SUPERVISOR’S DECLARATION

I hereby declare that I have checked this project and in my opinion, this project is adequate in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering.

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I hereby declare that the work in this project is my own except for quotations and summaries which have been duly acknowledged. The project has not been accepted for any degree and is not concurrently submitted for award of other degree.

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ACKNOWLEDGEMENTS

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ABSTRACT

The aim of this thesis is to study the methods of the lattice Boltzmann equations in order to apply in two types of D2Q4 model in thermal fluid flow problems. LBM has been found to be useful in application involving interfacial dynamics and complex boundaries. These methods utilize the statistical mechanics of simple discrete models to simulate complex physical systems. The theory of lattice Boltzmann method in nine and four velocity model are reviewed. The isothermal and thermal equation have been derived from the Boltzmann equation by descretiziton on both time and phase space. In this isothermal problem, a few simple isothermal flow simulation will be done by using the nine velocity model. The concepts of distribution function are considered beside the theory of Boltzmann equations. Then the derivations of Navier-Stokes equation from the Boltzmann equations are also presented. Some simulation results are performed, to highlight the important features of the isothermal LB model. The application of lattice Boltzmann scheme in thermal fluid problem is investigated in chapter 3. By using the derivation of the discretised density distribution function, a 4-velocity model is applied to develop the internal energy distribution function. This model is validated to simulate the porous couette flow problem for thermal fluid flow problems. The performance for both types D2Q4 microscopic model is demonstrated in the simulations of porous thermal couette flow and natural convection flow in a square cavity. The simulation of thermal fluids flow is applied to two different types of four-velocity model that are Azwadi model and old model. The same simulation test is performed for both types and the accuracy and stability analysis of both models are stated. These models are compared and discussed to ensure its validity.
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**LIST OF SYMBOLS**

- $f$: Density distribution function;
- $c$: Microscopic velocity
- $\Omega(f)$: Collision integral.
- $t$: Time
- $T$: Temperature
- $N$: Kinematic viscosity
- $A$: Thermal diffusivity
- $\beta$: Thermal expansion coefficient
- $T_C$: Cold temperature
- $T_H$: Hot temperature
- $u$: Horizontal velocity
- $\mathbf{u}$: Velocity vector
- $U$: Horizontal velocity of top plate
- $v$: Vertical velocity
- $V$: Volume
- $x$: Space vector
- $P$: Pressure
- $\tau_{f,g}$: Time relaxation
- $\nu$: Shear viscosity
- $\beta$: Thermal expansion coefficient
- $E$: Internal energy
- $\nu$: Kinematic viscosity
- $\alpha$: Thermal diffusivity
- $\mu$: Viscosity
- $k$: Thermal conductivity
- $\rho$: Density
- $T_\infty$: Infinity temperature
- $T_f$: Film temperature
- $T_s$: Surface temperature
- $A$: Area of contact $A$
- $\eta$: Proportionally constant
- $\chi$: Thermal diffusivity
- $\Omega$: Collision operator
- $T_m$: Average temperature
- $g$: Acceleration due to gravity
- $c$: Microscopic velocity
- $f^{eq}$: Equilibrium distribution function
- $f$: Distribution function
- $Pr$: Prandtl number
- $Ra$: Rayleigh number
- $Re$: Reynolds number
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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

1.1.1 Navier-Stokes equations

The Navier–Stokes equations describe the motion of fluid substances that is substances which can flow. These equations arise from applying Newton's second law to fluid motion, together with the assumption that the fluid stress is the sum of a diffusing viscous term (proportional to the gradient of velocity), plus a pressure term. The mathematical relationship governing equation fluid flow is the famous continuity equation and Navier Stokes equation given by. The Navier–Stokes equations dictate not position but rather velocity. A solution of the Navier–Stokes equations is called a velocity field or flow field, which is a description of the velocity of the fluid at a given point in space and time. Once the velocity field is solved for, other quantities of interest such as flow rate or drag force may be found (X.He and L.S.Luo). Some exact solutions to the Navier–Stokes equations exist. Examples of degenerate cases; with the non-linear terms in the Navier–Stokes equations equal to zero; are Poisuelle flow, Couette flow and the oscillatory Stokes boundary layer. But also more interesting examples, solutions to the full non-linear equations, exist; for example the Taylor–Green vortex. Note that the existence of these exact solutions does not imply they are stable: turbulence may develop at higher Reynolds numbers.
1.1.2. **Lattice Boltzmann Method (LBM)**

