

PERPUSTAKAAN UMP



0000092773

ANALYSIS OF BUCKLING BEHAVIOUR IN WEB GIRDER.

NORSHAMIERA AFZAN BINTI ISMAIL

Thesis submitted in fulfillment of the requirements for the award of the degree of
B.ENG (HONS.) CIVIL ENGINEERING

Faculty of Civil Engineering and Earth Resources
UNIVERSITI MALAYSIA PAHANG

JULY 2014

ABSTRACT

Normally, an economical and effective design of web plate girder requires thin webs. However, thin and very slender web plate girder will cause it to buckle. Corrugated web girder or Trapezoidal Web Plate (TWP) is used to overcome this situation. This project aims to produce a finite element model and studying the web profile of the beam by using LUSAS Modeller 14.0. There were three model produced to be analyse in this project. The dimension of each model are 1500mm x 200mm x 900mm. Each model is simply supported with fixed support and pinned support. The type of mesh used for each model is Quadrilateral Thin Shell with 8 nodes (QSL8). LUSAS Modeller 14.0 finite element software is able to carry out linear analysis to determine deformed mesh, maximum stress and strain under axial load. Besides, critical buckling load can be predicted under the linear buckling analysis. The results show that the TWP2 has highest strength compared to TWP1 and Flat Web beam. The results also show that TWP of web can sustain higher buckling load compared to Flat Web girder. Error! Bookmark not defined.

ABSTRAK

Biasanya, suatu reka bentuk web plat galang yang ekonomi dan berkesan memerlukan web nipis. Walau bagaimanapun, web plat galang yang nipis dan sangat langsing akan menyebabkan ia rebah. Galang web beralun atau Trapezoid Plate Web (TWP) digunakan untuk mengatasi keadaan ini. Projek ini bertujuan untuk menghasilkan satu model unsur terhingga dan mengkaji profil web rasuk dengan menggunakan pereka bentuk model LUSAS 14.0. Terdapat tiga model yang dihasilkan untuk dianalisis dalam projek ini. Dimensi setiap model adalah 1500mm x 200mm x 900mm. Setiap model yang disokong dengan sokongan tetap dan sokongan disematkan. Jenis mesh digunakan untuk setiap model yang Quadrilateral nipis Shell dengan 8 nod (QSL8). LUSAS 14.0 pereka bentuk model perisian unsur terhingga dapat menjalankan analisis linear untuk menentukan jaringan cacat, tegasan maksimum dan terikan di bawah beban paksi. Selain itu, beban lengkukan kritikal boleh diramalkan di bawah analisis lengkukan linear. Keputusan menunjukkan bahawa TWP2 mempunyai kekuatan paling tinggi berbanding dengan TWP1 dan Web rasuk datar. Keputusan juga menunjukkan bahawa TWP web boleh menampung beban lengkukan yang lebih tinggi berbanding dengan Web rasuk datar.

TABLE OF CONTENTS

	PAGE
SUPERVISOR'S DECLARATION	ii
STUDENT'S DECLARATION	iii
DEDICATION	iv
ACKNOWLEDGEMENTS	v
ABSTRACT	vi
ABSTRAK	vii
TABLE OF CONTENTS	viii
LIST OF TABLES	xi
LIST OF FIGURES	xii
LIST OF SYMBOLS	xiv
LIST OF ABBREVIATIONS	xv
CHAPTER 1 INTRODUCTION	
1.1 Introduction	1
1.2 Problem Statement	2
1.3 Objectives Of Study	3
1.4 Scope Of Study	3
1.5 Significant Of Study	4
CHAPTER 2 LITERATURE REVIEW	
2.1 Introduction	5
2.2 Advantages Of Flat Plate Girder	7
2.3 Advantages Of Trapezoidal Web Plate Girder	7
2.4 Mode Of Failure In Corrugated Web Girder	9
2.4.1 Local Buckling	10
2.4.2 Global Buckling	12
2.5 Analysis Of Buckling Behaviour In Web Girder	14

