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Prediction Modeling of Torque in End-milling

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Abstract—**This paper presents the development of mathematical models for torque in end-milling of AISI618 using coated carbides cutting tool. Response surface method was use to predict the effect of torque in the end-milling. From the model, the relationship between the manufacturing process factors including the cutting speed, feed rate, axial depth and radial depth with the torque can be developed. The effect of the factors can be investigated from the equation developed for first order to fourth order model. The acquired results show that the torque increases with decreases of the cutting speed and increases the feed rate, axial depth and radial depth. It found that the second order is more accurate based on the analysis of variance (ANOVA) and the predicted torque results is closely match with the experimental results. Third-and fourth-order model generated for response to investigate the 3- and 4-way interaction between the factors. The third and fourth order model show that 3- and 4-way interaction found less significant for the variables.**

***Keywords*:** torque, end milling, response surface method, cutting speed, AISI 618

I. INTRODUCTION

Response surface method (RSM) is a collection of statistical and mathematical methods that are useful for the modelling and optimization of the engineering problems. In this technique, the main objective is to optimize the responses that are influencing by various parameters [1]. RSM also quantifies the relationship between the controllable parameters and the obtained response. In modelling of the manufacturing processes using RSM, the sufficient data is collected through designed experimentation. In general, a second order regression model is developed because of first order models often give lack off fit. The study uses the Box-Behnken design in the optimization of experiments using RSM to understand the effect of important parameters. Box-Behnken Design is normally used when performing non-sequential experiments. That is, performing the experiment only once. These designs allow efficient estimation of the first and second –order coefficients. Because Box-Behnken design has fewer design points, they are less expensive to run than central composite designs with the same number of factors. The RSM is practical, economical and relatively easy for use and it was used by lot of researchers for modeling machining processes [2,3,4]. Mead and Pike [5] and Hill and Hunter [6] reviewed the earliest work on response surface methodology. Response surface methodology is a combination of experimental and regression analysis and statistical inferences. The concept of a response surface involves a dependent variable y called the response variable and several independent variables $x_1,x_2,...,x_k$ [7]. The main aim of the paper is to develop the first and second order model by using Surface Response Methodology. From this model, the relationship between the factors and the response can be investigated. The objective of this paper is to develop the mathematical model and predicted the torque in end-milling.

II. MATHEMATICAL MODELING

The proposed relationship between the responses (torque and torque) and machining independent variables can be represented by the following:

$$\tau = C\left(V^m F^n A_x^y A_r^z\right)\varepsilon' \qquad (1)$$

Where τ is the torque in Nm, V, F, A_x and A_r are the cutting speed (m/s), feed rate (mm/rev), axial depth (mm) and radial depth (mm). C, m, n, y and z are the constants. Equation (1) can be written in the following logarithmic form as in Equation (2):

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$$\ln \tau = \ln C + m \ln V + n \ln F + y \ln A_x + z \ln A_r + \ln \varepsilon' \quad (2)$$

Equation (2) can be written as a linear form:

$$y = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon \quad (3)$$

where y =lnτ is the torque, , x_0 = 1(dummy variables), x_1= ln$V$, x_2 = ln$F$, x_3 = lnA_x , x_4 = lnA_r and $\varepsilon = \ln \varepsilon'$,where ε is assumed to be normally-distributed uncorrelated random error with zero mean and constant variance, β_0 =ln$C$ and β_1, β_2, β_3, and β_4 are the model parameters. The second model can be expressed as:

$$y'' = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{44} x_4^2 + \beta_{11} x_1 x_2 + \beta_{12} x_1 x_3 + \beta_{13} x_1 x_4 + \beta_{14} x_2 x_3 + \beta_{15} x_3 x_4 \quad (4)$$

The values of β_1, β_2, β_3 and β_4 are to be estimated using the method of least squares. The basic formula can be expressed as in Equation (5):

$$\beta = \left(x^T x\right)^{-1} x^T y \quad (5)$$

where x^T is the transpose of the matrix x and $(x^T x)^{-1}$ is the inverse of the matrix $(x^T x)$ and y is the value from experiment. The details solution of this matrix approach is explained in [1]. The parameters have been estimated by the method of least-square using a Matlab computer codes.

III. Engineering Design

To develop the first-order, a design consisting 27 experiments were conducted. Box-Behnken Design method is normally used when performing the non-sequential experiments. These design allow the efficient estimation of the first and second-order coefficients because of the Box-Behnken Design has fewer design points, they are less expensive to run than central composite designs with the same number of factors. Box-Behnken Design no axial points, thus all design points fall within the safe operating. Box-Behnken Design also ensures that all factors are never set at their high levels simultaneously [8,9,10].Preliminary tests were carried out to determine the suitable cutting speed, federate, axial and radial depth of cut as shown in table 1.

