Numerical Simulation on the Integrated Shallow Water Flow Model

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ABSTRACT
Even the 2D model has become popular in a few areas of river modelling due to the availability of high-resolution data and development of computer technology, the 1D model has not lost its market. It has long been used in simulating fluvial hydrodynamics and the associated processes such as sediment transport, pollutant dispersion and flooding caused by overtopping or breaching of river banks. In some case of modelling a fluvial flood event, it is still difficult to resolve the problematic river reach in a 2D manner, thus it is desirable to have a 1D component that can deal with the highly dynamic and complex flow hydrodynamics under flood conditions. It is advantageous to have a model solving the fully 1D shallow water equations by the robust Godunov-type scheme so that the unsteady flow in different regimes, including shock-like flow discontinuity, can be reliable simulated (Toro, 2001). The same goes to the solute transport which is closely related to the water quality in shallow water bodies. Solute transport has great impacts on the local environment and ecosystem as it is a common process that take place in rivers, lakes and estuarine. When it is associated with an urban flood event it may also cause potential risk on public health. So, understanding the solute transport processes in shallow flows is thus of fundamental and practical importance to hydraulic and environmental engineering as it provides an essential tool for water quality management, environmental impact assessment and hydraulic design (Falconer, 1992).

KEYWORDS
Godunov-type scheme; 1D shallow water equations; solute transport, Riemann solver; source terms; well-balanced; wetting and drying

INTRODUCTION
Due to drastically increase in populations, flood incidence become more frequent especially in urban area. The occurrence of flood inundation is due to breach of flood defences or inadequate capability of the drainage systems after heavy rainfall. Flood risk is expected to increase significantly in and beyond the 21st century due to climate change and rapid urbanisation. It has been reported that flooding is one of the major natural disasters to human life and assets. One-third of all losses due to nature's forces can be attributed to flooding (Bates and Roo, 2000) and recently, losses generated by flood disaster have increased drastically.

As the computer modelling has now become the primary tool in simulating flood flows, therefore it is essential to develop or improve the flood modelling system in order to cope with greater urbanisation and climate change. As in a flood event, the flood flows can be a major source of pollution as well as it picks up potentially harmful substances from surfaces such as oil, household chemicals and faecal material. Those detrimental substances will then be transferred to urban watercourses. This excess foul poses risks to human health and impact to the environment. Due to this, contaminated flow and solute transport are found as important aspects to be considered in developing an intensive flow model.

This work presents a development of a 1D hydraulic model that has been extended to include the diffusion-advection process. The integrated model will be used to simulate the hydrodynamics flows in channels and rivers together with the evolution of flood flows in the large-scale floodplain. It is essential to have a reliable 1D fluvial flood model to simulate and provide an accurate description of the flow hydrodynamics in the river reach as in some cases it is difficult to resolve the problematic river reach in a 2D manner. As most of the 1D engines are based on the solution to an approximated form of the fully 1D shallow water equations (Cunge *et al*., 1980), thus it is desirable to have a 1D component that can deal with the highly dynamic and complex flow hydrodynamics under flood conditions, with full consideration of the convective and source terms.

METHODS

Development of 1D Integrated Shallow Water Flow Model

One of the main tasks is to develop the one-dimensional surface flow solved by the Godunov-type numerical scheme. The 1D open channel flow model will be integrated with the pollutant transport that flows together during and after the flood event. The dependent variables are the changes in water level; *h* and the flow; *q* along the channel. Those variables are predicted by numerically solving the St. Venant or so called shallow water equations; as they have been experimentally confirmed (Cunge *et al*., 1980) and are accepted for many practical applications in modelling the unsteady flow in either one or two dimensional approach (Brufau *et al*., 2002). The fundamental used in the mathematical modelling of rivers are formalized in the equation of unsteady one dimensional open channel flow and is written as in (1):

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}}{\partial x} = \mathbf{s} \qquad (1)$$