CHAPTER 3 METHODOLOGY

3.1	Introduction	16
3.2	Flow Chart	17
	3.2.1 Explanation of Flow Chart	17
3.3	Introduction To LUSAS	18
3.4	Finite Element Analysis	18
	3.4.1 Linear Analysis	19
	3.4.2 Linear Buckling Analysis	20
3.5	Finite Element Idealization	20
	3.5.1 Profile of Beam Component	21
	3.5.2 Mesh	22
	3.5.3 Geometry	24
	3.5.4 Material	25
	3.5.5 Boundary And Support Condition	26
	3.5.5.1 Support	26
	3.5.5.2 Loading	27

CHAPTER 4 RESULTS AND DISCUSSION

4.1	Introduction	28
4.2	Finite Element Models	29
	4.2.1 Flat Web	29
	4.2.2 Trapezoidal Web 1	30
	4.2.3 Trapezoidal Web 2	31
4.3	Displacement	32
	4.3.1 Displacement Of Flat Web	32
	4.3.2 Displacement Of Trapezoidal Web 1	33
	4.3.4 Displacement Of Trapezoidal Web 2	34
4.4	Maximum Stress	35
	4.4.1 Maximum Stress Of Flat Web	35
	4.4.2 Maximum Stress Of Trapezoidal Web 1	36
	4.4.3 Maximum Stress Of Trapezoidal Web 2	37
4.5	Maximum Strain	38
	4.5.1 Maximum Strain Of Flat Web	38
	4.5.2 Maximum Strain Of Trapezoidal Web 1	39
	4.5.3 Maximum Strain Of Trapezoidal Web 2	40
4.6	Buckling Behaviour	40

4.6.1	Buckling Behaviour Mode Of Flat Web Beam	41
4.6.2	Buckling Behaviour Mode Of Trapezoidal Web Plate 1	43
4.5.3	Buckling Behaviour Mode Of Trapezoidal Web Plate 2	45

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1	Introduction	50
5.2	Conclusion	50
5.3	Recommendation	51

REFERENCES

53

APPENDICES

LIST OF TABLES

Table No.	Title	Page
2.1	Lightweight fabrication by corrugated web I-beam (Hamada,1984)	6
3.1	The dimension properties of TWP models	22
3.2	Properties of material	26
4.1	Buckling load of flat web beam	43
4.2	Buckling load of trapezoidal web plate 1	45
4.3	Buckling load of trapezoidal web plate 2	47
4.4	Comparison between flat web, TWP1 and TWP2	48

LIST OF FIGURES

Figure No.	Title	Page
2.1	Local buckling mode	9
2.2	Global buckling mode	9
2.3	Interactive buckling mode	9
3.1	Flow chart of methodology	17
3.2	3D view of finite element model	21
3.3	Attribute< Mesh< Surface< Surface Mesh dataset	22
3.4	Thin shell element (QSL8)	23
3.5	Attribute< Geometry< Geometry surface	24
3.6	Attribute< Material< Material library< Material library dataset	25
3.7	Attribute< Support< Structural support dataset	26
3.8	Attribute< Loading< Global distributed	27
4.1	Finite Element Of Flat Web Beam	29
4.2	Finite Element Of Trapezoidal Web Plate Beam 1	30
4.3	Finite Element Of Trapezoidal Web Plate 2	31
4.4	Displacement Of Flat Web Plate	32
4.5	Displacement Of Trapezoidal Web Plate 1	33
4.6	Displacement Of Trapezoidal Web Plate 2	34
4.7	Maximum Stress Of Flat Web Plate	35
4.8	Maximum Stress Of Trapezoidal Web Plate 1	36
4.9	Maximum Stress Of Trapezoidal Web Plate 2	37
4.10	Maximum Strain Of Flate Web Plate	38
4.11	Maximum Strain Of Trapezoidal Web Plate 1	39
4.12	Maximum Strain Of Trapezoidal Web Plate 2	40
4.13	Buckling Behaviour 1 Of Flat Web Plate	41

