

Numerical Study of Free Convection Boundary Layer Flow on a Vertical Surface with Prescribed Wall Temperature, Heat Flux and Newtonian Heating Using Shooting Method

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Abstract – In this paper, the steady free convection boundary layer flow on a vertical surface with prescribed wall temperature (PWT) and prescribed heat flux (PHF) and for the case of Newtonian heating (NH), respectively, are investigated numerically via shooting method. The transformed boundary layer equations are solved numerically via shooting method. The transformed boundary layer equations are solved numerically, and numerical solutions are obtained for the skin friction coefficient, local heat transfer coefficient and wall temperature. The features of the flow and heat transfer characteristics for large range of values of the Prandtl number are analyzed and discussed. Comparisons with previous works by other methods for the cases of $Pr = 0.08, 0.1, 0.72, 1, 7$ and 10 show very good agreement.

INTRODUCTION

Free or natural convection flow is a flow in which no such externally induced flow exists and the flow arises “naturally” from the effect of a density difference, resulting from a temperature or concentration difference in a body force field such as gravity. The process is termed *natural* or *free* convection. The density difference gives rise to buoyancy forces due to which the flow is generated. A heated body cooling in ambient air generates such a flow in the region surrounding it. The buoyant flow arising from heat or material rejection to the atmosphere, heating and cooling of rooms and buildings, recirculating flow driven by temperature and salinity differences in oceans, and flows generated by fires are other examples of natural convection (Bejan and Kraus[3]).

The problem of obtaining similarity solutions for the free convection boundary layer equations governing the flow on a heated vertical plate was first considered by Sparrow and Gregg [4]. Merkin [1, 2, 5] used the similar technique to obtain similarity solutions for both the free convection boundary layer flow at low Prandtl number on a heated vertical plate with a prescribed power-law heating and the mixed convection at low Prandtl number on a vertical surface with a prescribed heat flux. Merkin [2] also extended this approach to the natural convection boundary layer flow on a vertical surface with Newtonian heating in which the heat transfer from the surface is proportional to the local surface temperature. Recently Salleh and Nazar [6, 7] studied the free convection boundary layer flow on a vertical surface with prescribed wall temperature, heat flux and Newtonian heating, respectively, and solved the problems numerically using Keller-box method.

Tong and Fu [8] derived the local condensate film thickness and the local Nusselt number using shooting method in order to investigate analytically into film wise condensation on a horizontal tube in a porous medium with suction at the tube surface. Anguir and Natesan [9] proposed a method for the numerical solution of singularly perturbed two-point boundary-value problems by combining the classical finite difference scheme, exponentially fitted difference, secant method and shooting method. Haran and White [10] declared that a commonly used numerical method for the solution of two-point boundary value problems in the chemical engineering is the shooting method, which implemented Maple to solve the problem numerically.

This present paper investigates numerically the steady free convection boundary layer flow on a vertical surface with PWT, PHF and NH, which was first considered by Merkin [1, 2] who obtained series solutions. The full boundary layer equations are derived and solved numerically using shooting method.

GOVERNING EQUATIONS

Consider a semi-infinite vertical flat surface with coordinates x and y measuring distances along and normal to the surface, respectively, with corresponding velocity components u and v . On employing the Boussinesq and boundary layer approximations as well as the non-dimensional transformation, the governing equations (non-dimensional) for the boundary layer flow on a vertical surface are (Merkin [1])

$$\square$$

(1a)



(1b)



(1c)

subject to the boundary conditions



at ,



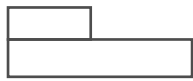
(a) (NH) or (2a)



(b) (PWT) or (2b)



(c) (PHF) (2c)



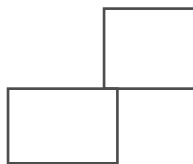
as , (2d)



where is a scaling factor, is the temperature difference, is the surface temperature, is the surface heat flux, k is the thermal conductivity and is the Prandtl number. The similarity transformation proceeds by selecting a stream function



such that



and (3)

so that the continuity equation (1a) is automatically satisfied. To do this we put



, (4)

Using (4), equations (1b) and (1c) become

$$\boxed{\phantom{\text{Equation (5a)}}}$$

(5a)

$$\boxed{\phantom{\text{Equation (5b)}}}$$

(5b)

and the boundary conditions (2) become

$$\boxed{\phantom{\text{Equation (6a)}}}$$

at

$$\boxed{\phantom{\text{Equation (6a)}}}$$

(a) (NH) or

(6a)

$$\boxed{\phantom{\text{Equation (6b)}}}$$

(b) (PWT) or

(6b)

$$\boxed{\phantom{\text{Equation (6c)}}}$$

(c) (PHF)

(6c)

$$\boxed{\phantom{\text{Equation (6d)}}}$$

as

(6d)

$$\boxed{\phantom{\text{Equation (6d)}}}$$

The quantities of physical interest in this problem are the reduced skin friction coefficient, the heat transfer coefficient

$$\boxed{\phantom{\text{Equation (6d)}}}$$

and the wall temperature.

