Next to that the pressure slope can be positive that is adverse pressure slope or negative that is positive pressure slope. It may be recognized that in the restrictive case of permanent plates, the flow is referred to as plane Poiseuille flow with a symmetric with course to the horizontal mid-plane of parabolic velocity outline.

2.7 POROUS COUETTE FLOW

The porous Couette flow is the flow in the middle of two counterpart plates that separated by a few gap as shown in figure 2.3. The top plate is moving and has cool temperature at the similar time as the bottom plate is permanent and has hot temperature. A stable normal flow of fluid is put in all the way through the underneath of hot plate. It is exposed at the related rate from the top plate.

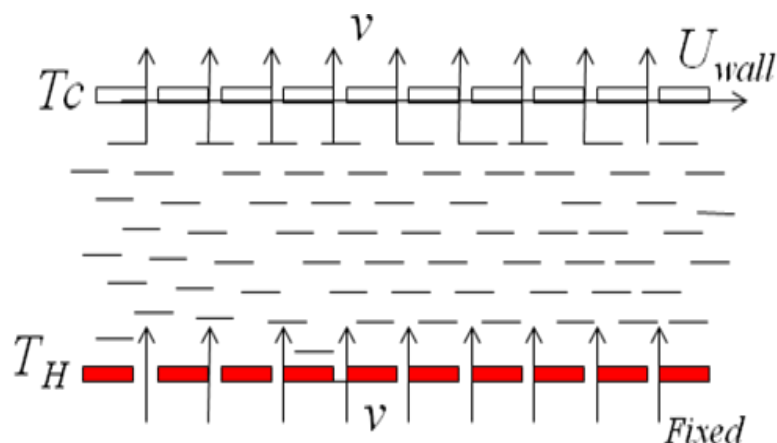


Figure 2.3: Porous Couette Flow

Source: A. R. M. Rosdzimin (2008)

2.8 NATURAL CONVECTION IN SQUARE CAVITY

Nowadays, many applications are connected to the natural convection in enclosures. The applications are such as double glazed window, solar collectors, cooling device for electronic instruments and building insulation. In many naturally occurring situation, the fluid motion results from buoyancy force due to the temperature gradient. Moreover, this condition is excellent comparison investigate for the performance estimation of numerical method models. The natural convection in square cavity is taken again to investigate the validity of finite difference thermal lattice Boltzmann via the recently build up four velocities for the inner energy density equilibrium density distribution function. The temperature contours at equilibrium condition expect for flows at various Rayleigh number. When the Rayleigh number is raise, a high degree of convection is monitored such that discrete thermal boundary layers begin emerge near the isothermal walls (A. R. M. Rosdzimin, 2008). The thickness of thermal boundary layer is reduce when Rayleigh number raise. The figure 2.4 shows the boundary conditions of the heated cylinders in square cavity enclosure. The two cylinders inside the enclosure is heated and placed in a cool enclosure.