The unsteady flow can be described by two dependant variables, the water depth, *h* and the discharge, $q=uh$ in a function of space, *x* and time, *t*. *u* is the average velocity. The flow variable vector; **u** , flux vector; **f** and the source term vector; **s** are given as (2);

$$\mathbf{u} = \begin{bmatrix} \eta \\ uh \\ ch \end{bmatrix}; \quad \mathbf{f} = \begin{bmatrix} uh \\ u^2h + \frac{1}{2}g(\eta^2 - 2\eta z_b) \\ uch \end{bmatrix}; \quad \boldsymbol{s} = \begin{bmatrix} -\frac{q}{b}\frac{\partial b}{\partial x} \\ -\frac{q^2}{bh}\frac{\partial b}{\partial x} - C_f u|u| - g\eta\frac{\partial z_b}{\partial x} \\ S_c \end{bmatrix} \qquad (2)$$

where η is the water level, Z_b denotes the bed elevation so that $h = \eta - Z_b$, g is the acceleration due to gravity, b is the channel width, $C_f = gn^2/h^{1/3}$ is the bed roughness coefficient with n being the Manning coefficient while ch is the conservative solute concentration with c being the solute concentration and s_c is a source or sink term for the solute concentration. Employing directly the water level η (instead of water depth h) as a flow variable, the above shallow water equations automatically provide well-balanced solutions for wet-bed applications and facilitate constructing a numerical scheme to deal with wetting and drying (Greenberg and LeRoux, 1996). It is a common practice to resort to St Venant shallow water equation in modelling (Birman and Falcovitz, 2006). The solute transport process is described by the advection-dispersion equations in one dimensional and is written together with the 1D shallow flow model so that they can be ran simultaneously. These equations consist of unsteady term which represents local time variation and the convective variation and adequately described the unsteady behaviour of the river flow whilst taking into account the longitudinal hydrostatic pressure gradient, the frictional resistance of the bed and the momentum of flow while retaining the mathematical balance between the flux gradient and source terms and also preserves steady state automatically.

Numerical simulations and benchmark tests

This work addresses numerical discretization using finite volume Godunov-type scheme. The interface fluxes prediction for the one-dimensional unsteady shallow water equation were directly obtained by implementing the Riemann problem as it is also form the bases of very efficient and robust Godunov-type method (Toro, 1999). The essential component of Godunov's method is the solution of the Riemann problem which describes the interaction of two different constant (velocity and pressure) states. By applying HLL Riemann solver with the second order Runge-Kutta time stepping method, the accuracy of computation is expected. Using a finite volume Godunov-type scheme, the following discretized formula as in (3) is used to update the flow variables from time level k to $k + 1$:

$$\mathbf{u}_i^{k+1} = \mathbf{u}_i^k - \frac{\Delta t}{\Delta x}\left(\mathbf{f}_{i+1/2} - \mathbf{f}_{i-1/2}\right) + \Delta t \mathbf{s}_i \tag{3}$$

where the subscript i denotes the cell index, Δx and Δt are respectively the cell size and time step, $\mathbf{f}_{i+1/2}$ and $\mathbf{f}_{i-1/2}$ are the interface fluxes through the two edges of cell i. In the context of a Godunov-type scheme, the fluxes are evaluated by solving local Riemann problems defined by the Riemann states on either sides of an interface. In this work, the HLL approximate Riemann solver (Amiram *et al*., 1983) is adopted for flux calculation due to its superior advantages in providing automatic entropy fix and facilitating wetting and drying. To improve the temporal accuracy of the scheme and the time marching, the second-order Runge-Kutta method is applied and (3) now becomes (4):

$$\mathbf{u}_i^{k+1} = \mathbf{u}_i^k + 0.5\Delta t\left(\mathbf{K}_i(\mathbf{u}^k) + \mathbf{K}_i(\mathbf{u}^*)\right) \tag{4}$$

The Runge-Kutta coefficients and the intermediate flow variables are defined by (5) and (6) respectively.

$$\mathbf{K}_i = -(\mathbf{f}_{i+1/2} - \mathbf{f}_{i-1/2}) / \Delta x + \mathbf{s}_i \tag{5}$$

$$\mathbf{u}_i^* = \mathbf{u}_i^k + \Delta t K_i(\mathbf{u}^k) \tag{6}$$

The numerical scheme or the developed model is then examined under a wide variety of physical conditions experienced in open channel. Taking the works done by previous researchers as a benchmark, simulation using the proposed method was done and discussed in the following section.