SHOOTING METHOD

Finite difference scheme

This paper presents the details of the finite difference scheme on free convection boundary layer flow over a

$$\boxed{\phantom{\text{Equation (7a,b,c)}}}$$

vertical surface with PWT, PHF and NH. We start by introducing the new dependent variables η , θ , and ψ so that equations (5) can be written as

$$\boxed{\phantom{\text{Equation (7a,b,c)}}}$$

(7a,b,c)

$$\boxed{\phantom{\text{Equation (7d)}}}$$

(7d)

determined from which the correction needed, δ , for the next iteration can be computed using equation (11).

The whole procedure is to be repeated the required number of times, each time using the new boundary condition until the difference satisfies the convergence criterion

(16)

where ϵ is a specified small number.

Euler's method

To linearize the nonlinear system of equations (7) using Euler's method, we introduce the following iterates:

(17a)

(17b)

(17c)

(17d)

(17e)

where

These calculations are repeated until some convergence criterion is satisfied and calculations are stopped when

(18)

ϵ

where ϵ is a small prescribed value.

RESULTS AND DISCUSSION

Equations (5) and (6) subject to boundary conditions (7) and (8) are solved numerically using the Shooting method for the cases of PWT, PHF and NH, respectively. The Shooting method used in this study is programmed in Maple. Values of Pr considered are Pr = 0.08, 0.1, 0.72, 1, 7 and 10.

For validation of the numerical results obtained in this study using shooting method, we first compare the

θ
 θ'

present results with the previously published results obtained by Merkin [1, 2] for various values of Pr. The values of

θ

or from equations (5) are given in Tables 1 and 2. We can see that the agreement between the present results and those of Merkin [1, 2] (using series solutions) and Salleh and Nazar [6, 7] (using Keller-box method) is very good. It is observed from Table 1 that for the cases of NH, PWT and PHF, as Pr increases, the reduced skin friction coefficient

$C_{f,w}$

decreases. On the other hand, for the case of PWT, as Pr increases, the temperature gradient or the heat transfer

θ'

θ''

coefficient also increases but for the case of PHF, as Pr increases, the wall temperature decreases. However, for the case of NH, as Pr increases, both the heat transfer coefficient and the wall temperature decrease.

Table 1: Comparison between the present results with previously published results for the cases of PWT and PHF with various values of Pr

	Pr	(PWT) or (PHF)			
		Merkin Nazar [1, 2] present	Salleh and [6, 7]	Merkin Nazar [1, 2] present	Salleh and [6, 7]
PWT	0.08	1.0120	1.01196	0.2323	0.23235
	0.1	0.9925	0.99243	0.2547	0.25470
	0.72	0.7791	0.7791	0.5332	0.5332
	1	0.77905	0.7791	0.53315	0.5332
	7	0.7395	0.7395	0.5951	0.5951
	10	0.73950	0.7395	0.59509	0.5951
		0.5157	0.5157	1.0771	1.0772
		0.51571	0.5157	1.07710	1.0772
		0.4790	0.47896	1.1915	1.19136
		0.4790	0.47896	1.1915	1.19136
PHF	0.08	2.4294	2.42929	3.2143	3.21434
	0.1	2.2548	2.25475	2.9868	2.98676
	0.72	1.1362	1.1362	2.9868	2.98676
		1.13620	1.1362	2.9868	2.98676
	1	1.0097	1.0097	1.6539	1.6539
	7	1.00969	1.0097	1.65393	1.6539
	10	0.4932	0.4932	1.5148	1.5147
		0.49323	0.4932	1.51472	1.5147
		0.4312	0.43120	0.9423	0.9423
		0.4312	0.43120	0.94231	0.9423
			0.8692	0.86929	

Table 2: Comparison between the present results with previously published results for the case of NH with various values of Pr

	Pr	(NH)			
		Merkin	Salleh and	Merkin	Salleh and

		Nazar	[7]	Nazar	[7]
		[2]		[2]	
		present		present	
	0.1				
		60.0770	60.07061	237.5770	237.65616
NH	0.7	5.1405	5.1402	12.3760	12.3744
	2	5.14061		12.37662	
	1	3.5088	3.50911	7.9725	7.97409
	7			0.7430	0.7429
	10	0.4127	0.4127	0.74295	
		0.41269		0.4964	0.49639
		0.2833	0.28325		

CONCLUSIONS

The steady free convection boundary layer flow on a vertical surface with PWT, PHF and NH, is investigated in this present study. The transformed boundary layer equations are solved numerically using Shooting method. The numerical results show that the previously published results and the present results using shooting method are in good agreement and thus, it can be concluded that the shooting method is an efficient method and it is easily adaptable to solve other boundary layer equations.

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