

BEHAVIOUR OF RC DEEP BEAMS WITH WEB OPENINGS AT THE TOP OF SHEAR SPAN NEAR SUPPORT-FINITE ELEMENT ANALYSIS

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ABSTRACT

This thesis deals with behaviour of RC deep beams with web openings at the top of shear span near support. The objectives of this thesis are to determine load deflection curve and crack pattern of RC deep beams with openings, identify the effects of opening size and shape and lastly comparing finite element analysis results with experimental results. In this thesis, two types of opening shapes were studied. The first one was circular and the other one was square. The opening sizes used were 150x150 mm, 200x200 mm and 250x250 mm. The structural twodimensional solid modeling of deep beams was developed using ATENA software. The finite element model of the structure was analyzed using nonlinear behaviour approach. Reinforcements used were 16 mm, 10 mm and 6 mm. Finally, crack pattern and load deflection curve which comprises of yield load, ultimate load and deflection of beam were obtained. Location of stress concentration could be detected. From the results, similar crack pattern was observed in beams with openings, both circular and square. All beams are failed in the same manner with diagonal cracks formed at the opening corner in square opening and around the circular opening. Beams failed in shear mode. As the opening size increases, beam capacity of deep beam decreases. For solid beam, the ultimate load is 65.09 kN. When comparing load capacity for deep beams with web openings, deep beam with 150x150 mm size opening has a higher ultimate load capacity compared to deep beam with the opening size of 200x200 mm and 250x250 mm. The ultimate load capacity decreases in the range of 47.76% to 73.31% as compared to control beam. Similarly, increasing in diameter of circular opening from Ø150 mm to Ø200 mm and Ø250 mm has reduced the ultimate load capacity of beam, about 34.98% to 56.08%. This signifies beam with circular opening demonstrate higher beam capacity as compared to square opening. Load deflection curve trend was observed to behave elastically under gradually increase load up to elastic limit and load decreased in plastic deformation phase. The results obtained from experimental work and finite element analysis in terms of ultimate load indicated close values. The highest percentage of difference was 56.53% due to the presence of square opening with the size of 240x240 mm. Whereas about 2.10% difference was observed in the results of beam with circular opening of 250 mm diameter. For crack pattern, fair agreement was observed between experimental work and FE analysis.

ABSTRAK

Tesis ini memperkatakan tentang kelakuan RC rasuk dengan bukaan web di bahagian atas span ricih berhampiran sokongan. Objektif tesis ini adalah untuk menentukan graf beban dan lenturan dan corak retak RC rasuk dengan bukaan, mengenal pasti kesan-kesan saiz pembukaan dan bentuknya ke atas kekuatan rasuk dan akhir sekali membandingkan keputusan analisis unsur dengan keputusan eksperimen. Dalam tesis ini, dua jenis bentuk dikaji. Yang pertama adalah bulat dan yang lain adalah persegi. Saiz bukaan yang digunakan adalah 150x150 mm. 200x200 mm dan 250x250 mm. Tesis menggambarkan teknik analisis unsur dalam pemodelan RC rasuk dengan bukaan web untuk menghasilkan corak retak dan graf beban dan lenturan. Struktur pemodelan pepejal dua dimensi rasuk telah dibangunkan menggunakan perisian Atena. Model unsur struktur telah dianalisis dengan menggunakan pendekatan tingkah laku tak linear. Tetulang vang digunakan adalah 16 mm, 10 mm dan 6 mm. Akhir sekali, corak retak dan graf beban dan lenturan yang terdiri daripada beban hasil, beban muktamad dan lenturan diperolehi. Lokasi tumpuan tekanan dapat dikesan. Lokasi tumpuan tegasan dapat dikesan. Daripada keputusan, corak retak yang sama diperhatikan dalam rasuk dengan bukaan, kedua-dua pekeliling dan persegi. Semua rasuk gagal dalam cara yang sama dengan retak pepenjuru terhasil di sudut bukaan di pembukaan persegi dan sekitar pembukaan bulat. Rasuk gagal dalam mode ricih. Seiring dengan peningkatan saiz pembukaan, kapasiti rasuk rasuk berkurangan. Untuk rasuk pepejal, beban muktamad adalah 65,09 kN. Kapasiti beban muktamad berkurangan dalam lingkungan 47,76% kepada 73.31% berbanding dengan kawalan pancaran. Begitu juga, pningkatan diameter pembukaan pekeliling dari Ø150 mm ke Ø200 mm dan Ø250 mm telah mengurangkan kapasiti beban muktamad rasuk, kira-kira 34.98% kepada 56,08%. Ini menunjukkan rasuk dengan pembukaan bulat menunjukkan kapasiti rasuk yang lebih tinggi berbanding dengan pembukaan persegi. Tend beban pesongan lengkung diperhatikan berkelakuan anjal secara beransur-ansur di bawah peningkatan beban sehingga had elastik dan beban berkurangan dalam fasa ubah bentuk plastik. Keputusan yang diperolehi daripada kerja uji kaji dan analisis unsur terhingga dari segi beban muktamad menunjukkan nilai dekat. Peratusan tertinggi perbezaan adalah 56.53% disebabkan oleh kehadiran pembukaan persegi dengan saiz 240x240 mm. Manakala perbezaan kira-kira 2.10% diperhatikan dalam keputusan rasuk dengan pembukaan bulat 250 mm diameter. Untuk corak retak, perjanjian saksama diperhatikan di antara kerja uji kaji dan analisis FE

