Numerical Study of Free Convection Boundary Layer Flow on a Vertical Surface with Prescribed Wall Temperature, Heat Flux and Newtonian Heating Using Shooting Method

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Abstract – In this paper, the steady free convection boundary layer flow on a vertical surface with prescribed wall temperature (PWT) and prescribed heat flux (PHF) and for the case of Newtonian heating (NH), respectively, are investigated numerically via shooting method. The transformed boundary layer equations are solved numerically via shooting method. The transformed boundary layer equations are solved numerically, and numerical solutions are obtained for the skin friction coefficient, local heat transfer coefficient and wall temperature. The features of the flow and heat transfer characteristics for large range of values of the Prandtl number are analyzed and discussed. Comparisons with previous works by other methods for the cases of Pr = 0.08, 0.1, 0.72, 1, 7 and 10 show very good agreement.

INTRODUCTION

Free or natural convection flow is a flow in which no such externally induced flow exists and the flow arises “naturally” from the effect of a density difference, resulting from a temperature or concentration difference in a body force field such as gravity. The process is termed *natural* or *free* convection. The density difference gives rise to buoyancy forces due to which the flow is generated. A heated body cooling in ambient air generates such a flow in the region surrounding it. The buoyant flow arising from heat or material rejection to the atmosphere, heating and cooling of rooms and buildings, recirculating flow driven by temperature and salinity differences in oceans, and flows generated by fires are other examples of natural convection (Bejan and Kraus[3]).

The problem of obtaining similarity solutions for the free convection boundary layer equations governing the flow on a heated vertical plate was first considered by Sparrow and Gregg [4]. Merkin [1, 2, 5] used the similar technique to obtain similarity solutions for both the free convection boundary layer flow at low Prandtl number on a heated vertical plate with a prescribed power-law heating and the mixed convection at low Prandtl number on a vertical surface with a prescribed heat flux. Merkin [2] also extended this approach to the natural convection boundary layer flow on a vertical surface with Newtonian heating in which the heat transfer from the surface is proportional to the local surface temperature. Recently Salleh and Nazar [6, 7] studied the free convection boundary layer flow on a vertical surface with prescribed wall temperature, heat flux and Newtonain heating, respectively, and solved the problems numerically using Keller-box method.

Tong and Fu [8] derived the local condensate film thickness and the local Nusselt number using shooting method in order to investigate analytically into film wise condensation on a horizontal tube in a porous medium with suction at the tube surface. Anguir and Natesan [9] proposed a method for the numerical solution of singularly perturbed two-point boundary-value problems by combining the classical finite difference scheme, exponentially fitted difference, secant method and shooting method. Haran and White [10] declared that a commonly used numerical method for the solution of two-point boundary value problems in the chemical engineering is the shooting method, which implemented Maple to solve the problem numerically.

This present paper investigates numerically the steady free convection boundary layer flow on a vertical surface with PWT, PHF and NH, which was first considered by Merkin [1, 2] who obtained series solutions. The full boundary layer equations are derived and solved numerically using shooting method.

GOVERNING EQUATIONS

Consider a semi-infinite vertical flat surface with coordinates and measuring distances along and normal to the surface, respectively, with corresponding velocity components and . On employing the Boussinesq and boundary layer approximations as well as the non-dimensional transformation, the governing equations (non-dimensional) for the boundary layer flow on a vertical surface are (Merkin [1])

(1a)

(1b)

(1c)

subject to the boundary conditions

at ,

(a) (NH) or (2a)

(b) (PWT) or (2b)

(c) (PHF) (2c)

as , (2d)

where is a scaling factor, is the temperature difference, is the surface temperature, is the surface heat flux, *k* is the thermal conductivity and is the Prandtl number. The similarity transformation proceeds by selecting a stream function such that

and (3)

so that the continuity equation (1a) is automatically satisfied. To do this we put

, (4)

Using (4), equations (1b) and (1c) become

(5a)

(5b)

and the boundary conditions (2) become

at

(a) (NH) or (6a)

(b) (PWT) or (6b)

(c) (PHF) (6c)

as (6d)

The quantities of physical interest in this problem are the reduced skin friction coefficient, the heat transfer coefficient

and the wall temperature.

SHOOTING METHOD

Finite difference scheme

This paper presents the details of the finite difference scheme on free convection boundary layer flow over a

vertical surface with PWT, PHF and NH. We start by introducing the new dependent variables , , and so that equations (5) can be written as

(7a,b,c)

(7d)

(7e)

To start the integration of equations (7), the values of v_0 and s_0 are needed at the starting points. Therefore, the initial value is guessed as the result in Merkin [2]. With the value of the initial guess, equations (7a,b,c,d,e) are integrated from 0 to ? to find the solutions v_1 and s_1 using Euler's method. Since the initial values, v_0 and s_0, were used only as guesses, the final values of v and s at *?* = ? denoted as v(?,X) and s(?,Y), may not satisfy the boundary condition of equations (6a,b,c,d) and may overshoot or under shoot the required solution. Then, trial for another guess for v_0 and s_0 (by making suitable correction to the earlier values of v_0 and s_0) to be used as the initial condition of v and s. Since the approach to obtain v_0 and s_0 is same, then we only stated the procedure to get v_0. By defining the function, h(X), as