RESULTS AND DISCUSSION

Simulations done in order to study the capability of the current model by applying a number of experimental and analytical tests involving changing in the channel width and bed profile. The model successfully handles all of the tests and produces results agreeing well with experimental measurements or analytical solutions, which implies its potential in more practical applications.

Tidal wave flow and flow over an irregular bed

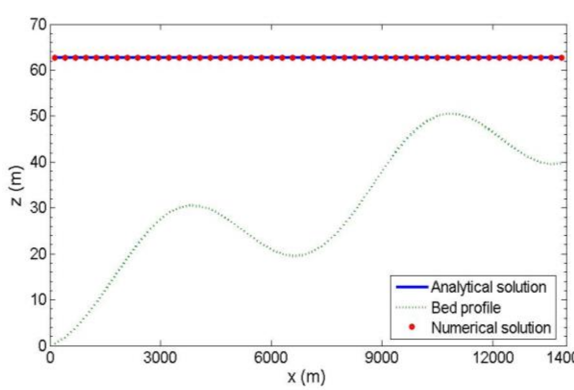
Presented here is test on tidal flow proposed by Bermudes and Vasquez (Bermudez and Vazquez, 1994). The tidal flow over an irregular bed which is defined by (7)

$$H(x) = 50.5 - \frac{40x}{L} - 10\sin\left[\pi\left(\frac{4x}{L} - \frac{1}{2}\right)\right] \tag{7}$$

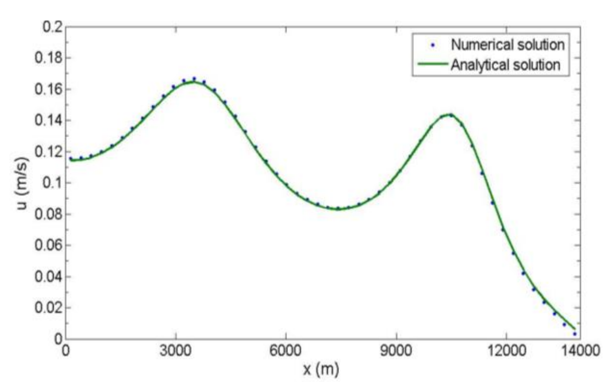
Where the channel length L is 14000 m. The initial condition are $h(x,0) = H(x)$ and $u(x,0) = 0$. The boundary conditions are as (8)

$$h(0,t) = H(0) + 4 - 4\sin\left[\pi\left(\frac{4t}{86400} + \frac{1}{2}\right)\right] \text{ and } u(L,t) = 0 \tag{8}$$

Having simulated using the proposed model, Figure 1 and 2 show the comparison of the numerical results with the analytical solution at t =7552.13 s. These excellent agreements suggest that the proposed scheme is accurate for tidal flow problems.



a) Comparison of water surface



b) Comparison of velocity

Figure 1: Tidal wave flow

In order to validate its performance on irregular bed, another simulation was done by taking the tabulated data in Table 1 as the bed profile of a channel. Replacing the $H(0)=16$, L=1500 m and $H(x)=H(0)-Z_b(x)$ while maintaining the previous initial and boundary conditions as (7) and (8), the tidal wave over an irregular bed is validated.