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LIST OF ABBREVIATIONS

2D 2-Dimensional

ACI American Concrete Institute

FE Finite Element

FEA Finite Element Analysis

FEM Finite Element Method

RC Reinforced Concrete

CHAPTER 1

INTRODUCTION

1.1 Background of study

Deep beams are defined as beams with span to depth ratio "L/d" equal to 2.5 for continuous beams and less or equal to 5 for simply supported beams according to ACI 318 building code (2002). In both cases, if the shear span of the beam is less or equal to twice its depth, it can be classified as a deep beam. In another definition, reinforced concrete deep beams are members in which a noticeable amount of the load is carried to the support by compression thrust joining the loading and reaction point.

Some references stated that deep beams should be supported on reaction points and loaded on loading points so that compression struts can develop between the supports and loads (Yanga, K.H, 2006). Due to its characteristics in transferring a noticeable amount of load, a deep beam has probability to have a large size which is usually 1.7m to 3.0 meters in height. Obstacle to mechanical service access has been arising due to such height.

From time to time, researchers tried to find the solution in achieving economical benefits in tall buildings by saving vertical space taken from beams. It has been found that deep beam with openings are one of the applicable solutions. Reinforced concrete (RC) deep beams with openings were often being used in various structural applications such as water tanks, deep girders, foundations, offshore structures and bunkers. Presence of web openings in such beams is required to provide accessibility such as doors and windows or to accommodate essential services such as ventilating and air conditioning ducts.

However, the presence of web openings in deep beam leads to many problems in beam behaviour, including excessive cracking and deflection, reduction in beam stiffness, and reduction in beam capacity. Moreover, inclusion of openings leads to high stress concentration around the openings especially at the opening corners. Simple beam behaviour can change to a more complex one by the reduction of area in the total cross sectional dimension of a beam (Mansur *et al.*, 1992; Mansur, 2006).

Due to these complexities, further analysis regarding the effects of web opening characteristics on the beam is done. Finite Element Analysis is one of the suitable methods which can be applied to model the beam besides experimental supervision. ATENA is a software package for dynamic analysis and advanced static of reinforced concrete. It can simulate the real behavior of the structure, including crack pattern of beam, ultimate load capacity as well as crushing of beam. It gives engineers the opportunities to check and verify their structures in accurate load. Modeling by using ATENA provides needed results regarding behaviour of RC deep beam with web openings thus come out with the best predicted solution for the complexities.

1.2 Problem statement

In construction practices, when web openings are provided in reinforced concrete deep beam to accommodate the passage of utilities or to provide access to equipment, geometric discontinuity within the beam and non-linear stress distribution over the depth of beam is occurred. Due to the reduction of concrete mass acting in compression, the ultimate strength of a deep beam is also decreasing. Besides that, openings are act as a stress raiser for shear crack propagation (Kong & Sharp 1977). Plenty investigations on various attributes of deep beam have been conducted by Tan *et al.* (1995, 1997a,b,c), Ashour (1997) and Foster and Gilbert (1998), to name a few. However, limited research has been done dealing with deep beams with openings (Maxwell and Breen 2000; Ashour and Rishi 2000; Tan *et al.* 2003). Current national Codes of Practice such as American Concrete Institute (ACI318-08) and Australian Standards (AS3600-09) do not include specifications

for web openings in deep beams. This has increased the need for an accurate and safe design methodology.