4.14	Buckling Behaviour 2 Of Flate Web Plate	41
4.15	Buckling Behaviour 3 Of Flate Web Plate	42
4.16	Buckling Behaviour 1 Of Trapezoidal Web Plate 1	43
4.17	Buckling Behaviour 2 Of Trapezoidal Web Plate 1	43
4.18	Buckling Behaviour 3 Of Trapezoidal Web Plate 1	44
4.19	Buckling Behaviour 1 Of Trapezoidal Web Plate 2	45
4.20	Buckling Behaviour 2 Of Trapezoidal Web Plate 2	45
4.21	Buckling Behaviour 3 Of Trapezoidal Web Plate 2	46
4.22	Pie Chart of Maximum Stress of Models	47
4.23	Pie Chart of Maximum Strain of Models	47
4.24	Pie Chart of Average Buckling Load	48

LIST OF SYMBOLS

L	Length/ Span of Beam
B	Width of Flange
D	Depth of Beam
t_w	Thickness of Web
h_r	Corrugation Depth
θ	Corrugation Angle
E	Young Modulus
ν	Poisson Ratio
σ_{\max}	Maximum Stress
ϵ_{\max}	Maximum Strain
kN	Force

LIST OF ABBREVIATION

LUSAS	London University Structural Analysis Software
TWP	Trapezoidal Web Plate
QSL8	Quadrilateral Thin Shell Element with 8 Nodes Clockwise

CHAPTER 1

INTRODUCTION

1.1 Introduction

In the past few decades, corrugated web girder or also called as Trapezoidal Web Plate (TWP) had represented as a new structural system. The corrugated girders have a corrugated shape profile, with aligned parts in two parallel planes in the longitudinal direction and tilted parts between them. The design of girder and beam requires thin webs if it want to be an economical design. However, the plate buckling problem may arise due to extremely slender of web. It is obvious that the usage of thicker plates or adding web stiffener will increase the cost of fabrication. As the increasing usage of corrugated web in the industry, it is shows that the corrugated web is success in replacing steel plate web. Corrugated web girder has a potential to replace the costly web stiffness.

The corrugated steel plates have their own characteristics. Be ignored the bending capacity and sufficient out of plane stiffness are the primary characteristics of the corrugated steel plates. This is because it has no stiffness perpendicular to the direction of corrugation. Lateral torsional buckling and local buckling of compression

flange are the failure modes that might occur for steel beam with corrugated web (Elgaaly et al. 1997).

Generally, trapezoidal profile is commonly used corrugation profile for corrugated web plates. The girder with the trapezoidal corrugated web has a higher load-carrying capacity compared to plate girders with the stiffened flat web. The girder with trapezoidal corrugated web also has small deflection compared with girder with the stiffened flat web. (Abdullah Tohamy et al. 2013).

Finite element analysis had widely used by the engineer to do an analysis and solve the structural problems or any engineering problems. The finite element model were applied to analyzed the maximum moment of set off lateral buckling of corrugated beam.

Previous research had shown that trapezoidal web can support a bigger loading than common plate girder of the same dimension and spacing.

1.2 Problem Statement

Studies on buckling resistance of web girder had been carrying out by previous researchers. Research on corrugated webs was started by Elgaaly (1989). Yi et al. (2008, p.13) states that shear buckling behavior depends on geometric characteristics of corrugated web. Research that had been done by Elgaaly (1996) found that the mechanism of failure of girder which under different loading modes, called shear mode, bending mode and compression patch load (Elgaaly et. al. 1996). Previous research done by Robert et al. (2006) found that the corrugated web had failed in shear by instability, and local and global buckling modes.