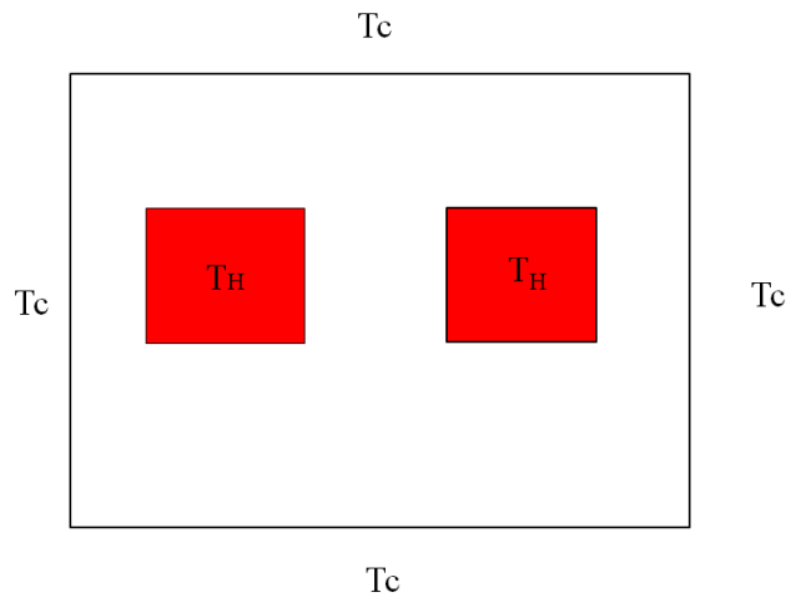


Figure 2.4: Heated cylinders in square cavity

CHAPTER 3

METHODOLOGY

3.1 ORIGINAL LBM ALGORITHM

LBM is a comparatively fresh simulation method designed for difficult fluid scheme plus has concerned attention as of researchers within computational physics. Nothing like the conventional CFD system, which explains the conservation equations of macroscopic possessions numerically, LBM models the fluid include of fictive element, and such unit carry out successive transmission and clash procedure in excess of a discrete lattice mesh. The algorithm flowchart for LBM is shown in figure 3.1. It consists of two processes; advection and collision process. The initial values of density distribution f are specified at each grid point. Then, the system evolves in the following steps. The first step of the algorithm is advection. The advection term is solved by applying the streaming process of the density distribution function.

Then the second step is the collision process that occurs during the simulation. The collision process is solved by BGK collision model. The next step is to define the boundary conditions according on the bounce back boundary conditions. After that, the convergence criterion is set to solve the simulation according to what is needed. The convergence criterion is according on steady state conditions. If the converge solution did not converge to the correct solution, the iteration is again made and the simulation is done from the advection until it converge to the right solution. If the iteration has come to the right solution, the simulation is then ended according to the convergence criterion. The flow of the LBM algorithm is shown clearly in figure 3.1. The main program iteration is done by the Compaq Visual Fortran. Main program is basically related to the theory of lattice Boltzmann.

