Journal of Structural Durability & Health Monitoring (SDHM), Tech Science Press, Vol. 4, no.1, 2008, pp.19-27, ISSN: 1930-2983 (print), ISSN: 1930-2991 (on-line).

Modeling of a surface contact stress for spur gear mechanism using static and transient finite element method

F. R. M. Romlay

(Faculty of Mechanical Engineering, Universiti Malaysia Pahang, 25000 Kuantan Pahang, Malaysia)

E-mail: fadhlur@ump.edu.my

Abstract: This paper presents a surface contact static stress of a spur gear system combined with dynamic characteristic using transient Finite Element Method (FEM). Traditionally, the static stress analysis is done separately with dynamic properties due to limitation of complex equation and avoiding of error occurred. However, in this paper, static stress information is combined with the dynamic mechanism due to the time consuming during the design and analysis stage. A transient FEM analysis is carried out to formulate and solve large systems of algebraic equations in order to obtain a relationship between the contact parameter and the kinematics function. The methodology of the research is started with static stress analysis on tooth surface contact of a pair of the spur gear. Finite element modeling is run by choosing a certain static condition. The loading conditions are applied suitable with the gear mechanism. Degree of freedom controlled is based on the transmission system. The process is repeated until diagnosing work is satisfied. The result of the surface contact stress is visualized at each condition. Modeling of spur gear system is continued by combining stress analysis with dynamic characteristic via transient finite element method. Analysis of gear mechanism is obtained by investigate the stress distribution on real time application. Time range is set at the beginning of the analysis. Duration of the analysis is depended on a time frame chosen. By the transient FEM analysis, the stress occur at each step of the work cycle is performed. Results of the kinematics functions are derived and qualitative kinematics variations due to contact changes in time-step domain is identified. The simulation results from static and transient FEM are compared due to the validating procedure. The finite element results are in good agreement compared with the theory calculation.

Key words: Gear mechanism, transient, Finite Elements Method

INTRODUCTION

Now days, there are so many mechanisms those involve with load and requirement to understand the stress in component is increased. The mechanisms and the stress always come together and they have a strong relation between each other.

In real application, displacement and stress are dependent with the dynamic criteria. The loads that apply to the model that got motion consider as transient load. This will involve kinetic and kinematic criteria of the model.

With the current trend now, modeling and simulation using computer is growing very fast and the demand is increasing dramatically by a year. Mathematical model through FEM is most suitable application in engineering design with high capability in analysis option. The performance of the FEM solver is influenced by the computer performance too. With the mature of the computer technology expansion, benefit for the computational mechanics is increased.

Presently, the model is analyzed by finite element method to identify the stress and displacement. The same model need to be design again or is possible, is exported to the dynamic analyzer for a purposed of dynamic study. This needs a long time to finish a simulation work. The CAD design also can be damage during the file transfer process. This factor can affect the accuracy of the result.

Using of finite element result for the dynamic analysis input is very difficult because of the involvement of a complex mathematical equation and high possibility of error to be occurred. That is why both of the analysis needs to be combined in a single analyzer.

GEAR SYSTEM

Gear system is very common component in mechanical system and world wide usage in machine driving mechanism. The function of the gear system is to increase or decrease a load transfer in machine component. As a critical part for load transfer and transmission system, gear structure must be robust enough. The reliability of the load transfer system depends on the gear performance during the operation.

For gear system, modeling stage is very important to determine the increasing or decreasing of the load ratio. In design stage, size, type of gear and material properties play an important rule in giving the specific output for the gear system as a torque. Modeling will help the designer to fabricate a suitable characteristic for a specific application.

In gear system, a contact of a gear tooth is very complex problem to solve. The complexity is increase with the influence of manufacturing and assembly technique. The basic errors that usually happen are lead crowning and shaft misalignment. Over contact stress is happened by these errors. A partial mating between the teeth is caused by improper alignment (Bensely et al., 2006). It develops high stress between the teeth in contact, leading to larger load acting on a very small area during sliding. This resulted in teeth chipping all around the edges of the crown wheel.