In heavy construction, girders are used for very long spans. Corrugated web girders have been recognized as effective load carrying members. The plate web girder has to use thick web plate when construct. The thickness of corrugated web is lesser than plate web when construct. Thus, corrugated web girder replaces the plate web girder in order to be competitive and cost effective. To better understand the buckling behavior of web girder, an analysis must be done. Besides that, the study of the strength of corrugated web girder also can be analyzed.

For corrugated web, analysis can be done to prove the strength of corrugated web girder is more than plate web girder. It is expected that corrugated web girder provides high strength and stiffer than plate web girder.

1.3 Objectives Of Study

The objectives of this analysis are:

- i. To investigate the buckling behavior of web girder.
- ii. To identify the load carrying capacity in web girder.
- iii. To compare the buckling behavior of corrugated web girder and flat web girder.

1.4 Scope Of Study

The scope of study of this research is to do an analysis on simply supported web girders subjected to shear load. This analysis will be conducted by using software which is London University Stress Analysis System, (LUSAS). The scope of this research is to study different type of corrugated shape and compare with the flat plate girder.

1.5 Significant Of Study

Generally, the usage of corrugated web girder is not widely used in Malaysia compared to flat web girder. This is because trapezoidal web shape is complicated shape which require it to use the art machine to design it. Therefore, the production of corrugated web is quite expensive. Using finite element analysis, time and cost of products can be saved compared to laboratory test. The significance of this study is to analyze the performance of the corrugated web girder. This analysis can anticipate the shear capacity and strength capacity of the corrugated web girder and plate web girder. Thus, the shear capacity and shear capacity of the web girder can be compared.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Girders are supported beam used in construction. Plate girders are used in bridges and buildings where heavy loads and large spans are required. Normally, the plate girders are designed by welding together to form an I-section. Generally, plate girders are required for the span above 15 metres and recently numerous plate girders which span 60 to 1000 metres have been constructed. Reinforcing the web may use the stiffeners. (Clarke et.al, 1987). It have been notice that using stiffened thin plate in girder have higher manufacturing cost and possibility reducing life span due to fatigue cracking which may begin at the welded connection between the stiffeners and flange.

Using stiffened thin plates in girders may cause to some disadvantages. It have been noticed that high fabrication cost and life expectancy may be reduced due to fatigue cracks that start at a welded connection between the stiffeners and the flanges. Just recently, the uses of fairly thin corrugated webs have been possible due to advanced welding technology. The corrugations in the beam act as transverse stiffeners that allow the use of a thin plate with a significant weight reduction. Nevertheless, the high

slenderness ratio of corrugated web plate leads to primary concerned for the stability in shear buckling. (H.W, Mok 2007)

Research has shown that when the girders with corrugated webs are compared with stiffened flat webs, the trapezoidal corrugation in the web permit the use of thinner webs and reduce the cost of using expensive web stiffeners. It was found that corrugated beams have 9% to 13% less weight than current traditional stiffened girders with flat webs. (Hamada,1984).

Table 2.1: Lightweight fabrication by corrugated web I-beam, (Hamada,1984)

Welded I-beam depth, web width, web thickness, flange thickness (mm)	Corrugated web I-beam (Corrugation width) (mm)	Section modulus ratio per unit width (corrugated web I-beam)
H200 x 100x 3.2 x 4.5	200 x 100 x 1.6 x 2.5 (150)	1.09
H250 x 125 x 4.5 x 6.0	250 x 125 x 2.0 x 6.0 (180)	1.13
H300 x 150 x 4.5 x 6.0	300 x 150 x 2.3 x 6.0 (220)	1.10
H400 x 200 x 4.0 x 12.0	400 x 150 x 2.7 x 12.0(300)	1.09

It is found that corrugated plates had increased in its application as girder webs. The corrugated web only required stiffening on its supports. Local buckling and global buckling must be counted in order to obtain shear capacity. (Lindner et.al, 1991).