Table 1. Bed elevation, z_b at point x

x	0	50	100	150	250	300	350	400	425	435	450	475	500	505
$\boldsymbol{Z_b}$	0	0	2.5	5	5	3	5	5	7.5	8	9	9	9.1	9
x	530	550	565	575	600	650	700	750	800	820	900	950	1000	1500
$\boldsymbol{Z_b}$	9	6	5.5	5.5	5	4	3	3	2.3	2	1.2	0.4	0	0

Figure 2 shows a comparison between the predicted surface and the analytical solution at t = 10800 s. A comparison of velocities is depicted in Figure 3. Again, excellent agreement is obtained between the numerical and the analytical solutions. Hence confirms that the proposed scheme is also accurate for tidal flow over an irregular bed.

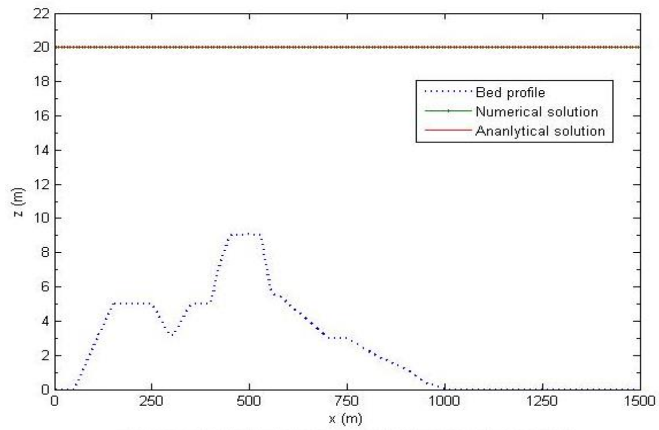


Figure 2: The irregular bed profile

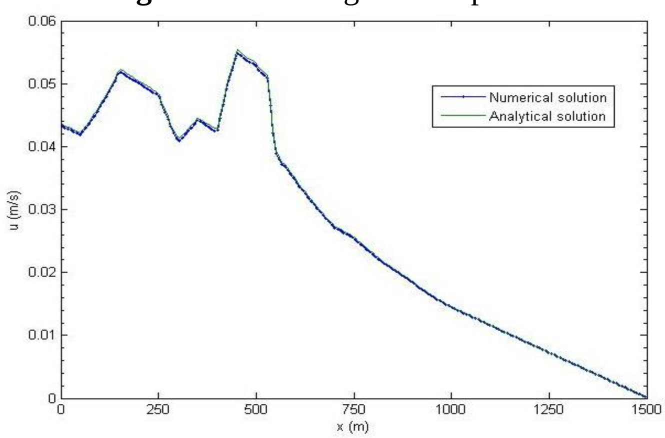


Figure 3: Comparison of velocity between numerical and analytical solutions at t=10800s

Flows along a horizontally and vertically contracted channel.

Along a 3 m horizontally and vertically contacted domain, the bottom topography is given by (9).

$$Z_b = \begin{cases} 0.1\cos^2[\pi(x-1.5)] & \text{for}\,|x-1.5|<0.5 \\ 0 & \text{otherwise} \end{cases} \qquad (9)$$

while the channel width varies as (10).

$$b(x) = \begin{cases} 1.0 - 0.1\cos^2[\pi(x-1.5)] & \text{for}\,|x-1.5|<0.5 \\ 1.0 & \text{otherwise} \end{cases} \qquad (10)$$

The first test case fixed the outflow at 1 m depth. When a unit-width discharge of q = 1.566 m^2/s is imposed at the upstream boundary, thus a subcritical flow is developed as in Figure 4. Simulation starts from q = 1.566 m^2/s and η = 1 m throughout the whole domain and run until the steady-state solution is reached. Figure 5 illustrates that the flow is critical at the narrowest section along the channel at x = 1.5 m as the inflow changed to q = 1.879 m^2/s. As the water continues flowing downstream, it turns into a supercritical condition.