Some time, tooth modification is needed in order to fix the tooth assembly. A precise theoretical method to be able to calculate surface contact stress and root bending stress of a pair of spur gears with considering manufacturing error, assembly error and tooth modification is provided (Li, 2007).

With the demand of modeling using computational mechanics is increased, the researchers were developed a gear mechanism simulation using computer aided design (CAD) software (Brauer, 2000 and Lunin, 2001). How ever, in term of time consume, is not very efficient. It is because a lot of mesh involved in a 3D modeling.

In order to expand the fundamental of gear modeling, Brauer was provided a general finite element model of involute gear. The model present was complete with the mathematical description include the root surface. The capability of addendum modification is also provided for the flexibility of gear modeling (Brauer, 2004).

Studying a contact of a pair of gear must be considering static stress. Static stress exists because of the load applied and by the same load, the part move simultaneously. That means the equation of the dynamic mechanism is dependent with the static stress result. Consider this factors, modeling the part by finite element method will make simulation become easier. The stress and displacement results will come together with the kinetic and kinematic information.

The critical part that needs a particular observation and investigation is only at contact area or surface. Some advance interpolation technique is needed to model the critical area. Prior to this section, this research focus on understanding of stress distribution in gear system. This paper will presents a modeling of surface contact stress between a pair of spur gear using finite element method. The results is validate with deterministic calculation.

SPUR GEAR

Spur gear as shown in Fig.1 has teeth parallel to the axis of rotation and used to transmit motion from one shaft as shown in Fig. 2 to another parallel shaft. Of all types, the spur gear is the simplest and for this reason, will be used to develop the primary kinematics relationship of the tooth form. [Shigley and Mischke, 2003].

Combination more that one gear is called gear system. The force transmitted is base on gear system ratio. Gear ratios express a mathematical relationship of one gear to another. Gear ratio depends on the geometry of the gear, means influence by diameter and number of teeth of the gears.

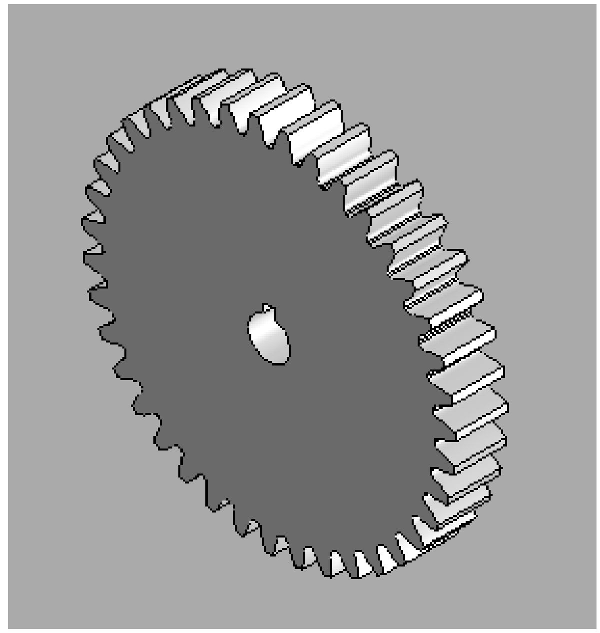

Fig. 1 Spur gear with the teeth parallel to rotational axis.

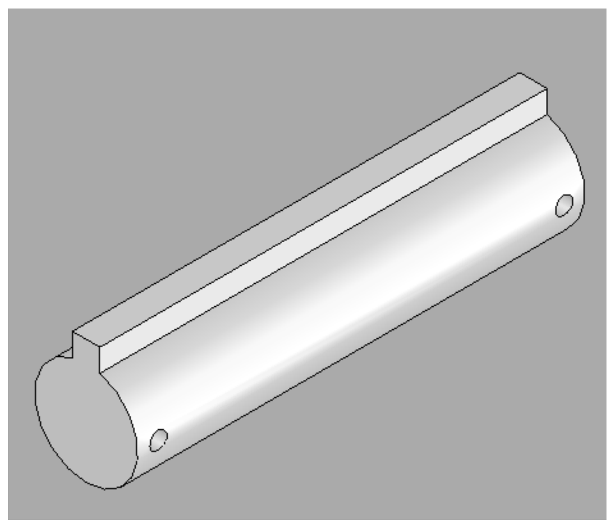

Fig. 2 Shaft used for gear rotating and force transmitting.