Lattice Boltzmann methods (LBM) is a class of computational fluid dynamics (CFD) methods for fluid simulation. Instead of solving the Navier–Stokes equations, the discrete Lattice Boltzmann (LB) equation is solved to simulate the flow of a Newtonian fluid with collision models such as Bhatnagar-Gross-Krook (BGK). LB scheme is a scheme evolved from the improvement of lattice gas automata and inherits some features from its precursor, the Lattice Gas Automata (LGA).

The main motivation for the transition from LGA to LBM was the desire to remove the statistical noise by replacing the Boolean particle number in a lattice direction with its ensemble average, the so-called density distribution function. Accompanying this replacement, the discrete collision rule is also replaced by a continuous function known as the collision operator. In the LBM development, an important simplification is to approximate the collision operator with the Bhatnagar-Gross-Krook (BGK) relaxation term. This lattice BGK (LBGK) model makes simulations more efficient and allows flexibility of the transport coefficients.

Lattice Boltzmann models can be operated on a number of different lattices, both cubic and triangular, and with or without rest particles in the discrete distribution function. A popular way of classifying the different methods by lattice is the DnQm scheme. Here "Dn" stands for "n dimensions" while "Qm" stands for "m speeds". For example, D3Q15 is a three-dimensional Lattice Boltzmann model on a cubic grid, with rest particles present.

Although LBM approach treats gases and liquids as systems consisting of individual particles, the primary goal of this approach is to build a bridge between the microscopic and macroscopic dynamics. It is by deriving macroscopic equations from microscopic dynamics by means of statistic, rather than to solve macroscopic equations.
1.2 PROBLEM BACKGROUND

The lattice Boltzmann method is an alternative approach to the finite difference, finite element, and finite volume techniques for solving Navier-Stokes equations. LB scheme is a scheme evolved from the improvement of lattice gas automata and inherits some features from its precursor, the LGA. The implementation of the Bhatnagar-Gross-Krook (BGK) approximation has been made for LB method to improve its computational efficiency. The algorithm of LBM is simple, and easily modified to allow for the application of other.

LBM originated from lattice gas automata (LGA) which is based on concepts from the kinetic theory of gases. Two dimensional, four velocity model is one of the most widely used today to modeling macroscopic flow phenomena. LGA views fluids as arrays of discrete particles living on a discrete lattice, evolving some interactive such as propagation and collision rules. Collision between the particles in D2Q4 model will occur, and the change in velocity of each particle including its performance will be found out.

1.2.1 Project Objective

To find out the performances for both types D2Q4 microscopic velocity models.

1.2.2 Project Scopes

The scopes of this project are limited to D1Q4 microscopic velocity model, at low Rayleigh number using heat transfer mechanism.
1.3 THEESIS OUTLINE

The aim of this thesis is to study the methods of the lattice Boltzmann equations in order to apply in two types of D2Q4 model in thermal fluid flow problems. These methods utilize the statistical mechanics of simple discrete models to simulate complex physical systems. The theory of lattice Boltzmann method in nine and four velocity model are reviewed. Then the new concern here is in chapter 4. Two types of four velocity model are studied and will be evaluated at low Rayleigh number heat transfer. The performance for both types D2Q4 microscopic model is demonstrated in the simulations of porous thermal couette flow and natural convection flow in a square cavity. Comparison of both models will be analyzed as the final result.

In chapter 2, the isothermal fluid flows problem will be the main subject. The concepts of distribution function are considered beside the theory of Boltzmann equations. Then the derivations of Navier-Stokes equation from the Boltzmann equations are also presented. Some simulation results are performed, to highlight the important features of the isothermal LB model.

Then in chapter 3, the application of lattice Boltzmann scheme in thermal fluid problem is investigated. By using the derivation of the discretised density distribution function, a 4-velocity model is applied to develop the internal energy distribution function. This model is validating to simulate the porous couette flow problem for thermal fluid flow problems. The accuracy and stability analysis of the model are discussed.