Table 1: Levels of independent variables

Input cutting parameter	Coding of Levels		
	-1	0	1
Speed, V_c (m/s)	100	140	180
Feed, f (mm/rev)	0.1	0.2	0.3
Axial depth of cut, a_a, (mm)	1	1.5	2
Radial depth of cut, a_r, (mm)	2	3.5	5

The AISI 618 stainless steel workpieces were provided in fully annealed condition in sizes of 65×170 mm. The tools used in this study are carbide inserts PVD coated with one layer of TiN. The inserts are manufactured by Kennametal with ISO designation of KC 735M.They are specially developed for milling applications where stainless steel is the major machined material. The end-milling tests were conducted on Okuma CNC machining centre MX-45VA Every one passes (one pass is equal to 85mm), the cutting test was stopped. The same experiment has been repeated for 3 times to get more accurate results.

IV. Results and Discussion

A. First-Order Torque Model

The first order torque model can be expressed as in Equation (6):

$$y' = 2.6215 - 0.1308 x_1 + 0.2292 x_2 + 0.140 x_3 + 0.2142 x_4 \quad (6)$$

The transforming equations for each of the independent variables are:

$$x_1 = \frac{\ln(V) - \ln(v)_{\text{centre}}}{\ln(v)_{\text{high}} - \ln(v)_{\text{centre}}}\ ;\ x_2 = \frac{\ln(F) - \ln(f)_{\text{centre}}}{\ln(f)_{\text{high}} - \ln(f)_{\text{centre}}}\ ;\ x_3 = \frac{\ln(A_x) - \ln(a_x)_{\text{centre}}}{\ln(a_x)_{\text{high}} - \ln(a_x)_{\text{centre}}}\ ;\ x_4 = \frac{\ln(A_r) - \ln(a_r)_{\text{centre}}}{\ln(a_r)_{\text{high}} - \ln(a_r)_{\text{centre}}} \quad (7)$$

The torque model can be expressed as Equation (8):

$$T = 315.23(V^{-0.5204} F^{0.796719} A_x^{0.489432} A_r^{0.60055}) \quad (8)$$

Table 2 shows the 95% confidence interval for the experiments and analysis of variance. For the linear model, the P-value for lack of fit is 0.196 and the F-statistics is 5.1033.Therefore, the model is adequate. The experimental and predicted torque results for the first order model are given in Table 3.

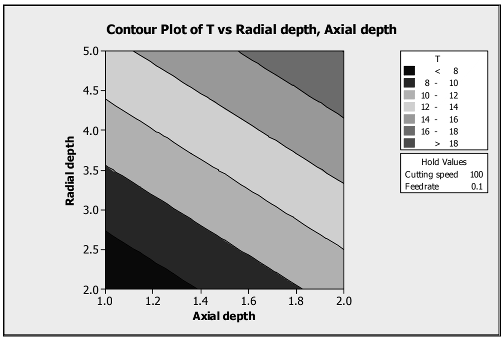
Table 2: Analysis of Variance (ANOVA) for first-order torque model with 95% confidence interval

Source	DOF	Seq. SS	Adj. SS	Adj, MS	*F*	*P*
Regression	4	434.746	434.746	108.687	186.37	0
Linear	4	434.746	434.746	108.687	186.37	0
Residual Error	22	12.830	12.830	0.583	-	-
Lack-of-Fit	20	12.830	12.830	0.642	5.1033	0.196
Pure Error	2	0.000	0.000	0.1258	-	-
Total	26	447.576	-	-	-	-

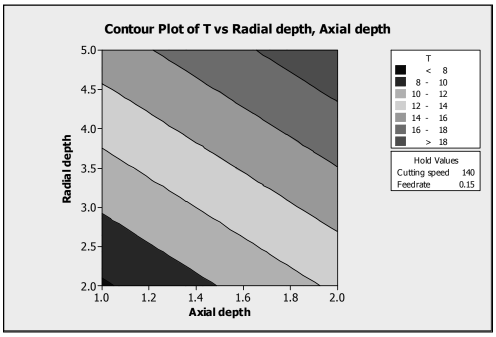
Table 3: The predicted result for first order torque model