A gear ratio also expresses the amount of torque multiplication between a pair of gears. The ratio is obtained by dividing the diameter or number of teeth of the driven gear by the diameter or teeth of the drive gear. The rotation number of driving gear for one rotation of driven gear is told by gear ratio.

The system that developed by more than two gears is categorized as planetary gear.

Calculating the gear ratio of the planetary gear set requires the use of different formulas than used to calculate the ratio of two gears in mesh. The planetary gear ratio is derived base on the Equ. (1). [Waldron and Kinzel, 2004].

$$\text{Gear ratio} = \frac{\text{Drive gear} + \text{Driven gear}}{\text{Drive gear}}. \quad (1)$$

STRESS CALCULATION

Stress distribution around the gear teeth contact is investigated by numerical and validated by deterministic method. The numerical is run by FEM. The investigation of static and transient conditions is considered as a major part of the stress analysis.

Surface contact static stress of the spur gear

Hertz formula is often used to calculate surface contact stress of gears when tooth load is applied. However, the formula is very complex to use if assembly error, machining error and tooth modification are considered. So this paper calculates the surface contact stress of gears with a ''Unit Force'' method. It means, calculation is generated by define the tooth load distributed on unit contact area of a tooth surface. When the tooth load distributions are obtained, the result is derived a stress and compare with FEM results (Shuting, 2007).

K is an arbitrary reference point on driving gear and *K'* is a responsive contact point on tooth surface of driven gear. *K* and *K'* are used as a pair of contact points in 2D view as shown in Fig.3 and expand as contact surface in 3D model.

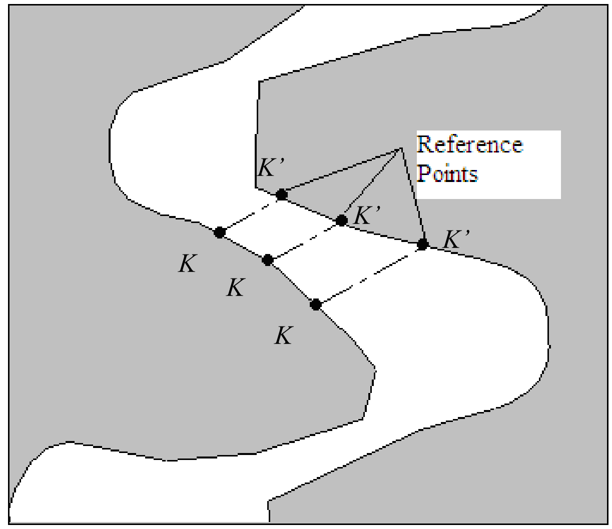

Fig. 3 Contact point of spur gear system

The contact force is transferred through the contact point along the gear curve and directly become a surface contact force. The contact between the gears surfaces are considered as elastic contact bodies. In gear study, an external force, *P* is assumed to be equal to the sum of all the contact force, F_j, (j=1 to n). The relationship between the external force, *P* and contact force, F_j, are given by the equation below:

$$P = \sum_{i=1}^{n} F_i \quad (2)$$

The contact surface area on specific time for spur gear is very small. By this condition, it is assumed that all the common normal lines of the contact point pairs are approximately parallel with the external load (Shuting, 2007).

Transient stress for gear mechanism

Usually, stress analysis for the dynamic component is done by stand alone analyzer using finite element method. Using a traditional method, the stress of the dynamic mechanical part is estimated by separated time step using quasi-static stress analysis approach (Haiba et al., 2002). The equation is given by Eq. (3), (4) and (5).

$$\sigma_x(t) = \sum_{i=1}^{n} \sigma_{xi} F_i(t) \tag{3}$$

$$\sigma_y(t) = \sum_{i=1}^{n} \sigma_{yi} F_i(t) \tag{4}$$

$$\tau_{xy}(t) = \sum_{i=1}^{n} \tau_{xyi} F_i(t) \tag{5}$$

where n is the number of applied load histories and $\sigma_{xi}(t)$ $\sigma_{yi}(t)$ $\tau_{xyi}(t)$ are the stress due to a unit load in a function of time. The stress is applied at a specific nodal and a same direction with the load history $F_i(t)$.