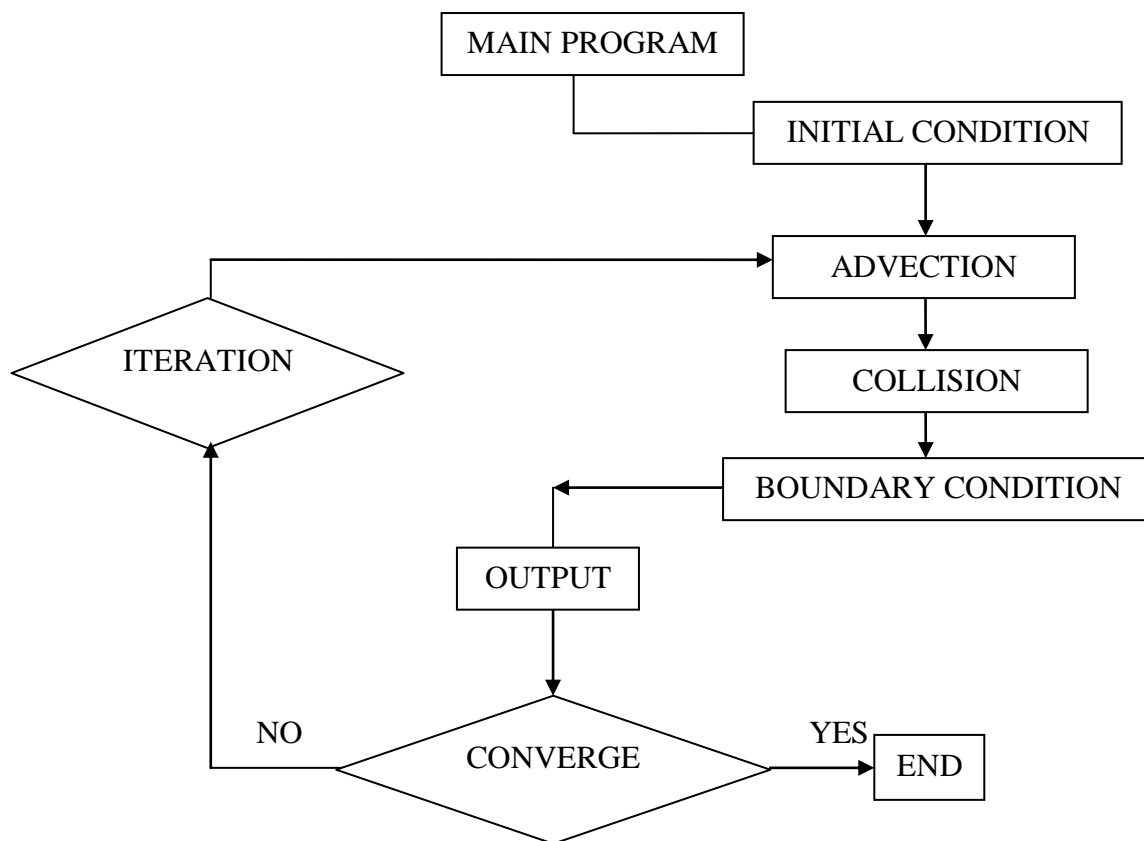


Figure 3.1: LBM algorithm flow chart

3.2 SIMULATION OF ISOTHERMAL LATTICE BOLTZMANN

The macroscopic equation for isothermal equation from Nor Azwadi C.S. (2007) is express as in Eq. (3.1)

$$\nabla \cdot u = 0$$

$$\frac{\partial u}{\partial t} + u \nabla \cdot u = -\nabla P + \left(\frac{2\tau - 1}{6} \right) \nabla^2 u \quad (3.1)$$

$$\nu = \frac{2\tau - 1}{6} \quad (3.2)$$

u = velocity

P = pressure

ν = kinematic shear viscosity

τ = relaxation time

3.2.1 Poiseuille Flow

Simulation of the flow in pipe (Poiseuille flow) is done by the Compaq Visual Fortran. The numerical simulation for the Poiseuille flow driven by a pressure gradient was carried out to test the validity of the isothermal lattice Boltzmann model. The pressure gradient is put between the inlet and the outlet finish of the channel. The densities are put at dissimilar values between the two ends. The bounce back boundary condition is applied at the top and bottom walls. The flow pattern is analyzed.

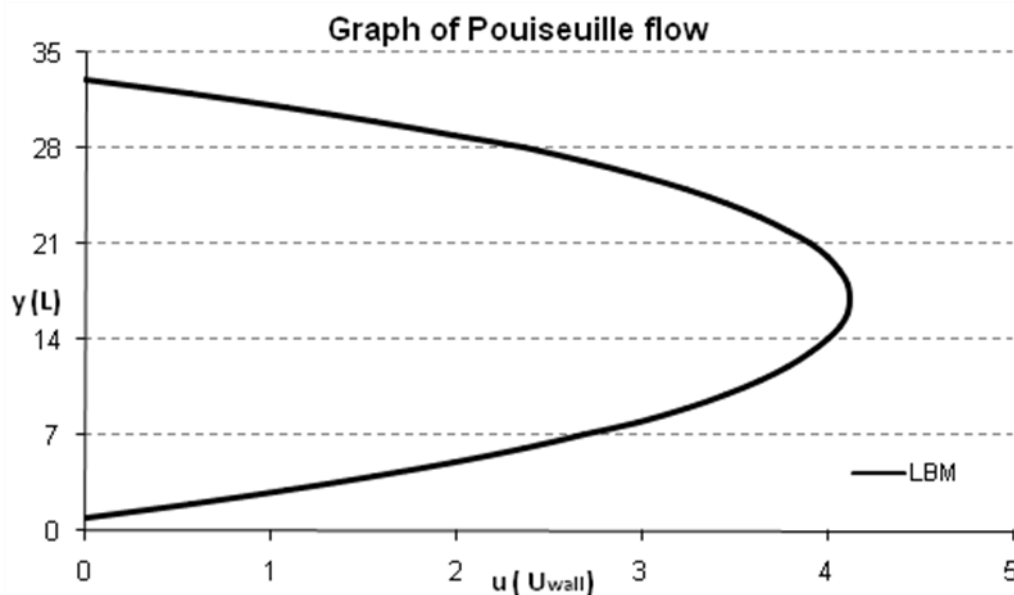


Figure 3.2: Poiseuille flow graph

3.2.2 Couette Flow

Simulation of the Couette flow is done by the Compaq Visual Fortran. The numerical simulation involving the time evolution of the Couette flow is presented. The top plate moves with constant velocity, while the bottom plate is permanent. The early conditions are in contact to a null velocity all over excluding on the top boundary, where the velocity is $u = (1, 0)$. The x-component of the velocity on the top plate is

retain at $U = 1.00$ (top plate boundary condition in LBM units), whereas the underneath one is at rest. In this case no pressure gradient. The flow is observed and judge against with different time, t .