Preliminary study about the profiled web is more on the vertically trapezoidal corrugation. Khalid et. al., in his study said that there are three failure mechanism found in the profiled web beam which are shear mode, bending mode and compressive patch load. Patch load failure relies on the position of loading and the corrugation's parameters. (Khalid et. al., 2004). In other research by Li et. Al., which is experimental testing, it is found that the corrugated profiled web girder has higher buckling resistance compared to conventional web type.

2.2 Advantages Of Flat Plate Girder

A plate girder has an I-type cross section. Rather than being hot-rolled, however the girder is constructed from steel plate elements which are connected together with welds, bolts, or rivets. A greater economy of material can be obtained as the designer has the ability to specify the section properties of the stringer to accommodate the local forces. The development of highly automated workshop in recent years has reduced the fabrication costs of plate girders very considerably. Plate girders are aesthetically more pleasing than trusses and are easier to transport and erect (Bashar S.). However, due to features of corrugation, the application of corrugated web leads to more advantageous for bridge structures.

2.3 Advantages Of Trapezoidal Web Girder

Many researchers had implemented the study to identify the shear capacity of corrugated web. The application of corrugated web leads to many advantages for composite bridge structures due to the features of corrugation. Other advantages of using corrugated steel web are prestressing force remain in concrete chords due to small stiffness in longitudinal direction of the web. (Kovesdi et. al., 2010).

A lot of researches that have been performed brings to the improvement of the structure by using new types of profiled girders. For example, the research that has been done in Japan in 1965 until 1986 brings to the first construction of Shinkai bridges using the corrugated web, in 1993. In 1980's the corrugated web bridge developed in France replaces the web of conventional pre-stressed concrete bridges. Basically, replacing the concrete webs with corrugated steel plates can reduce the self weight of the main girder, improved efficiency of the prestress and improved shear resistance.

Previous research shows that corrugated web girder has less deflection than that of the equivalent in weight stiffened flat web girder. Since bridges represent the main subject for the use of girders due to the noticeable reduction in material and labor costs, then a testing and an analysis should be more suitable to evaluate the behavior of corrugated web girders. It is shown that vertical corrugated web gave a sturdy support against the flange buckling, compared to horizontally corrugated web and flat web girders. Recently, a finite element analysis had been done to investigate the behavior of beam with trapezoid web steel section (Fatimah et. al.,2010). Finite elements were used to model beams with flat web, horizontally and vertically corrugated webs. The horizontally corrugated web more weak to support against the flange buckling, compared to girder with vertical corrugated web and flat web. Earlier research by Elgaaly focused on the failure mode of corrugated beams under some loading modes which are bending mode, shear mode and compressive patch loads. Elgaaly found that the failure of the corrugated webs under bending mode and shear mode was sudden and due to buckling of the web (Elgaaly et. al.,1997).

The shear strength of corrugated web girder is primarily a function of the web height and thickness, the corrugation geometry, and material properties, although initial web geometric imperfections may also play significant role. The corrugations gives solidity to the web, removing the need for the transverse stiffeners that have a primary impact on the shear strength of conventionally stiffened flat web plate girders (Robert G. et. al.,2006)

According to Sayed-Ahmed, the flexural strength of a steel girder with corrugated web plate is provided by the flanges with almost no contribution from the web and with no interaction between flexure and shear behavior. The corrugated web only provides the shear capacity of girders where the shear strength is controlled by buckling or steel yielding of the web. The flanges provide boundary supports for the web which lie somewhere between a simply supported boundary and a clamped one. (Sayed-Ahmed,2007)

2.4 Mode of Failure in Corrugated Web Girder.

It is to be found that corrugated web girder had failed in shear with three failure mode, local buckling, global buckling and interactive buckling mode. Local buckling represents the buckling in the sub panel, while global buckling represents the buckling in the whole web. Interactive buckling which involved a few sub panels happened because of the interaction between local and global buckling (Yi et. al., 2008).