In this condition, the analysis is chosen just for a certain moment from the whole mechanism. Usually engineer need to predict the critical time and result for the whole process cannot be performed.

MODELING STRATEGY

Prior to this section, this research focus on understanding of stress distribution in gear mechanism. The spur gear model was developed as illustrates in Fig. 4, Fig. 5 and Fig. 6.

Computer aided design of gear system

The modeling is started with a design in computer aided design (CAD) software. The design of the gear must properly construct in order to avoid a modeling error. The error occur at design stage will dramatically increase in analysis stage especially by the involvement of multiple function (Ariffin & Romlay, 2004).

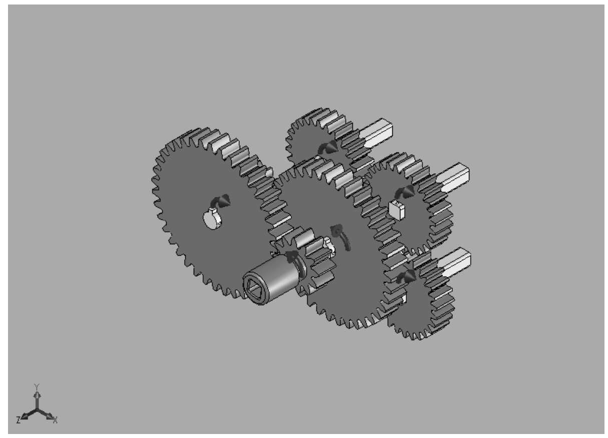

Figure 4 Isometric design view and direction of the motion

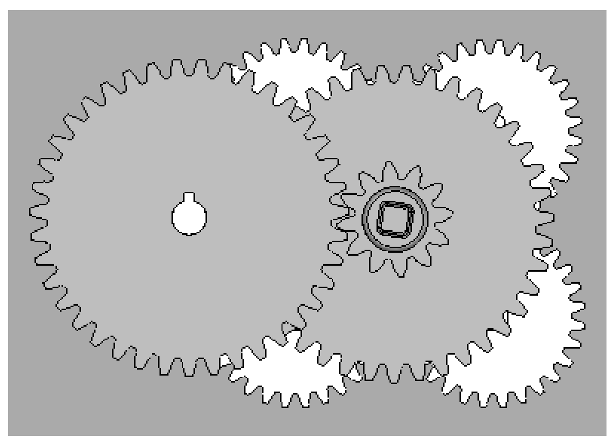

Fig 5 Front view of spur gear system

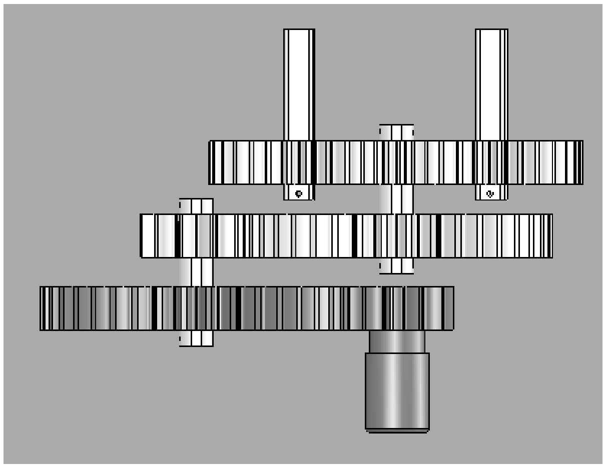

Fig 6 Top view of spur gear system which divide by 3 level.

The gear system is developed to decrease the force. It is using nine type of gear which is variable in size. The gear ratios are decreasing the torque to 21.125 times and there have three levels of gears.

