Competing risk models in reliability systems, a gamma distribution model with bayesian analysis approach

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Abstract. *In this paper our effort is to introduce the basic notions that constitute a competing risks models in reliability analysis using Bayesian analysis approach with Gamma distribution as our model and presenting their analytic methods. The Gamma distribution is widely used in reliability analysis and it is known as an natural extension of the exponential distribution. The cases are limited to the models with independent causes of failure, only the scale parameter is a random variable, and uniform prior distribution is used in our analysis. This model describes the likelihood function and follows with the description of the posterior function and the estimations of the point, interval, hazard function, and reliability. The net probability of failure if only one specific risk is present, crude probability of failure due to a specific risk in the presence of other causes, and partial crude probabilities are also included.*

Keywords. *Competing risks, Likelihood function, Posterior/prior function, Hazard function, and Net/Crude/partial crude probability*

1. Introduction

The weakness of most reliability theories that the system and its components are described and explained as simply functioning or failed. However, depending on the age and the environment of the system (and/or its components), the failures may be from various causes. Suppose that units under observation experience any one several distinct failure types. Time to failure and cause of failure are recorded. Failure may be correspond to breakdown of a component where there are some possible causes of failure such as tread wear, puncture, or defective side walls of an automobile tire. In engineering area this phenomena is called as *competing risks.* Woman who is using birth control device might facing several risks such as getting pregnant accidentally, removal the device for medical or personal reasons, and the device rejection. In this case we might interested to estimate the lenght of time that woman to use this device when the rejection of it can be removed from the system. In bio-medicine and engineering areas, to describe this phenomena, people use the term *competing risks.* The competing risks theory describes how the performance of a system affected by many causes of failure that act together. Applications of this theory also occur in actuarial and demography sciences and it is called as *multiple-decrement analysis.*

Estimation of the parameters of the assumed failure models based on the classical sampling theory has been used broadly in many reliability areas. However, this classical methods have been observed unsatisfactory for many instances especially when analysis based on the scarcity of data or small sample size. The increasing cost of reliability test and the changing of engineering design also contribute to this problem. In contrast, Bayesian estimation methods ability to combine past knowledge or experience in the form of an apriori distribution with life test data to accomplish the task with smaller sample sizes allow us to cope with that problem.

Competing risks models and Bayesian estimation have been studied since the middle of 17th century by numerous scientists and researchers. These models have been used extensively in area of bio-medicine but only a few in engineering; the estimation methods, however, have been classical. Bayesian estimation has been used widely in reliability analysis but the application so far is limited to single failure-mode models. The combination of the two is the contribution of this paper.

In this paper we select our investigation by using the Gamma distribution as our model. We assume here all causes of failure are independent. This distribution is one of some other common failure models such as