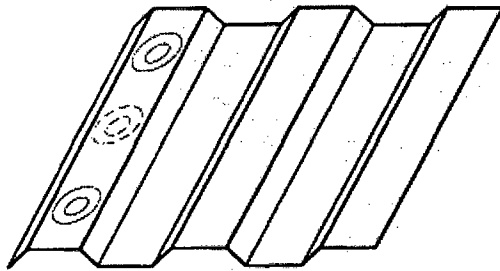


Figure 2.1: Local Buckling Mode

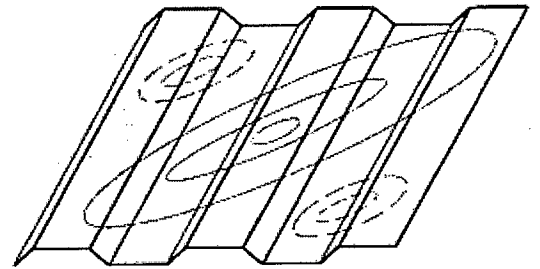


Figure 2.2: Global Buckling Mode

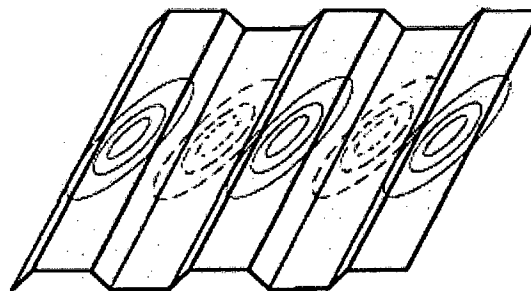


Figure 2.3: Interactive Buckling Mode

Normally, several types of local buckling begin the buckling process. Nevertheless, a local buckling mode, which started the buckling can reduce shear post-buckling capacity. Furthermore, in the post-buckling stages, local buckling, either directly develop and transform to global buckling mode, causing failure or starting from a sub-panel to another, the first buckling mode of the zone and then change to a so-called tension field over the entire depth of the girder. (H. W. Mok, 2007).

The corrugated steel web plate failure occurs by the steel yielding of the web under a pure shear stress state. It also can occur by web buckling due to either local instability of any panel between two folds or overall instability of the web over two or more panels. Another possibility of failure mode that could be occurring between these different failure criteria is an interactive failure mode. Another criterion which affects the design strength of girders is local buckling of compression flange with corrugated web. Usually, local buckling of the compression flange of an I-section mainly depends on the flange out-stand-to thickness ratio. Limits are placed on this ratio such that the critical stress initiating local flange buckling will not be reached before reaching the yield stress (Sayed-Ahmed, 2007).

2.4.1 Local Buckling.

This is review on local buckling stress that had been obtained from many observations and from some experimental result and theoretical on corrugated web girder. Previous researcher, (Driver et. al. 2006), had conducted a numerical and testing analysis on corrugated webs under shear. It was found that the specimens failed in local buckling mode. In the local buckling mode, the trapezoidal webs act as a series of flat sub-panels that supported each other along their vertical and are supported by flanges at horizontal edges. It is investigated that local buckling are using the elastic shear stress written as:

$$\tau_{cr,l} = k_l \frac{\pi^2.E}{12.(1-\nu^2)} \cdot \left(\frac{w}{t_w}\right)^2 \quad (2.1)$$

Where:

E = elastic modulus

ν = Poisson's ratio

k_l = local shear buckling coefficient

t_w = thickness of web

w = maximum fold width.

In this case, k_1 lies between 5.34 (by assume the fold has simply supported edge) and 8.98 (by assume fixed edges).