For the first level, it has two type of gear; the spur gear with 12 teeth and other one is gear with 39 teeth. Ratio for the load transmission is 3.25. For the second level, it contains two types of gear which are the spur gear with 12 teeth and a gear with 39 teeth. The ratio reduction for this level is also 3.25. For the third level, it is attached by five types of gears. A single spur gear at the middle of the level and four gears connect this gear in rectangle. The spur gear is fabricated with 14 teeth and the rest four gears are fabricated with 28 teeth. Ratio for this gear system is two. Therefore, the overall ratio is 21.125.

An equation control is used for the purpose of geometry correction and updating. A complete design file is exported to computer aided engineering (CAE), as a first step for analyze the reliability of the model.

Computer aided engineering by FEM

The gear assembly was defined by finite element method. The CAD model was meshed and defined using brick element as shown in Fig. 7. The constraint was declared by a pin at the centre of the gear. A torque is applied at the pin to drive the gear mechanism and at the same time applied the load to the part.

Consider assembly of the main and pinion gear, the affect of the contact surface will result the stress around the gear part. To make it happen, surface contact need to be identify and it is driven by actuator element. It should be noted that, actuator elements are used to specify the relative motion of two points of a structure or mechanism.

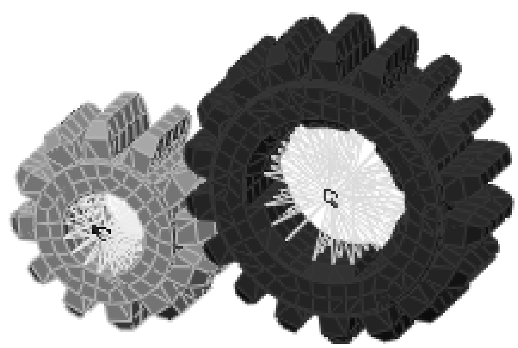

Fig. 7 Modeling of main and pinion gear with respect to the torque at a pin for each gear.

Consider the dynamic affect direct to the model, the angular velocity is applied and the time-step needs to be defined. All these kinetic and kinematic parameters were contributed the mechanism of gear motion.

Transient modeling

Transient dynamic analysis is used to predict the motion behavior of the component. The dynamic modeling includes a path and trajectory. To be clear, the path only considers the coordinate and vector of the line, while the trajectory integrated a time domain of the path.

Modeling the mechanism of the part by transient analysis parallel with the finite element method will make simulation become easier. The stress and displacement results will come together with the kinetic and kinematics information. The model is verified by repeating the program in an iteration form which is defined by time-step function.

Static stress analysis exist because of the load applied and by the same load, the part got a motion. That means the equation of the dynamic mechanism is dependent with the static stress result. Consider this factor, the error occur from static stress analysis will dramatically increase the error of the dynamic modeling especially by the involvement of multiple function. That is why the performance of error control is very important in combining static stress analysis and transient dynamic modeling.

DETERMINISTIC AND FEA RESULTS OF SURFACE CONTACT STATIC STRESS OF THE SPUR GEAR SYSTEM

This calculation is to determine surface contact stress between a pair of spur gear. In this analysis, the result from the deterministic method is compared with the finite element analysis (FEA).

Level 3 of the gear system analysis

Denoting by F, the magnitude of the tangential force between gear teeth, F = 3711.56 N. The area of the gear contact surface is $8.27*10^{-5}\,m^2$. The stress is determined as $449.96*10^5\ Nm^{-2}$. Force at gear with 28 teeth is calculated as 220472 N while force at gear with 14 teeth is equal to 93333 N.

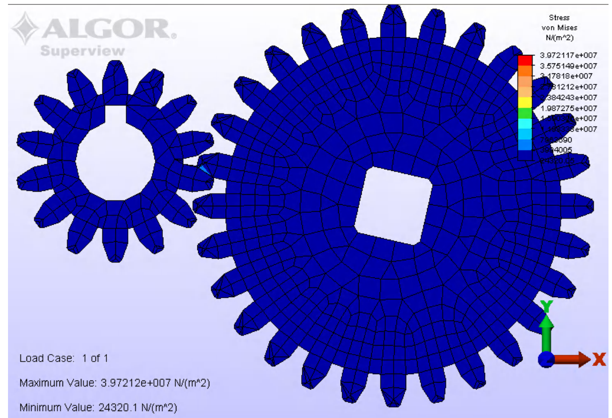

Fig. 8 Stress analysis of level 3

Fig. 8 shows the stress analysis of spur gear (Level 3) which is the maximum stress is $449.96x10^5\ Nm^{-2}$. The value stress from calculation is $397.21\ x10^5\ Nm^{-2}$ as shown in Table 1.

