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ND SIDE WALLS IN AN

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#### ABSTRACT

Coefficients of roughness, n are characterized as parameters representing the channel roughness and flow resistance. The Manning equation has been recognized as the most proper formulae to represent the open channel flow application. This study is about determining the mean roughness coefficients for an open channel with different bed and side walls roughness. The experiments were conducted at the Hydraulic and Hydrology Laboratory in Universiti Malaysia Pahang. The experiments were performed on a rectangular open channel with glass sides and a flat bed. The size of the rectangular open channel was 10m long, 0.3 m wide and 0.46m deep. The experiments were carried out using two different types of roughness; 5mm gravel and 2 mm gravel. There were two channel conditions, bed and sidewalls having the same roughness, and only the channel bed had the roughness while the sidewalls were smooth. For both conditions; experiments were conducted with certain three fixed slope gradients which are 1:100, 1:300, and 1:500. The fixed flow rates,  $4m^3/s$ ,  $8m^3/s$  and  $12m^3/s$  were also set by adjusting the water pump. The data obtained was converted into graph form before performing the analysis. The heights and velocities were recorded at the upper stream and lower stream of the open channel under different roughness conditions. The results showed that the coefficient of roughness for the open channel with wall roughness is higher than the channel without sidewall roughness and larger grain size will give higher roughness coefficient. It also can be concluded that channel slope and surface roughness were the main factors in determining the roughness coefficient.

#### ABSTRAK

Pekali kekasaran, n mempunyai ciri-ciri sebagai parameter yang mewakili kekasaran saluran dan mengalir rintangan. Persamaan Manning telah diiktiraf sebagai formula paling sesuai untuk mewakili permohonan aliran saluran terbuka. Kajian ini adalah kirakira menentukan pekali kekasaran min bagi saluran terbuka dengan katil yang berbeza dan dinding sebelah roughness. This eksperimen dijalankan di Hidraulik dan Hidrologi Makmal di Universiti Malaysia Pahang. Kajian ini menggunakan satu saluran segiempat tepat dengan sisi kaca dengan katil yang rata. Saiz segi empat tepat saluran terbuka adalah 10m panjang, 0.3 m lebar dan 0.46m dalam. Kajian ini telah dijalankan dengan menggunakan dua kekasaran yang berbeza itu '; Kerikil 5mm dan 2 mm kerikil. Akan ada dua syarat saluran, katil dan sisi mempunyai kekasaran yang sama dan hanya katil mempunyai permukaan kekasaran whiles yang sisi adalah permukaan licin. Bagi keduadua keadaan, eksperimen dijalankan dengan kecerunan tiga cerun tetap tertentu yang 1:100, 1:300, dan 1:500. Kadar aliran tetap, 4 m3 / s, 8 m3 / s dan 12 m3s juga diwujudkan dengan melaraskan tahap pam air. Data yang diperolehi ditukar kepada bentuk graf sebelum ia analisis. Ketinggian dan halaju direkodkan pada aliran atas dan aliran lebih rendah saluran terbuka dengan syarat kekasaran yang berbeza. Keputusan menunjukkan bahawa saluran kekasaran pekali dengan dinding adalah lebih tinggi daripada saluran tanpa sisi dan saiz butiran yang lebih besar mempunyai pekali kekasaran yang lebih tinggi. Ia juga boleh membuat kesimpulan bahawa cerun saluran dan kekasaran permukaan yang kelihatan sebagai faktor utama dalam menentukan pekali kekasaran.

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### LIST OF SYMBOLS

| Α              | Cross section, area                     |
|----------------|---|
| b              | Width of open Channel                   |
| С              | Chezy coeeficient                       |
| f              | Darcy-Weisbach friction factor          |
| Fr             | Froude number                           |
| g              | Acceleration due to gravity: $g = 9.81$ |
| h              | Depth flow                              |
| h <sub>f</sub> | Head loss                               |
| k              | Roughness height                        |
| κ              | Von Karman constant: $\kappa = 0.4$     |
| k <sub>c</sub> | Critical roughness                      |
| Ł              | Characteristic length                   |
| L              | Length of the channel                   |

| n                     | Manning's coefficient                      |  |
|-----------------------|--|--|
| ρ                     | Density of fluid                           |  |
| Re                    | Reynolds number                            |  |
| Rh                    | Hydraulic radius                           |  |
| S                     | Surface slope                              |  |
| t                     | Time                                       |  |
| τ                     | Total shear stress                         |  |
| τ <sub>0</sub>        | Bed shear stress                           |  |
| τ <sub>t</sub>        | Turbulent shear stress                     |  |
| U                     | Mean flow velocity                         |  |
| u(z)                  | Flow velocity at the depth z               |  |
| <i>u*</i>             | Friction velocity (wall friction velocity) |  |
| v                     | Kinematic consistency of water             |  |
| <i>z</i> <sub>0</sub> | Elevation corresponding to zero velocity   |  |

#### **CHAPTER 1**

#### INTRODUCTION

#### 1.1 BACKGROUND

Water asset projects and hydraulic engineering design works are rapidly being developed all over the world, therefore forecast of channel roughness coefficients is turning into a significant criterion for the planning and design of hydraulic related structures like open channels and dams. Understanding open channel hydraulics is crucial for all engineers in the design and planning when it comes to hydraulic structures.