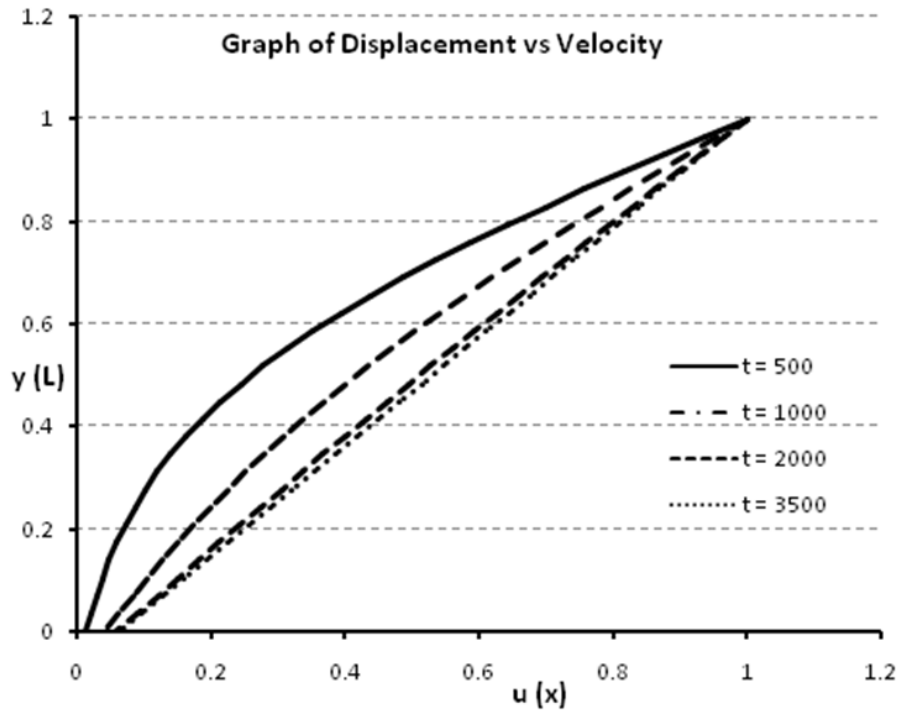


Figure 3.3: Velocity profiles across the normalized channel width at different times

3.3 SIMULATION OF THERMAL LATTICE BOLTZMANN

The macroscopic equation for thermal express from H.N. Dixit and V. Babu (2006) as in Eq. (3.2) and (3.3)

$$\nabla \cdot u = 0$$

$$\frac{\partial u}{\partial t} + u \nabla \cdot u = -\nabla P + \left(\frac{2\tau_f - 1}{6} \right) \nabla^2 u \quad (3.2)$$

$$\frac{\partial T}{\partial t} + \nabla \cdot (uT) = \left(\tau_g - \frac{1}{2} \right) \nabla^2 T \quad (3.3)$$

$$\tau_f = 3\nu + \frac{1}{2} \quad (3.4)$$

$$\tau_g = \chi + \frac{1}{2} \quad (3.5)$$

u = velocity

ν = kinematic shear viscosity

τ = relaxation time

χ = thermal diffusivity

3.3.1 Porous Couette Flow

Simulation of Porous Couette flow is via the Compaq Visual Fortran. The porous Couette flow is the flow between two parallel plates that divided by a few space. The top plate is moving and has cool temperature at the same time as the lower plate is permanent and has hot temperature. A steady normal flow of fluid is inserted through the bottom of hot plate. It is withdrawn at the equivalent rate from the higher plate. Periodic boundary conditions are used at the access and way out of the channel, and the non-equilibrium bounce back boundary conditions for velocity. For temperature boundary condition, the non-equilibrium bounce back boundary condition is applied. The flow pattern is observed with different Reynolds number, Re .