Level 2 of the gear system analysis

Denoting by F, the magnitude of the tangential force between gear teeth, F = 4241 N and the surface contact stress equal to $1.31*10^5\,Nm^{-2}$. Force at gear with 39 teeth is determined as 373334 N while force at gear 12 tooth is equal to 114867 N.

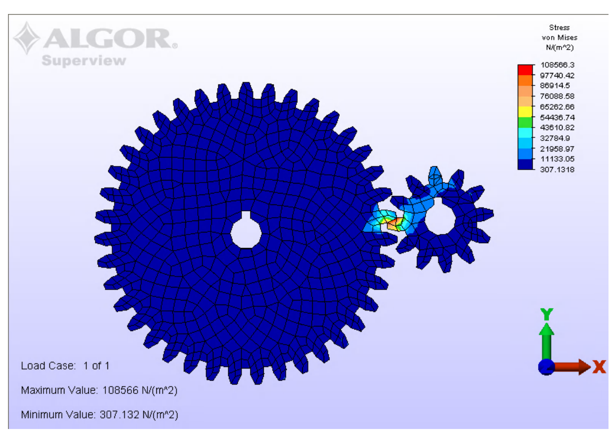

Fig. 9 Stress analysis of level 2

Fig. 9 shows the stress analysis of spur gear (Level 2) which is the maximum stress is $1.31\text{x}10^5$ Nm $^{-2}$. The value stress from calculation is $1.09\ \text{x}10^5$ Nm $^{-2}$ as shown in Table 1.

Level 1 of the gear system analysis

Finally, for level one, the magnitude of the tangential force between gear teeth, F = 1303 N and the surface contact stress, σ = 398471 Nm $^{-2}$. Force at gear 39 teeth is calculated as 114867 N and force at gear 12 teeth, is determined as 35343 N.

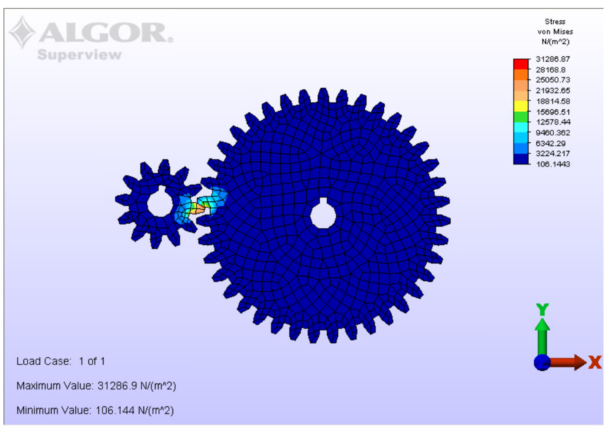

Fig. 10 Stress analysis of Level 1

Fig. 10 shows the stress analysis of spur gear (level 1) which is the maximum stress is $3.98\text{x}10^5$ Nm $^{-2}$. The value stress from calculation is $3.12\ \text{x}10^5$ Nm $^{-2}$ as shown in Table 1. The results comparison of maximum stress between deterministic and FEM show that the minimum error is 11.49 and the maximum error is 21.68.

The error occur because of the area calculation is not very precise. The touching area cannot specify accurately base on the mesh shape is not symmetry.

Table 1. Comparison results of the deterministic and FE maximum stress at the contact surface of the spur gear.