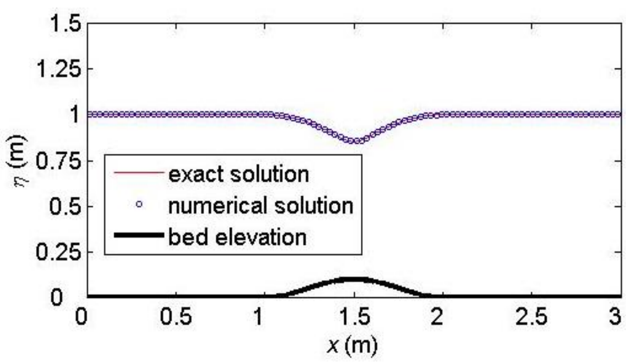


Figure 4: Subcritical flow

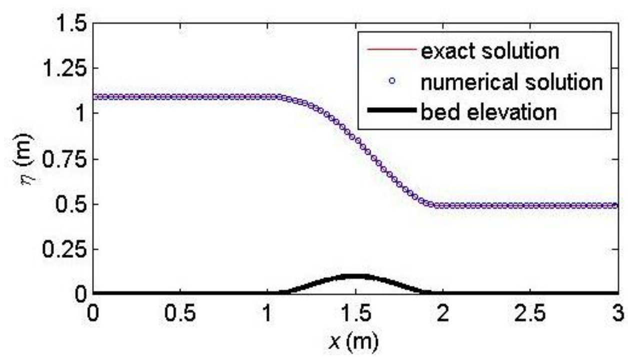


Figure 5: Transcritical flow

Figure 6 shows the supercritical flow developed when a greater amount of discharge is imposed at the upstream end. For all of the three cases, the numerical predictions match perfectly the analytical solutions.

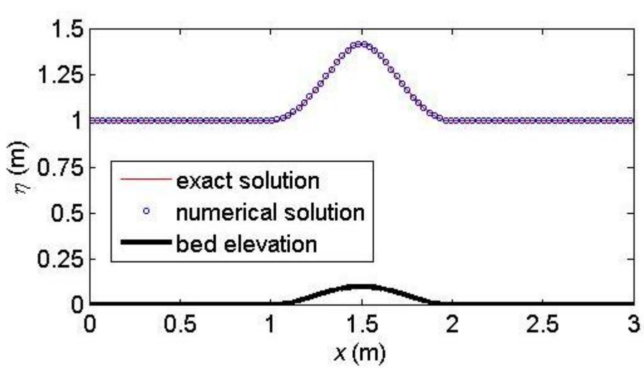


Figure 6: Supercritical flow

Fate of pollutant on a frictionless dry bottom

For the time being, in order to check the efficiency of the integrated model in dealing with the fate of pollutant, the proposed integrated model has been tested on a simple bench mark test. Hence, a laboratory test done by Concerted Action on Dam Break Modelling (CADAM) is chosen to be simulated by the proposed model. The results are compared with the analytical solutions.

The laboratory test CADAM takes place in a 2000m long by 200 m width channel. The upstream reservoir is separated with the downstream valley by a dam that is located 1000m away from the upstream end of the channel. The still water in the reservoir is polluted and the well-mixed solute has a concentration of 1. As there is no diffusion considered, the solute concentration is 1 wherever there is water and 0 over the dry bed and thus the analytical solution for q_c is the same as that of water depth in magnitude. The simulation is carried out on a uniform grid. Figure 7 presents the numerical water depth plotting against the analytical solution at t = 50s. The predicted depth-averaged velocity is shown in Figure 8 and agrees closely with the analytical solution in most of the domain. Figure 9 demonstrates the solute concentration, which matches perfectly the analytical solution. The results confirm the capability of the current numerical scheme on simulating the pollutant concentration.