The exact analysis of reinforced concrete deep beams with web openings presents terrified problems. Existing methods either semi-empirical equations or elastic theory, neither of which is satisfactory in predicting the ultimate strength of deep beams (Mohammad, K. I, 2007). Prediction of magnitude and location of tensile stresses due to openings in deep beams have been used by using elastic methods of analysis. Elastic theory does not predict the failure load although it is useful for obtaining an understanding of the way in which the beams carry load. Once the first crack has appeared in the beam, the assumption of a homogeneous elastic beam is no longer handy. Some recommendations have been made following experimental investigations of the subjects. Truss analogy is a note which led to the development of a semi-empirical design equation for ultimate strength but these equations are found complicated. It cannot be used to predict the failure modes of deep beams (Tan, et al. 2003). It is recognized that finite element methods can provide satisfactory solutions for the nonlinear behavior of reinforced concrete structures (Mohammad, K. I, 2007). Finite element method can be used to predict the behavior of reinforced concrete deep beam subjected to load in modeling material characteristics. Using this method, the load deflection curve, failure mode and crack pattern can be predicted with an accuracy that is acceptable for engineering purposes and safety.

1.3.1 Objectives:

- 1. To determine load deflection curve and crack pattern of RC deep beams with openings.
- 2. To identify the effects of opening size and shape.
- 3. To compare finite element analysis results with experimental results.

1.4 Scope of Study

The scopes of this research are to model RC deep beams with web openings by using ATENA software.

- 1. 2D finite element analysis is applied to all deep beams.
- 2. There are seven beams consist of one controlled beams and six variable beams.
- 3. There are two types of opening's shape which are circular and square.
- 4. Square openings have the sizes of 150x150 mm, 200x200 mm and 250x250 mm while for circular openings the sizes are 150Ø mm, 200Ø mm and 250Ø mm.
- 5. The openings are located at the top of shear span near support.
- 6. The dimension of deep beams is 100x500 mm and 1200 mm length.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction to RC Deep Beams

A beam is generally classified as a deep beam when its shear span, a, to depth ratio, d, is less than 2.5 based on (ACI-ASCE Committee 426). The same definition of deep beam is also shared by the (ACI 318-08) code. It specifies that deep beam has either clear spans less than or equal to four times the overall member depth or in other cases region with concentrated loads is within twice the member depth from the face of the support. RC deep beams are mainly present in building structures in which it works as a water tank, transfer girders, shear wall and also foundation where the walls act as vertical beams spanning between column supports (Yousif, 1986). Particularly, the use of deep beams at the lower levels in tall buildings for both commercial and residential purposes has ascended rapidly because of their convenience and economical efficiency.

2.2 RC Deep Beams with Openings

Referring to Mohamed H.A (2013), the purposes of providing web openings in RC deep beam are for electrical conduits and mechanical or even for passageways, such as openings for doors and hallways in buildings. In addition, openings are provided to reduce the material and construction cost due to minimizing storey's height of the building (Mansur, 2006) since it is compulsory for the buildings to have the service ducts.

2.3 Characteristics of Web Openings of RC Deep Beams

RC deep beam with web opening is an avoidable structure in most of the buildings nowadays. There are numerous shapes of openings available in RC deep beams such as diamond, circular, trapezoidal, rectangular and even irregular shapes based on the design. According to Mohamed H.A (2013), the most common types of openings being used are rectangular and circular openings shown in Figure 2.1. In fact, the size of the web openings are varies depending on the purpose, classified as small or large openings. Besides the sizes, the locations of the web openings also vary depending on the design strength.