In chapter 4, the simulation of thermal fluids flow is applied to two different types of four-velocity model. The same simulation test is performed for both types and the accuracy and stability analysis of both models are also discussed. These models are compared and tested to ensure its validity.

Finally in chapter 5, conclusions and discussion on future studies are presented.
CHAPTER 2

LITERATURE STUDY

2.1 LATTICE BOLTZMANN METHOD

The lattice Boltzmann method is a powerful technique for the computational modeling of a wide variety of complex fluid flow problems including single and multiphase flow in complex geometries. It is a discrete computational method based upon the Boltzmann equation. It considers a typical volume element of fluid to be composed of a collection of particles that are represented by a particle velocity distribution function for each fluid component at each grid point (X.HE, 1997). The time is counted in discrete time steps and the fluid particles can collide with each other as they move, possibly under applied forces. The rules governing the collisions are designed such that the time-average motion of the particles is consistent with the Navier-Stokes equation.

This method naturally accommodates a variety of boundary conditions such as the pressure drop across the interface between two fluids and wetting effects at a fluid-solid interface. It is an approach that bridges microscopic phenomena with the continuum macroscopic equations. Further, it can model the time evolution of systems. Lattice Boltzmann Method can be reviewed as a numerical method to solve the Boltzmann equation. In LB method, the phase space is discretized. In a LB model, the velocity of a particle can only be chosen from a velocity set, which has only a finite number of velocities.

The Lattice Boltzmann Equation (LBE) method is described for simulating micro- and meso-scale phenomena. The method is employed to study multiphase and multicomponent flows in microchannels.
The primary goal of LBM is to build a bridge between the microscopic and macroscopic dynamics rather than to deal with macroscopic dynamics directly. In other words, the goal is to derive macroscopic equations from microscopic dynamics by means of statistics rather than to solve macroscopic equation.

The Boltzmann equation for any lattice model is an equation for the time evolution of $f_i (x,t)$, the single-particle distribution at lattice site $x$:

$$f(x + c\Delta t, t + \Delta t) - f(x,t) = -\frac{f - f^{eq}}{\tau}$$  \hspace{1cm} (2.1)
2.2 COLLISION INTERGRAL, $\Omega( f )$

Basic principle of LBM is including streaming step and collision step. The particles move to another place in the variable direction with their velocities (streaming step) and after they meet to each other, the collision happens (collision step) and the particles will separate again. (streaming step) (Xiaoyi et al., 1996). We also can take an example from the ‘snooker’. From this situation, means when a ball hit to another ball, its can firstly streaming and then its will collision and it become streaming again. Collisions between particles change their velocities, and make them move in and out of the domain. A collision term describes the net increase of the density of the number of particles in the domain due to the collision. One of the simplest collision models is the Bhatnagar, Gross and Krook (BGK) simplified collision model.

$$\Omega \ f = \frac{1}{\tau} \ f^{eq} - f$$

(2.2)

Boltzmann came out with the H-theorem where the value of distribution function will always tend to the equilibrium distribution function, $f^{eq}$ during collision process. The distribution function $f$ can be relate to $f^{eq}$.

2.3 BGK (Bhatnagar, Gross, and Krook)

In BGK model, the nonlinear collision term of the Boltzmann equation is replaced by a simpler term and the model makes the derivation of the transport equations for macroscopic variables much easier. A problem, which is easily solved by the BGK model, is that of relaxation of a state of a fluid to equilibrium.

$$f \ x + c \Delta t, t + \Delta t = f \ x, t - \frac{1}{\tau} \left[ f \ x, t - f^{eq} \ x, t \right]$$

(2.3)

2.4 BOUNDARY CONDITION

The set of conditions specified for the behavior of the solution to a set of differential equations at the boundary of its domain. Boundary conditions are
important in determining the mathematical solutions to many physical problems. In a numerical simulation, it is impossible and unnecessary to simulate the whole universe. Generally we choose a region of interest in which we conduct a simulation. The interesting region has a certain boundary with the surrounding environment. Numerical simulations also have to consider the physical processes in the boundary region. In most cases, the boundary conditions are very important for the simulation region's physical processes. Different boundary conditions may cause quite different simulation results. Improper sets of boundary conditions may introduce nonphysical influences on the simulation system, while a proper set of boundary conditions can avoid that. So arranging the boundary conditions for different problems becomes very important. While at the same time, different variables in the environment may have different boundary conditions according to certain physical problems. Commonly there are several different types of boundary conditions.