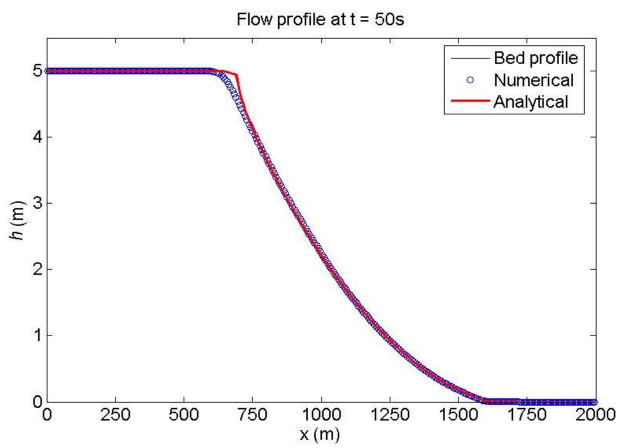


Figure 7: Flow profile along the 2000 m channel

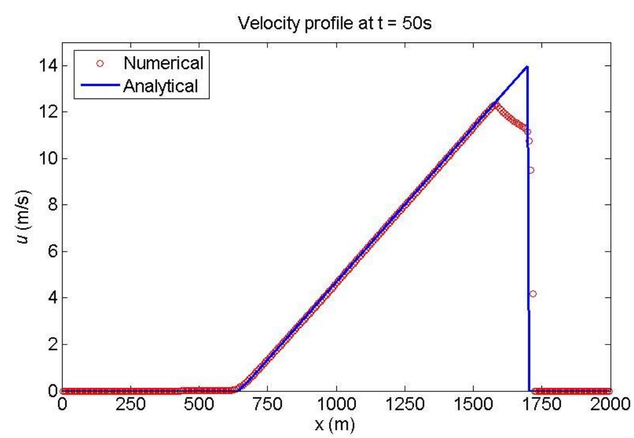


Figure 8: Velocity profile along the channel

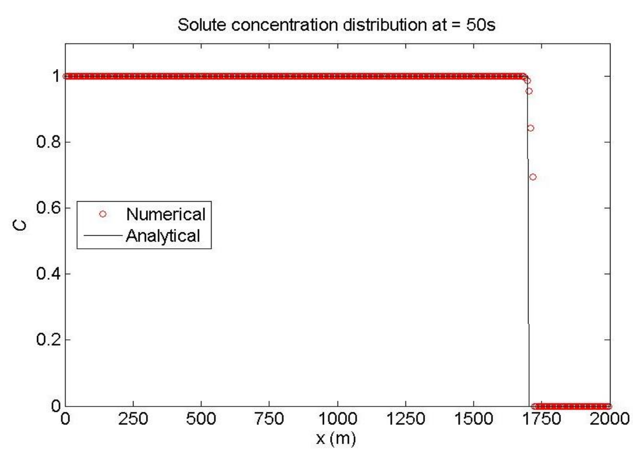


Figure 9: Solute concentration distribution

CONCLUSIONS AND RECOMMENDATIONS

Up to now, a 1D hydraulic model is developed by solving the governing St Venant equations with varying width using a Godunov-type finite-volume scheme in conjunction with the HLL approximate Riemann solver. HLL approximate Riemann solver is chosen to find the direct approximation of fluxes through the cell interfaces. The higher-order accuracy of the numerical approach is then achieved using a 2nd Order Runge-Kutta time integration method. The model has been validated against several benchmark tests of open channel flow, where the numerical predictions are compared with analytical solutions and experimental data available in literature. Close agreement has been achieved for all the tests being considered and this confirms the effectiveness of the current 1D code.

In the next stage, the 1D hydrodynamic model that integrated with the solute transport model will be used to simultaneously solving more test cases the. The integrated model later will be used to simulate an idealised real-world flood event. The specific contributions of these studies include:

- A fully 1D hydrodynamic model that can deal with the highly dynamic and complex flow hydrodynamics under certain flood conditions.
- An extended 1D model is expected to be capable to simulate the fate of pollutant simultaneously for high flood event in high dense areas.
- The schemes used are able to deal with flow over irregular topography and some other issues as discussed in literature.

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