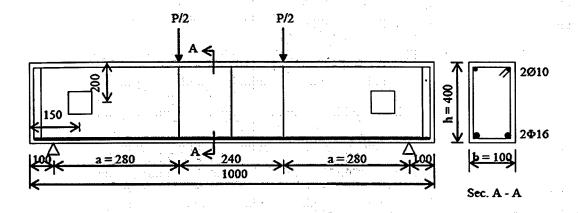


Figure 2.1: Rectangular shape of openings (Mohamed, H.A, 2013)

2.4 Behaviour of RC Deep Beam with Web Openings

Presence of web openings in RC deep beam leads to some problems mainly regarding the beam behaviors which are cracking, whether it is tension cracks or diagonal cracks, deflection and beam capacity as shown in Figure 2.2. It is found out that the presence of large openings often interrupt the load transfer by concrete struts and finally lead to the decrease in ultimate strength. In addition, not only the sizes that matters, but the location and shape of the web openings also play roles in determining RC deep beam behaviour.



Figure 2.2: Crack Pattern (Alsaeq, H.M, 2013)

2.5 Effect of Web Openings Location

Location of web openings plays as important role in contributing to structural strength of RC deep beam. Large openings, if located between the support and the loading point, the flow of force transfer will disrupt and usually the load-carrying capacity is reduced (Ray 1990). Regardless of all the effect, the most suitable location of the web openings is far from flexure and arching action region, which is near the upper corners of the beam (Alsaeq, H.M, 2013).

When opening is located at or extended towards rigid zone, the effect of opening on ultimate strength was insignificant compared when opening is located at Flexural zone. Ultimate strength was largely affected by the opening (Tae, M.Y, 2011). Figure 2.3 shows some different locations of the opening. The opening moves horizontally either towards the support point or loading point. 10% and 30% of percentage values in the Figure 2.3 indicate the distance between bottom left corner of the opening with respect to point P.

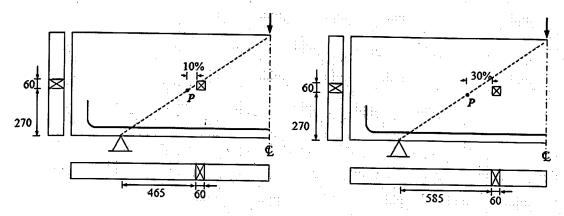


Figure 2.3: Opening with different location (Tae, M.Y, 2011)

2.6 Effect of Web Openings Sizes

Opening size is another factor which may affect the behavior of the RC deep beam. Web openings size depends on the purpose of use. As the opening sizes being used become larger, the structural strength of the beam becomes weaker. The beam action may be assumed to be used for deep beams which have small openings size. Hence, the design and analysis of beam with small openings may follow the similar course of action as that of a solid beam (Mansur, 2006). The theory is also similar to Tae, M.Y (2011). Load-carrying capacity decreased equivalent when web opening's size increased. Figure 2.4 shows opening with different sizes. The size increases in both horizontal and vertical directions from 10% to 15% as the location of the centre of opening remains at mid-depth of beam.

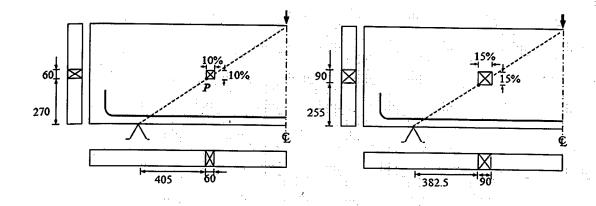


Figure 2.4: Opening with different sizes (Tae, M.Y, 2011)

2.7 Effect of Web Openings Shape

Based on the research, the best shape for web openings of deep beam is the narrow rectangular with long sides extended in the horizontal direction as shown in Figure 2.6. In other words, the length represents the long sides while height represents the short one. However, the application of this shape might not suitable for some cases. By comparing both shapes of circular and square, the use of circular shape as shown in Figure 2.5 has more advantages over square one regarding the structural strength of deep beam (Alsaeq, H.M. 2013)

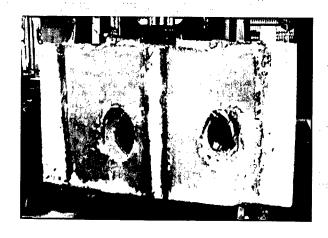


Figure 2.5: Circular openings (G, 2012)

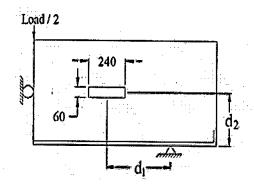


Figure 2.6: The narrow rectangular with long sides extended in the horizontal direction.