Flow in open channel is divided into two categories, steady flow and unsteady flow. Furthermore, the steady flow is subdivided into uniform and varied flow. The theory of uniform flow is fundamental to the understanding and solution for most issues in open channel hydraulics. In most cases, open channels, from natural stream beds to lined artificial channels, show distinctive and unique coefficient of roughness, depending upon the state of the channel

The roughness characteristics of open channel have been widely studied and applied. Coefficients of roughness, n are characterized a parameters representing the channel roughness and flow resistance. The Manning equation has been recognized as the most proper formulae to represent open channel flow application.

Understanding the effects of roughness coefficient on flow conditions has become an integral part of river restoration projects. Roughness compositions are vital components in natural ecological systems. Channel properties, obstruction, vegetation, silting and scouring are examples of factors that may create different roughness conditions. Velocity distribution relies on the condition and roughness of the channel. Height, velocity and bed roughness are important to determine the calculation of the velocity profile.

In open channel conditions, Manning's equation has been widely studied and applied to determine the roughness coefficient, n. The influence of bed conditions will affect the flow rate and also the roughness characteristics. This equation has been studied and verified by many engineers but there are still many uncertainties remaining that concern the effect and determination the precise channel roughness coefficients value for computation of discharge in open channel hydraulics.

#### **1.2 PROBLEM STATEMENT**

The determination of n turned out to be a challenge because the values cannot be figured equivalent for all types of open channel. There are many laws of friction put forward by Chezy, Darcy, Sticklers and other hydraulic engineers for uniform roughness of the whole surface of closed or open conduits but it is difficult when it comes to determining the roughness when the channel has different roughness of bed and sidewalls.

#### 1.3 **OBJECTIVES**

- i. To obtain the mean roughness coefficients for an open channel with different bed and side walls roughness.
- ii. To investigate the relationship between roughness coefficient, discharge and bed slope.

### 1.4 SCOPE OF STUDY

The experiments were conducted at the Hydraulic and Hydrology Laboratory in Universiti Malaysia Pahang. The experiments used a rectangular open channel with glass sides and a flat bed. The size of the rectangular open channel is 10m long, 0.3 m wide and 0.46m deep. The experiments were carried out using two different roughnesses with a certain fixed slope gradient and fixed flow rate by adjusting the water pump level. Two types of conditions were tested; the bed and sidewalls with the same roughness and the bed is rough while the sidewalls are smooth.

#### 1.5 IMPORTANCE OF THE STUDY

It is difficult when it comes to determination of n because different types of open channels do not compute the same. According to the law of friction by Chezy, Darcy, Strickler and other hydraulic engineers, the equation provided were given for uniform roughness of the whole surface of closed or open conduits. Hence a study is needed to determine the roughness of an channel when it has different roughness between bed and sidewalls as well as the factors that affects the roughness coefficients

#### **CHAPTER 2**

#### LITERATURE REVIEW

#### 2.1 THEORETICAL REVIEW

In this part incorporates a point of view to the theories about the velocity profile and roughness height. Moreover, depiction of flow resistance is given in this chapter. At the beginning of this chapter, there is a review on the common and basic theories based on the project tittle and after that the specifics details theories and equation used for the research experiment.

#### 2.2 TYPES OF FLOW

There are three basic methods to depict the types of flow. The methods are individual and could be utilized independently. The decision which technique ought to be utilized relies on upon the context. The methods are briefly depicted below.

#### 2.2.1 Change of Depth Versus Time and Space

If the time is the characterization criteria, then a flow could be delegated being either steady, which suggests that the depth of flow (Figure 2.1) does not change with time  $(\partial h/\partial t = 0)$ , or unsteady, which suggests that the depth does change with time  $(\partial h/\partial t \neq 0)$ . Though if space is utilized as the criteria, then a flow could be delegated uniform if the depth of flow does not change with distance  $(\partial h/\partial x = 0)$  or as nonuniform if the depth differs with distance  $(\partial h/\partial x \neq 0)$  (French, 1986).