Expt. No.	Cutting speed (m/s)	Feed rate (mm/rev)	Depth of cut(mm)		Torque (N.m)	
			Axial	Radial	Experimental	Predicted
1	140	0.15	1	2	10	8.06
2	140	0.2	1	3.5	13	14.18
3	100	0.15	1	3.5	16	13.43
4	180	0.15	1	3.5	13	9.89
5	140	0.1	1	3.5	8	8.16
6	140	0.15	1	5	16	13.97
7	100	0.15	1.5	2	16	11.71
8	140	0.1	1.5	2	7	7.11
9	100	0.2	1.5	3.5	17	20.60
10	140	0.15	1.5	3.5	14	13.75
11	180	0.2	1.5	3.5	18	15.17
12	180	0.15	1.5	2	12	8.62
13	140	0.2	1.5	2	13	12.36
14	140	0.2	1.5	5	18	21.42
15	140	0.15	1.5	3.5	13	13.75
16	180	0.1	1.5	3.5	8	8.73
17	100	0.1	1.5	3.5	14	11.86
18	100	0.15	1.5	5	22	20.29
19	140	0.1	1.5	5	14	12.33
20	180	0.15	1.5	5	15	14.95
21	140	0.15	1.5	3.5	18	13.75
22	140	0.15	2	5	20	19.61
23	140	0.2	2	3.5	23	19.91
24	140	0.1	2	3.5	13	11.46
25	140	0.15	2	2	11	11.31
26	100	0.15	2	3.5	23	18.86
27	180	0.15	2	3.5	16	13.89

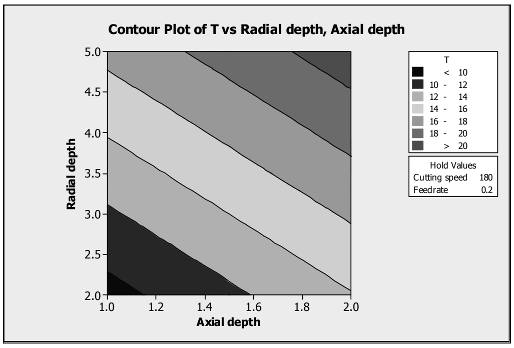
Equation (8) shows that the torque increases with decreases of the cutting speed while increases of the feed rate, axial and radial depth of cut. It also indicates that the feed rate has the most significant effect on the torque, follow by radial and axial depth of cut and cutting speed. Equation (8) is utilized to develop torque contour at selected cutting speed and feed rate. Figure 1 shows that the torque contours in the axial-radial depth plane for different cutting speed and feed rate. It is helpful to predict the torque at any experimental zone. It is clearly shown that the cutting speed, feed rate, axial depth of cut, radial depth of cut and feed rate are strongly related with the torque in end-milling. It can be seen that the increases of torque with increases of cutting speed and feed rate. The torque obtained the highest value about 25 N at cutting speed 180 m/min.



(a) V_c = 100 m/min and f = 0.1 mm/rev



(a) V_c = 140 m/min and f = 0.15 mm/rev



(a) V_c = 180 m/min and f = 0.2 mm/rev

Figure 1: Torque contours in the axial-radial depth plane for different cutting speed and feed rate

B. Second-Order Torque Model

The second-order model was postulated in obtaining the relationship between the cutting force and the machine independent variables. The second order model equation can be expressed as in Equation (9):

$$y'' = 2.05074 - 0.031x_1 + 47.37x_2 + 2.97x_3 + 1.60x_4 + 0.00029x_1^2 - 50.17x_2^2 - 0.78x_3^2 - 0.14x_4^2 \\ - 0.29x_1x_2 - 0.018x_1x_3 - 0.0094x_1x_4 + 24.3x_2x_3 + 12.8x_2x_4 + 0.80x_3x_4 \quad (9)$$

Table 4 is given the 95% confidence interval for the experiments and analysis of variance. For the second-order model, the *P*-value for lack of fit is 0.221 and the *F*-statistics is 4.5249. Therefore, the model is adequate. The second-order model is more adequate because of the predicted result is much more accurate than the first model. The *P*-value higher than the first order predicted value. The predicted torque results for second order model are given Table 5. Equation (9) is utilized to develop the contour plots for torque which is shown in Figure 2(a) at the maximum axial depth and radial depth.