Source: Tae M.Y. (2011)

2.8 Introduction to Finite Element Analysis

Finite Element Analysis (FEA) represents a numerical method, which provides solution to difficult problem. Numerical analysis investigations were performed with commercial software ATENA. This software designed for computer simulation of concrete structures. The graphical user interface in ATENA provides an effective and powerful environment to solve many structural problems. (Cervenka, et al. 2002).

2.9 Modeling of Structure

Modeling of structure is based on topology which consists of geometrical joint, geometrical lines, macroelement, opening and reinforcement. To complete the analysis, load cases must be completed to come out with load deflection curve and crack pattern of beam. (Cervenka, et al. 2006).

2.10 Verification on Finite Element Program

To validate results obtained by the finite element analysis, both results from the FE analysis and experimental work need to be compared. Predicted crack pattern in FEM shows a similar crack pattern as obtained by the experimental testing (Chin, et al. 2012).

CHAPTER 3

METHODOLOGY

3.1 Introduction

Finite Element Analysis (FEA) represents a numerical method, which provides solution to problems that difficult to be solved. The numerical analysis investigations were performed with commercial software ATENA. This software is designed for computer simulation of concrete structures. The graphical user interface in ATENA provides an efficient and powerful environment to solve many structural problems. Load deflection curve as well as crack pattern can be presented as desired.

3.2 Details of the Study

For this study, seven deep beams have been modeled as simply supported beams using 2-Dimensional Engineering ATENA and analyzed under two points load. All beams which have an identical cross sectional of 100 mm x 500 mm and 1200 mm length have been modeled using ATENA software. The clear span for these beams is 1000 mm. All these beams have two different shapes and three different sizes of web openings. For the square opening, the sizes used are 150x150 mm, 200x200 mm and 250x250 mm. For circular opening, the sizes are Ø150 mm, Ø200 mm and Ø250 mm.

Bottom steel reinforcement consists of four 16 mm diameter bars while top steel reinforcement consists of two 10 mm diameter bars. Stirrups consist of 6 mm diameter bars each spaced by 150 mm center to center. A clear cover of 20 mm is controlled for all beams. One beam is acted as a controlled beam without openings (solid beam). Variable beams are having openings with circular and square shapes. These study parameters are the opening size and shape in which all the openings are placed at the top of shear span near support.

3.3 Analysis of RC Deep Beams Using ATENA

For this study, two-dimensional nonlinear finite element analysis was conducted by using ATENA to model deep beams with and without web openings. This analysis is done to come out with load deflection curve and crack pattern results. Four main steps involved in this research methodology are material modeling, topology, load cases and solution parameter. Details of every step are explained in further section.

3.3.1 Selecting Material Model

Material types and their parameter are determined. Material types used in this study are steel plate, reinforcement and concrete. Steel Plate used 210000 Mpa as its elastic modulus value. For reinforcement, multilinear is chosen as its type. There are three different size of bars used which are 16 mm, 10 mm and 6 mm. All of them are inserted with their stress strain value. In concrete parameter, rotated crack model is chosen as crack model with F_{cu} of 35 Mpa.

3.3.2 Topology

This section provides steps for building geometrical model. This process consists of joints, lines, macroelements, openings and reinforcements. As referred in the Table 3.1, specific data for each object are listed.

Table 3.1: Specific data of objects

Object	Specific data
Joint	Coordinates
Line	Joints
Macroelement	Lines
Reinforcement	Coordinates
Opening	Coordinates

3.3.2.1 Geometrical Joint

Geometrical joint is decided based on coordinate. Each coordinate for beam, opening and steel plate is stated according to experimental beam dimension. It includes clear cover for the beam.

3.3.2.2 Geometrical Lines

Geometrical lines are created by connecting selected points. Each point needs to be connected one by one in order to avoid any error during the meshing process. By creating these lines, the shape of the beam and steel plate will appear eventually. No refinement is selected for refinement method.

3.3.2.3 Macroelement

In ATENA 2D, region is called macro-element. This step is completed by deciding mesh type, element size, thickness and quadrilateral element type. For all seven beams,