An analysis on trapezoidal webs under shear is continued to study the buckling mode were performed by Elgaaly and Hamilton (1996). It is found that in the local buckling mode, the corrugated web acts as series of flat panels that support each other along their vertical (longer) edges and the panels supported by the flanges along their horizontal (shorter) edges. The elastic buckling stress given by:

$$\tau_{cr,l} = k_s \frac{\pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{w}{t}\right)^2 \quad (2.2)$$

Where:

E = elastic modulus

μ = Poisson's ratio

k_s = buckling coefficient

t = thickness of web,

w = flat panel width

k_s is given by:

$$k_s = 5.34 + 2.31 \left(\frac{w}{h}\right) - 3.44 \left(\frac{w}{h}\right)^2 + 8.39 \left(\frac{w}{h}\right)^3, \quad (2.3)$$

for the simply supported, longer edges and short edges clamped.

$$k_s = 8.98 + 5.6 \left(\frac{w}{h}\right)^2, \quad (2.4)$$

for the all edges are clamped.

In case $\tau_{cre} > 0.8\tau_y$, inelastic buckling will happen and inelastic stress can be calculated by:

$$\tau_{cri} = (0.8 \times \tau_{cre} \times \tau_y)^{0.5} \leq \tau_y. \quad (2.5)$$

From the previous experimental test done by Elgaaly, a conclusion was made that if corrugation of the web is coarse, the buckling of the panel is classified by local buckling of the flat folds if it is corrugation. The buckling stress formula for flat plate can be used to calculate the strength of trapezoid webs with a very good degree of accuracy. The finite element model was used in the numerical study that held by Elgaaly. This finite element model used to run a parametric study to investigate other corrugation configuration and panel dimension as well as different flange to web thickness ratio. The predicted shear capacity from the numerical analysis showed a connection between the analytical and experimental results.

2.4.2 Global Buckling

This is review on global buckling stress that had been obtained from many observations and from some experimental result and theoretical on corrugated web girder. Sayed-Ahmed in 2007 (Sayed-Ahmed, 2007) said that the critical shear stress of global buckling mode can be calculated from (Galambos, 1988):

$$\tau_{cr,g} = k_g \cdot \frac{D_y^{1/4} \cdot D_x^{3/4}}{h_w^2 \cdot t_w} \quad (2.6)$$

Where:

K_g = global shear buckling coefficient

D_x, D_y = flexural stiffness

It is stated that k_g is 36 for steel girders (Sayed-Ahmed 2007). The factors of D_x and D_y are defined as follows:

$$D_x = \frac{EI_x}{c} = \frac{E}{b+d} \left(\frac{bt_w[d \tan\alpha]^2}{4} \right) + \left(\frac{t_w[d \tan\alpha]^3}{12 \sin\alpha} \right) \quad (2.7)$$

$$D_y = \left(\frac{c}{s}\right) \left(\frac{Et_w^3}{12}\right) = \left(\frac{b+d}{b+d/\cos\alpha}\right) \left(\frac{Et_w^3}{12}\right) \quad (2.8)$$

Where:

I_x = second moment area of one “wave-length”

c = projected length

s = actual length

t_w = web thickness

b = panel width

d = horizontal projection of inclined panel

α = Corrugation angle

Elgaaly et. al. in 1996 had performed a test on corrugated web girder where global buckling was in controls in the fine corrugation (Elgaaly et. al., 1996). The buckling stress for overall corrugated web panel can be calculated by using orthotropic-plate buckling theory when the global buckling in controls. The global buckling stress can be calculated as:

$$\tau_{cre} = k_s \frac{(D_x)^{0.25} (D_y)^{0.75}}{th^2} \quad (2.9)$$

Where:

$$D_x = \frac{(q/s)Et^3}{12} \quad (2.10)$$

$$D_y = \frac{EI_y}{q} \quad (2.11)$$

$$I_y = 2bt \left(\frac{h_t}{2}\right)^2 + \left[\frac{t(h_r)^3}{6 \sin\theta}\right] \quad (2.12)$$

k_s = buckling coefficient, equals to 31.6 for simply supported boundaries and 59.2 for clamped boundaries.