Figure 2.1: Cross section of channel bed. A is the projected area of the flow [m2], h is distance of water surface from mean bed level (depth of flow) [m], Rh is the hydraulic radius [m], and B is the width of the channel [m]. (Smart, 2001)

#### 2.2.2 Turbulence of Stream

Turbulence may be created by over flow rates, rough surfaces and curves in the channel. Chow (1959) characterizes that the flow is turbulent if the viscous forces are feeble with respect to the inertial forces. The impact of viscosity with respect to inertias can be written by the Reynolds number, stated as:

$$R_e = \frac{UR_h}{v} \tag{2.1}$$

Where,

U = Mean speed of stream [m/s];

*Rh* = The pressure driven range [m];

Data obtained from the experiment laboratory for slope 1:500

The flow Reynolds number is utilized to group the flows as shown as follows:

Table 2.1: Flows according to Reynolds number

| Laminar stream      | $\text{Re} \le 500$              |  |
|---------------------|----------------------------------|--|
| Transitional stream | $500 \le \text{Re} \le 12\ 500.$ |  |
| Turbulent stream    | $Re \ge 12\ 500.$                |  |

In laminar flow, the water particles seem to move in clear smooth ways, or streamlines and imperceptibly thin layers of liquid appear to slide over contiguous layers. In turbulent flow, the water particles move in spasmodic ways, which are not smooth or altered, yet which in the total still represent the forward movement of the whole stream. Between the laminar and turbulent states there is a transitional state (Chow, 1959). In natural channel, all flows are essentially turbulent.

#### 2.2.3 Impact of Gravity

As indicated by Chow (1959) the impact of gravity upon the state of flow is represented by a degree of inertial forces to the gravity forces. This degree is represented by the Froude number, characterized as

$$F_r = \frac{U}{gR_h} \tag{2.2}$$

Where,

g = speeding up because of gravity  $[m^2/s]$ 

The Froude number is utilized to characterize the flow as stated (Chow, 1959; French, 1986; Graf, 1998) as shows in Table 2.2.

| a pie 2.2: Characteristic of nov | Table 2.2: | Characteristic | of flow |
|----------------------------------|------------|----------------|---------|
|----------------------------------|------------|----------------|---------|

| Subcritical (fluvial) flow      | Fr < 1 |
|---------------------------------|--------|
| Supercritical (torrential) flow | Fr > 1 |
| Critical flow                   | Fr = 1 |

In subcritical state of flow, the gravity forces is more purported; consequently the flow has a low velocity and is regularly portrayed as tranquil and streaming (Chow, 1959). As indicated by French (1986) a wave can propagate upstream against the flow, and upstream zones are in hydraulic contact with the downstream zones. While in supercritical state of flow, the inertial forces get predominant; so the flow has a high velocity and is normally depicted as rapid, shooting, and torrential (Chow, 1959). As per French (1986) a wave cannot propagate upstream against the flow, and the upstream zones of the channel are not in hydraulic contact with the downstream zones. Between the subcritical and supercritical states there is a critical state (Chow, 1959).

#### 2.3 VELOCITY DISPERSION

In an open-channel, the flow velocity is not uniformly dispersed. In this research, only vertical velocity profile is examined. If there an occurrence of gravel bed and turbulence flow, vertical velocity profile is regularly thought to be logarithmically conveyed (Chow, 1959; French, 1986; Graf 1998; and Ferro, 1999).

To simplify the calculations, the velocity could be thought as uniform over the entire cross section. This assumption will simplify the calculations in correlation with the logarithmically distributed circumstances and the results are still satisfactory for a few circumstances. On the other hand, for most calculations and figuring this assumption gives off base results and distorts the characteristics of the channel.

#### 2.3.1 Introduction

Velocity dispersion depends more to the shape and roughness of the channel. Friction velocity is a vital term in the estimation of velocity profile and roughness. The following is a portrayal of the fundamental speculations about the vertical velocity profile.

#### 2.3.1.1 Friction Velocity

The friction velocity  $u^*$  is the liquid elevation at elevation,

$$z = z_o e^k, \tag{2.3}$$

Where,

 $z_o$  = the height comparing the zero velocity [m]  $\kappa$  = the von Kármán steady:  $\kappa = 0.4$ .

This is the trademark velocity for turbulent flows at a given wall shear stress (Schlichting and Gersten, 2000).

As indicated by Chow (1959) the friction velocity could be characterized as

$$u^* = \sqrt{\frac{\tau_o}{\rho}} \tag{2.4}$$

Where,

 $\tau_o$  = bed-shear stress (bottom part shear stress) [Pa]  $\rho$  = thickness of liquid  $[m^3/s]$ .