Table 4: Analysis of Variance (ANOVA) for second-order model with 95% confidence interval

Source	DOF	Seq. SS	Adj. SS	Adj, MS	F	P
Regression	14	447.358	447.358	31.954	1758.88	0.000
Linear	4	447.358	447.358	108.687	5982.52	0.000
Square	4	2.922	2.922	0.731	40.21	0.000
Interaction	6	9.690	9.690	1.615	88.90	0.000
Residual Error	12	0.218	0.218	0.018	-	-
Lack-of-Fit	10	0.218	0.218	0.022	4.5249	0.221
Pure Error	2	0.000	0.000	0.00486	-	-
Total	26	447.358	-	-	-	-

Table 5: The predicted result for second order torque model

Cutting speed (m/s)	Feedrate (mm/rev)	Axial depth (mm)	Radial depth (mm)	Torque (N.m)	
				Experimental	Predicted
140	0.15	1.0	2.0	10	7.94
140	0.2	1.0	3.5	13	14.21
100	0.15	1.0	3.5	16	13.51
180	0.15	1.0	3.5	13	9.97
140	0.10	1.0	3.5	8	8.09
140	0.15	1.0	5.0	16	13.98
100	0.15	1.5	2.0	16	11.84
140	0.10	1.5	2.0	7	6.98
100	0.20	1.5	3.5	17	20.45
140	0.15	1.5	3.5	14	13.75
180	0.20	1.5	3.5	18	15.05
180	0.15	1.5	2.0	12	8.72
140	0.20	1.5	2.0	13	12.39
140	0.20	1.5	5.0	18	21.55
140	0.15	1.5	3.5	13	13.75
180	0.10	1.5	3.5	8	8.87
100	0.10	1.5	3.5	14	11.97
100	0.15	1.5	5.0	22	20.20
140	0.10	1.5	5.0	14	12.30
180	0.15	1.5	5.0	15	14.82
140	0.15	1.5	3.5	18	13.75
140	0.15	2.0	5.0	20	19.73
140	0.20	2.0	3.5	23	19.98
140	0.10	2.0	3.5	13	11.44
140	0.15	2.0	2.0	11	11.30
100	0.15	2.0	3.5	23	18.78
180	0.15	2.0	3.5	16	13.81

C. Third-Order Torque Model

The third-order model as shown below was use is obtained to investigate the 3-way interaction between the variables.

$$y''' = c + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x^2{}_1 \beta 6 x^2{}_2 + \beta_7 x^2{}_3 + \beta_8 x^2{}_4 + \beta_9 x^3{}_1 + \beta_{10} x^3{}_2 + \beta_{11} x^3{}_3 + \beta_{12} x^3{}_4 + \beta_{13} x_1 x_2 + \beta_{14} x_1 x_3 + \beta_{15} x_1 x_4 + \beta_{16} x_2 x_3 + \beta_{17} x_2 x_4 + \beta_{18} x_3 x_4 + \beta_{19} x_1 x_2 x_3 + \beta_{20} x_1 x_2 x_4 + \beta_{21} x_1 x_3 x_4 + \beta_{22} x_2 x_3 x_4 \quad (11)$$

From this model the most important points are the main effects, 2-way interaction and 3-way interaction. The third order torque model can be presented as in Equation (12):

$$y''' = c + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{13} x_1 x_2 + \beta_{14} x_1 x_3 + \beta_{15} x_1 x_4 + \beta_{16} x_2 x_3 + \beta_{17} x_2 x_4 + \beta_{18} x_3 x_4 + \beta_{19} x_1 x_2 x_3 + \beta_{20} x_1 x_2 x_4 + \beta_{21} x_1 x_3 x_4 + \beta_{22} x_2 x_3 x_4 \quad (12)$$

The third-order model parameters can be solved using least-squares method. *β's* are the model parameters, x_1 = cutting speed, x_2 =feedrate, x_3 =axial depth and x_4 =radial depth. The third order model for torque can be rewrite in Equation (13):

$$y'''_T = -176.95 + 1.3922 x_1 + 1103.97 x_2 - 7.6632 x_3 + 56.7540 x_4 - 7.8022 x_1 x_2 - 0.05 x_1 x_3 - 0.4237 x_1 x_4 + 50 x_2 x_3 - 353.753 x_2 x_4 + 3.4863 x_3 x_4 + 2.4792 x_1 x_2 x_3 \quad (13)$$

Table 6 is given the analysis of variance (ANOVA) for the third-order torque model. The model adequate and significant of 3-way interaction can be seen form Table 6. From the ANOVA both model not significant to the 3-way interaction since the *P*-value>0.05.The third-order model adequate for torque since the *P*-value for lack of fit for torque is 0.818 and the *F*-statistics is 0.52. It indicates that this model is not suitable as much as second-order torque model.