Gear	$\sigma_{Deterministic}$ (10^5 Nm^{-2})	σ_{FEM} (10^5 Nm^{-2})	**Error**
Level 3	449.96	397.21	11.49
Level 2	1.31	1.09	17.06
Level 1	3.98	3.12	21.68

The result of the maximum stress is happened at the contact surface of a pair of spur gear. The maximum force is transmitted by the maximum stress along the area contact. The calculation of the contact area needs to specify very well. Overload is directly effected the contact surface between the gear and probability of failure is among this region.

SIMULATION OF TRANSIENT FINITE ELEMENT

Fig. 11 illustrated the result of the stress distribution for the forth time-step. The Von-Mises stress result was represent by time-step within 0.0002 seconds for a single time-step. The maximum Von Mises stress is 234029 lbf/in^2. The maximum Von Mises stress is increase to 3596.89 lbf/in^2 at eighth time-step as shown in Fig. 12. However, the value decreases to 2893.84 lbf/in^2 at twelfth time-step as shown in Fig. 13. The value continues reduce to 1827.17 lbf/in^2 at sixteenth time-step as shown in Fig. 14.

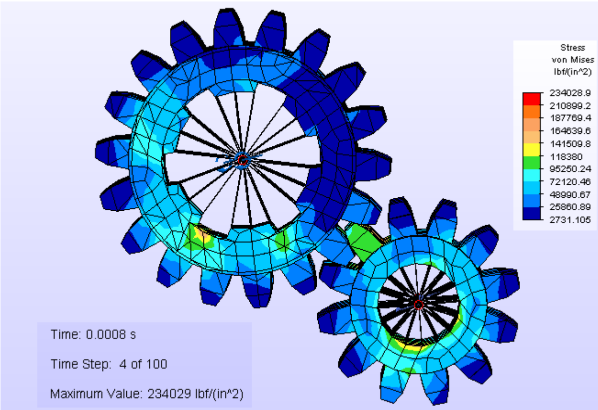

Fig. 11 Stress distribution for fourth time-step

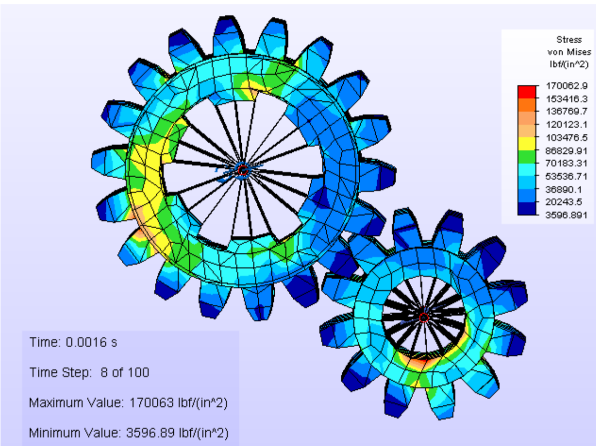

Fig. 12 Stress distribution for eighth time-step

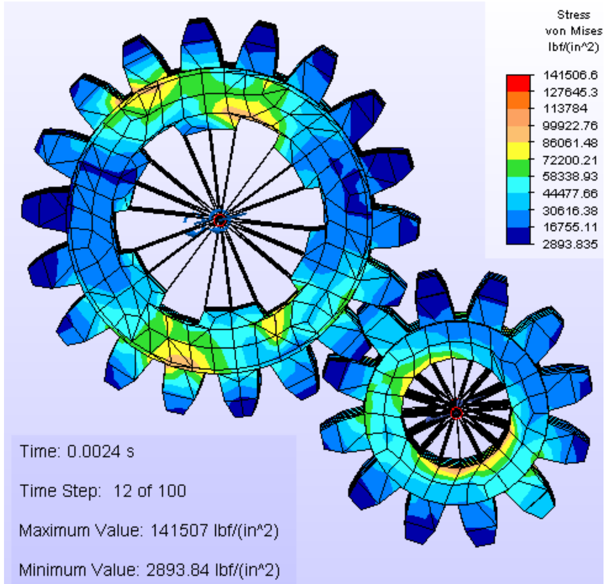

Fig. 13 Stress distribution for twelfth time-step

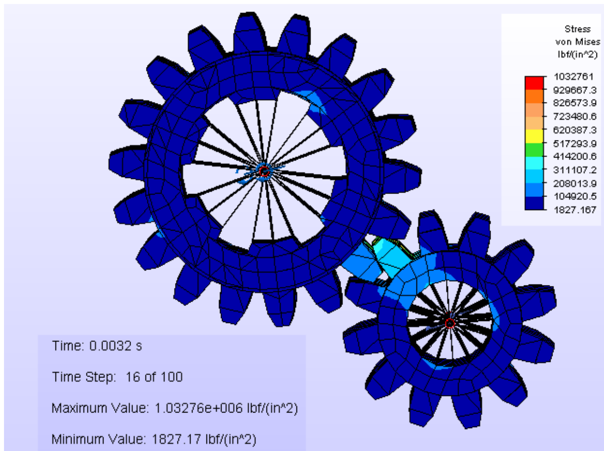

Fig. 14 Stress distribution for sixteenth time-step

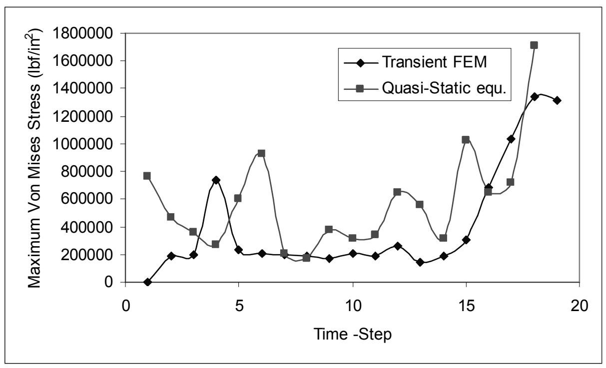

Fig. 15 Graph of maximum Von Mises Stress versus time-step.

Fig. 15 shown the result of transient finite element method validate with deterministic approach by quasi-static equation. Both the curve is not in uniform shape and the maximum Von Mises stress is variable along the time step of the event.

CONCLUSIONS