If there should be an occurrence of wide and shallow channel, the bed shear stress is:

$$\tau = \rho g h S \tag{2.5}$$

Where,

h = depth flow[m]

 $S = \text{surface slope} = \tan \beta \text{ [m/m]}$  like outlined in Figure 2.2



Figure 2.2: Fluid forces in case of wide channel. Surface slope S is assumed to be tan  $\beta$ . Bed shear stress  $\tau_0 = \tau_z = 0$ 

Inserting eq (2.4) into to eq (2.3), we get to the friction velocity

$$u^* = \sqrt{ghS} \tag{2.6}$$

# 2.3.1.2 Flow Over Smooth or Rough Surface

The flow might be using hydraulically either smooth or rough. Hydraulic smooth flow happens when the surface irregularities are small to the point that all roughness components are altogether submerged in the laminar sub-layer (Chow, 1959). Hence, the bed roughness would not influence the velocity dispersion. As indicated by Graf (1998) and Schlichting & Gersten (2000) the stream is smooth if

$$0 < \frac{u^*k}{v} < 5 \tag{2.7}$$

Where,

u\* = friction velocity [m/s]

k = the roughness height [mm]

The flow is rough when bed roughness is large to the point that it produces whirlpools near to the bottom (Liu, 2001). There is no viscose sub-layer and the velocity dispersions are influenced just by bed roughness. As per Graf (1998) and Schlichting and Gersten (2000) the flow is hydraulically rough when

$$70 < \frac{u^*k}{v} \tag{2.8}$$

For the term using hydraulically rough, also term completely rough is utilized (Schlichting and Gersten, 2000). The flow is in the transition region when

$$5 < \frac{u^*k}{v} < 70 \tag{2.9}$$

Hence, the velocity dispersion is influenced by bed roughness and viscosity (Chow, 1959)

## 2.3.2 Classification of Flow Layers

As indicated by Liu (2001) there are two sorts of arrangements of flow layers: scientific and engineering.

In the scientific arrangement, the flow profile has been separated into four layers (Figure 2.3). Those are from the bottom to up:

- 1. Viscous sub-layer: flow is laminar.
- 2. Transition layer: viscosity and turbulence are just as equally important.

- 3. Turbulent logarithmic layer: viscous shear stress might be disregarded in this layer. It is expected that the turbulent shear stress is consistent and equivalent to lowest part shear stress.
- 4. Turbulent outer layer: velocities are practically steady.





In the engineering classification, rather a turbulent layer with the logarithmic velocity profile covers the transitional layer, the turbulent logarithmic layer and the turbulent external layer, Figure 2.4 (Graf, 1998). In examination, the vertical speed profile in instance of laminar flow and consistently appropriated velocity profile are represented in Figure 2.5. (Graf, 1998) presents additionally that in the circumstances of uniform, hydraulically rough flow there are internal and external area in the profile. Internal area's tallness is z = 0.2h and the profile is not logarithmically appropriated in that layer.



Figure 2.4: Engineering classification of flow region in case of turbulent flow (Layer thickness is not in scale). (Liu, 2001)



Figure 2.5: Vertical velocity profiles in case of laminar flow and uniformly distributed velocity. (Graf, 1998)

## 2.3.3 Velocity Dispersion in the Turbulent Layer

The velocity dispersion in an uniform channel stream will become steady when the turbulent limit layer is completely created. In the turbulent limit layer, the dispersion might be indicated to be nearly logarithmic (Chow, 1959). The total stress increase simultaneously with depth refers to (Eq. 2.5).

$$\tau_t(z) = \tau_0 \left( 1 - \frac{z}{h} \right)$$
(2.10)

Where,

z = separation from the bottom [m]

(Figure 2.2)

By Prandtl's mixing hypothesis, the shear stress at any point in a turbulent flow moving over a solid surface is stated as:

$$\tau_t = \rho \ell^2 (\frac{du}{dx})^2 \tag{2.11}$$

Where,

€ = Characteristic length / the mixing length [m]
du/dz = velocity inclination at a typical distance z from the solid surface
(Chow, 1959)

As per Liu (2001) the mixing length might be characterized as

$$\ell = \kappa z \left( 1 - \frac{z}{h} \right)^{0.5} \tag{2.12}$$

Where,

 $\kappa = \text{von Karman constant} (\kappa = 0.4)$ 

For the district close to the solid surface, Prandtl presented two suspicions: (1) that the blending length is corresponding to z, and (2) that the shearing anxiety is consistent. The second presumption gives  $\tau = \tau_0$ . From these two assumptions and from mathematical statements (2.4), (2.10), (2.11) and (2.12) we get:

$$\frac{du}{dx} = \frac{\sqrt{\tau_o/\rho}}{\kappa z} = \frac{u^*}{\kappa z}$$
(2.13)