Table 6: Analysis of Variance (ANOVA) for third-order torque model

Source	DOF	Seq. SS	Adj. SS	Adj, MS	F	P
Main effects	4	294.98	311.38	77.845	11.80	0.000
2-way Interactions	6	51.30	73.36	12.226	1.85	0.156
3-way Interasctions	1	26.13	26.13	26.131	3.96	0.065
Residual Error	15	99.00	99.00	6.600	-	-
Lack-of-Fit	12	67.00	67.00	5.583	0.52	0.818
Pure Error	3	32.00	32.00	10.667	-	-
Total	26	471.41	-	-	-	-

D. Fourth–Order Torque Model

The fourth-order model as shown below is obtained to investigate the 4-way interaction between the variables.

$$y'''' = c + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x^2{}_1 \beta_6 x^2{}_2 + \beta_7 x^2{}_3 + \beta_8 x^2{}_4 + \beta_9 x^3{}_1 + \beta_{10} x^3{}_2 + \beta_{11} x^3{}_3 + \beta_{12} x^3{}_4 + \beta_{13} x^4{}_1 + \beta_{14} x^4{}_2 + \beta_{15} x^4{}_3 + \beta_{16} x^4{}_4 + \beta_{17} x_1 x_2 + \beta_{18} x_1 x_3 + \beta_{19} x_1 x_4 + \beta_{20} x_2 x_3 + \beta_{21} x_2 x_4 + \beta_{22} x_3 x_4 + \beta_{23} x_1 x_2 x_3 + \beta_{24} x_1 x_2 x_4 + \beta_{25} x_1 x_3 x_4 + \beta_{26} x_2 x_3 x_4 + \beta_{27} x_1 x_2 x_3 x_4 \quad (14)$$

From this model the most important points are the main effects, 2-way, 3-way and 4-way interactions. So the fourth order model can be reduced as in Equation (15):

$$y'''' = c + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_6 x_1 x_2 + \beta_7 x_1 x_3 + \beta_8 x_1 x_4 + \beta_9 x_2 x_3 + \beta_{10} x_2 x_4 + \beta_{11} x_3 x_4 + \beta_{12} x_1 x_2 x_3 + \beta_{13} x_1 x_2 x_4 + \beta_{14} x_1 x_3 x_4 + \beta_{15} x_2 x_3 x_4 + \beta_{16} x_1 x_2 x_3 x_4 \quad (15)$$

This model parameters can be solved using least squares method. *β's* are the model parameters, x_1 = cutting speed, x_2 = feed rate, x_3 = axial depth and x_4 = radial depth. The fourth order torque model can be presented as in Equation (16):

$$y''' = -216 - 0.0625 x_1 + 512 x_2 + 30 x_3 + 63.5 x_4 + 5.5 x_1 x_2 + 0.7031 x_1 x_3 - 0.0625 x_1 x_4 + 444 x_2 x_3 - 155 x_2 x_4 - 3.75 x_3 x_4 - 10 x_1 x_2 x_3 - 1 x_1 x_2 x_4 - 0.2266 x_1 x_3 x_4 - 139 x_2 x_3 x_4 + 2.75 x_1 x_2 x_3 x_4 \quad (16)$$

Table 7 is given the analysis of variance (ANOVA) for the fourth-order torque model. The model adequate and significant of 4-way interaction for both model are also presented in Table 7. It can be seen that from the ANOVA analysis both model not significant to 4-way interaction since the *P*-value > 0.05. The third-order model adequate for torque since the *P*-value for lack of fit for torque is 0.818 and the *F*-statistics is 0.52. It indicates that this model is not suitable as much as second-order torque model.

Table 7: Analysis of Variance (ANOVA) for fourth-order torque model

Source	DOF	Seq. SS	Adj. SS	Adj, MS	F	P
Main effects	4	359.667	176.667	44.167	6.38	0.007
2-way Interactions	6	28.000	22.827	3.804	0.55	0.761
3-way Interasctions	4	7.629	4.327	1.082	0.16	0.956
4-way Interactions	1	0.000	0.000	0.000	0.00	1.000
Residual Error	11	76.112	76.112	6.919	-	-
Lack-of-Fit	9	62.112	62.112	6.901	0.99	0.599
Pure Error	2	14.000	14.000	7.000	-	-
Total	26	471.407	-	-	-	-

V. Conclusions

Reliable torque model have been developed and utilized to enhance the efficiency of the milling 618 stainless steel. The torque equation show that feed rate, cutting speed, axial depth and radial depth plays the major role to produce the torque. The higher the feed rate, axial depth and radial depth, the torque generates very high compare with low value of feed rate, axial depth and radial depth. Contours of the torque outputs were constructed in planes containing two of the independent variables. These contours were further developed to select the proper combination of cutting speed, feed, axial depth and radial depth to produce the optimum torque. The third order model and fourth order model very important to investigate the 3-way interaction and 4-way interaction. The third order model and fourth order model, shows that the 3-way interaction and 4-way interaction not significant.

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