(8)

And expanding $h(X)$ in Taylor series about the current value of X gives

(9a)

Assuming the use of instead of X as the initial condition in equations (7) satisfies the final boundary condition on v, equations (9a) equals to zero by retaining only the first two terms:

(9b)

where (9c)

Noting that equation (8) gives

(9d)

Equation (9b) can be rewritten as

(10)

Thus the new guess value after initial condition on v_0 becomes

(11)

where X is the old guess value and is given by equation (11). The partial derivative, needed for the computation of in equation (10) can be obtained by partially differentiating the differential equations (7) as

(12a)

and

(12b)

where .

By defining the sensitivity functions

(13)

equations (12) can be expressed as

(14a)

(14b)

The initial conditions for g_1 and g_2 can be obtained from definition given in equations 6

(15a)

and

(15b)

Equations (7a,b,c,d,e) are to be solved (integrated from 0 to ?) as a system of simultaneous differential equations using the initial conditions of equations (6) and (15). After integrating these equations , the values of X and   can be determined from which the correction needed, , for the next iteration can be computed using equation (11).

The whole procedure is to be repeated the required number of times, each time using the new boundary condition until the difference satisfies the convergence criterion

(16)

where ? is a specified small number.

Euler's method

To linearize the nonlinear system of equations (7) using Euler's method, we introduce the following iterates:

(17a)

(17b)

(17c)

(17d)

(17e)

where

These calculations are repeated until some convergence criterion is satisfied and calculations are stopped when

(18)

where is a small prescribed value.

RESULTS AND DISCUSSION

Equations (5) and (6) subject to boundary conditions (7) and (8) are solved numerically using the Shooting method for the cases of PWT, PHF and NH, respectively. The Shooting method used in this study is programmed in Maple. Values of Pr considered are Pr = 0.08, 0.1, 0.72, 1, 7 and 10.

For validation of the numerical results obtained in this study using shooting method, we first compare the present results with the previously published results obtained by Merkin [1, 2] for various values of Pr. The values of or from equations (5) are given in Tables 1 and 2. We can see that the agreement between the present results and those of Merkin [1, 2] (using series solutions) and Salleh and Nazar [6, 7] (using Keller-box method) is very good. It is observed from Table 1 that for the cases of NH, PWT and PHF, as Pr increases, the reduced skin friction coefficient decreases. On the other hand, for the case of PWT, as Pr increases, the temperature gradient or the heat transfer coefficient also increases but for the case of PHF, as Pr increases, the wall temperature decreases. However, for the case of NH, as Pr increases, both the heat transfer coefficient and the wall temperature decrease.

Table 1: Comparison between the present results with previously published results
for the cases of PWT and PHF with various values of Pr

				(PWT) or (PHF)	
	Pr				
		Merkin	Salleh and	Merkin	Salleh and
		Nazar		Nazar	
		[1, 2]	[6, 7]	[1, 2]	[6, 7]
		present		present	
	0.08				
		1.0120	1.01196	0.2323	0.23235
PWT	0.1				
	0.72	0.9925	0.99243	0.2547	0.25470
	1	0.7791	0.7791	0.5332	0.5332
	7	0.77905		0.53315	
	10	0.7395	0.7395	0.5951	0.5951
		0.73950		0.59509	
		0.5157	0.5157	1.0771	1.0772
		0.51571		1.07710	
		0.4790	0.47896	1.1915	1.19136
	0.08				
		2.4294	2.42929	3.2143	3.21434
PHF	0.1	2.2548	2.25475		
	0.72	1.1362	1.1362	2.9868	2.98676
		1.13620			
	1	1.0097	1.0097	1.6539	1.6539
	7	1.00969		1.65393	
	10	0.4932	0.4932	1.5148	1.5147
		0.49323		1.51472	
		0.4312	0.43120	0.9423	0.9423
				0.94231	
				0.8692	0.86929

Table 2: Comparison between the present results with previously published results
for the case of NH with various values of Pr

				(NH)	
	Pr				
		Merkin	Salleh and	Merkin	Salleh and

		Nazar		Nazar	
		[2]	[7]	[2]	[7]
		present		present	
	0.1				
		60.0770	60.07061	237.5770	237.65616
NH	0.7	5.1405	5.1402	12.3760	12.3744
	2	5.14061		12.37662	
	1	3.5088	3.50911	7.9725	7.97409
	7			0.7430	0.7429
	10	0.4127	0.4127	0.74295	
		0.41269		0.4964	0.49639
		0.2833	0.28325		

CONCLUSIONS

The steady free convection boundary layer flow on a vertical surface with PWT, PHF and NH, is investigated in this present study. The transformed boundary layer equations are solved numerically using Shooting method. The numerical results show that the previously published results and the present results using shooting method are in good agreement and thus, it can be concluded that the shooting method is an efficient method and it is easily adaptable to solve other boundary layer equations.

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