2.4.1 Periodic boundary condition

Periodic boundary conditions (PBC) are a set of boundary conditions that are often used to simulate a large system by modeling a small part that is far from its edge. Periodic boundary conditions are particularly useful for simulating a part of a bulk system with no surfaces present. Moreover, in simulations of planar surfaces, it is very often useful to simulate two dimensions (e.g. x and y) with periodic boundaries, while leaving the third (z) direction with different boundary conditions, such as remaining vacuum to infinity. This setup is known as slab boundary conditions.

![Figure 2.2: Periodic Boundary Condition](image)
2.4.2 Bounce Back Boundary condition

The bounce-back boundary condition for lattice Boltzmann simulations is evaluated for flow about an infinite periodic array of cylinders. The bounce-back boundary condition is used to simulate boundaries of cylinders with both circular and octagonal cross-sections. The convergences of the velocity and total drag associated with this method are slightly sublinear with grid spacing. Error is also a function of relaxation time, increasing exponentially for large relaxation times. However, the accuracy does not exhibit a trend with Reynolds number between 0.1 and 100. The bounce-back boundary condition is shown to yield accurate lattice Boltzmann simulations with reduced computational requirements for computational grids of 170×170 or finer, a relaxation time less than 1.0 and any Reynolds number from 0.1 to 100. For this range of parameters the root mean square error in velocity and the relative error in drag coefficient are less than 1 % for the octagonal cylinder and 2 % for the circular cylinder.

2.4 DISCRETIZATION OF MICROSCOPIC VELOCITY

For the discretization of microscopic velocity, from the Gauss-Hermitte integration, we can integrate from the continuous velocity to the 9-discrete velocity and also 4-discrete velocity. We focused on the two type of discrete velocity [S. Harris]. For the (poiseuille and couette flow) isothermal fluid flow, we are focusing on 9-discrete velocity model and for the (porous couette flow) thermal fluid flow, we are using 4-discrete velocity model.
2.5.1. Isothermal Fluid Flow

Figure 2.2 showed the 9-discrete velocity model that is going to be used in simulation of isothermal fluid flow (poiseulle flow and coutte flow).

2.5.1.1 The Macroscopic Equation for Isothermal

By using chapmann-enskog expansion procedure, we can have the navier-stroke equations accurate in continuity equations and in momentum equation:

\[
\nabla \cdot \mu = 0
\]  
\[
\frac{\partial \mu}{\partial t} + \mu \nabla \cdot \mu = -\nabla P + \left( \frac{2\tau - 1}{6} \right) \nabla^2 \mu
\]

The relation between the time relaxation \( \tau \), in microscopic level and viscosity of fluid \( \nu \), in macroscopic level is;

\[
\nu = \frac{2\tau - 1}{6}
\]
2.5.2 Thermal Fluid Flow

![Continuous velocity](image)

**Figure 2.4:** 4-Discrete Velocity Model

Figure 2.3 showed the 4 discrete velocities model for that are going to be used in simulation of thermal fluid flow (porous couette flow).

2.5.2.1. The Macroscopic Equation for Thermal

By using the chapmann-enskog expansion procedure, we can get the derivation equation for the energy equation.

\[
\frac{\partial T}{\partial t} + \nabla \cdot uT = \left( \tau_g - \frac{1}{2} \right) \nabla^2 T
\]

(2.7)

Where:

\[
\tau_g = \chi + \frac{1}{2}
\]

(2.8)

2.6 THEORY OF REYNOLD NUMBER

Reynolds number can be defined for a number of different situations where a fluid is in relative motion to a surface. These definitions generally include the fluid properties of density and viscosity, plus a velocity and a characteristic length or characteristic dimension. This dimension is a matter of convention - for example a radius or diameter is equally valid for spheres or circles, but one is chosen by convention. For flow in a pipe or a sphere moving in a fluid the diameter is generally used today. Other shapes (such as rectangular pipes or non-spherical...