From the simulation done, the graph of porous couette have been plotted with different Reynolds number and different Prandtl number. These graphs have shown the profile and the pattern of the couette flow. The normalized temperature profile for $Pr = 0.71$, $Ra = 100$ and $Re = 5, 10, 20$ and 30 is shown in the figure 3.4.

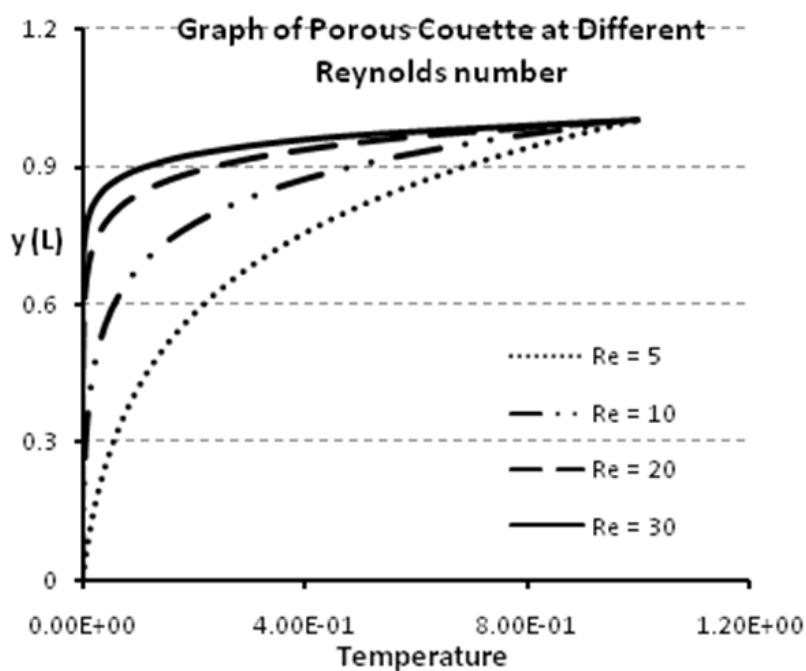


Figure 3.4: Porous Couette flow graph at different Reynolds number

Results from the simulation for $Ra = 100$, $Re = 10$ and $Pr = 0.2, 0.8$ and 1.5 is shown on the figure of 3.5.

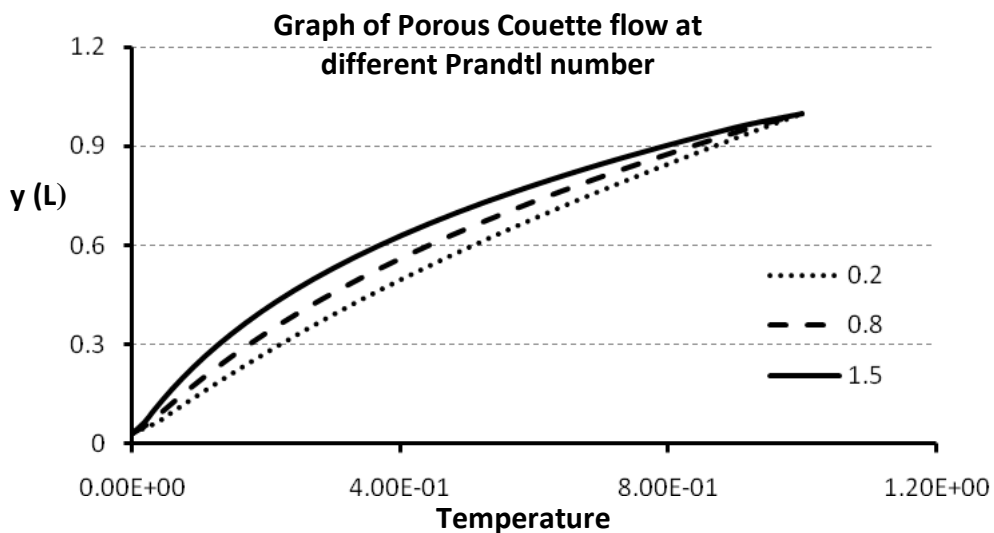


Figure 3.5: Porous Couette flow graph at different Prandtl number