The result of the analysis has shown that the factor of dynamic properties will affect the stress distribution of the component by showing the changing of the stress value in time domain function. By using transient FEM, the stage of the analysis is shorts and creates an efficient work of design and modeling stage compare to static analysis by FEM. This method is useful and practical to be applied for a product development and fabrication of a new product.

References

Aberšek, B., *Failure analysis gear*, Faculty of Mechanical Engineering, University of Maribor, Maribor, Slovenia, 2004.

Ariffin, A.K. & Romlay, F.R.M., *Deterministic and Probabilistic Approach in Modelling of Fatigue Crack Propagation*, International Conference on Computational Methods, Singapore, 15-17 Dec. 2004.

Bensely, A., Jayakumar, S.S., Lal D.M., Nagarajan, G. and Rajadurai A., Failure investigation of crown wheel and pinion **,** *Engineering Failure Analysis, Volume 13, Issue 8, December 2006, Pages 1285-1292.*

Brauer, J., A general finite element model of involute gears, *Finite Elements in Analysis and Design,* 40 (2004) 1857–1872.

Brauer, J., Towards a dynamic model of a conical involute gear transmission, Licentiate Thesis, KTH, Stockholm, Sweden, 2000.

Haiba, M., Barton, D.C., Brooks, P.C. & Leveslay, M.C., *Review of life assessment techniques applied to dynamically load automotive components*, Computer and Structures, **80,** 2002, pp. 481-494.

Lunin, S., New method of gear geometry calculation, JSME International Conference on Motion and Power Transmissions, Fukuoka, Japan, 2001, 472–477.

Myszka, D.H., *Machines And Mechanisms, Applied Kinematic Analysis*, University of Dayton, Second Edition, 2002, 349 – 375.

Shigley, J.E., Mischke, C. R., *Mechanical engineering design*, International Edition 2003, Sixth Metric Edition, 825 – 856.

Shuting L., Finite element analyses for contact strength and bending strength of a pair of spur gears with machining errors, assembly errors and tooth modifications, *Mechanism and Machine Theory* 42 (2007) 88–114.