CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

This chapter provides preliminaries concepts of this research. It consist of the probability theory background, stochastic processes, stochastic integrals, formulation of DDEs, SDEs and SDDEs, the numerical methods and parameter estimation of SDDEs.

3.2 PROBABILITY THEORY BACKGROUND

The fundamental background of probability theories which are required in this research is presented in this section. Those definitions, theorems, principles and basic relations associated with this research have been taken from Gardiner, (1989), Kloeden and Platen (1992), Mao (2008) and Mikosch (1998).

3.2.1 Basic Concept of Probability Theory

Random quantity in mathematics is interpreted as random variables, most frequently denoted as $X(\omega)$. Random variables are measured on its probability space (Ω, F, P). Ω corresponds to set of all possible outcomes, also known as a sample space. Each possible outcomes in a sample space is denoted as $\omega \in \Omega$.

A and A^{c} are two distinct outcomes of trial, A^{c} is complement of A, which are subset of Ω. Not all events in Ω are observable or interesting events. A collection of all observable or interesting events is denoted as *F*. An ordered pair (Ω, F) is a measurable space and the elements of F are called *F*-measurable sets. These interpretations are defined mathematically in the following definitions.

Definition 3.1: Random Variable (Kloeden and Platen, 1992)

A random variable is a real function $X(\omega),\ \omega \in \Omega$ and measurable with respect to a probability measure *P*.

Definition 3.2: Probability Measure (Mao, 2008)

Probability measure P on sample space (Ω, F) is a function $P: F \to [0,1]$ such that

(i) $\forall A \in \Omega,$ then $0 \le P(A) \le 1$

(ii) $P(\Omega) = 1$

(iii) $P(A) + P(A^{c}) = 1$

(iv) Assume that $A_1, A_2, A_3, \ldots A_n, \ldots$ are random events which are belonging to Ω. If

$$\{(A_i \cap B_j) = \phi, \text{ for } i \ne j\} \text{ then } P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n) = 1$$

Definition 3.3: σ - algebra (Mao, 2008)

A family *F* is called σ - algebra which is subset of Ω. If the following properties hold

(i) $\phi \in F$, where ϕ illustrates empty set.

(ii) $A \in F \Rightarrow A^{C} \in A$, where $A^{C} = \Omega - A$ is complement of A in Ω.

(iii) For any sequence $A_n \subseteq F$, $\bigcup_{n=1}^{\infty} A_n \in F$.

Definition 3.4: Probability Space (Kloeden and Platen, 1992)

The triple (Ω, F, P) is a probability space which comprises of Ω (a set all of possible outcomes), a σ - algebra F of subsets Ω, called events and a probability measure is P on F .

The elementary events are grouped together in a set, Ω. σ - algebra is very important in studying a stochastic process because it aids as to communicate with the process situation (past, present and future). Modelling using SDEs and SDDEs involve continuous random variable. Hence, the following definition of continuous random variable and stochastic process are required.

Definition 3.5: Continuous Random variable (Mikosh, 1998)

$X(\omega)$ is a continuous random variable if there exist density function $f(x)$ such that

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(iii) $F(x) = \int_{-\infty}^{x} f(u)\,du$

Definition 3.6: Stochastic Process (Kloeden and Platen, 1992)

A stochastic process is a family of random variable $X = X(t, \omega)$ of two variables $t \in T$ and $\omega \in \Omega$ on probability space (Ω, F, P) which assumes real values and is $P-$measurable as a function of ω for fixed t. the $X(t, \cdot)$ is a random variable on Ω. While $X(\cdot, t)$ indicates trajectory or sample path of stochastic process.