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Contributed Paper

A New Efficient Approximation for Concentration Parameter of Circular Data

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ABSTRACT

New and efficient approximations of the concentration parameter of circular data using two approaches are proposed in this paper. First, we consider the power series expansion of mean resultant length and the estimate of concentration parameter may be obtained by the roots of the polynomial function. Secondly, we consider the power series expansion of the reciprocal of a Bessel function in the log-likelihood function of the concentration parameter and the estimate of concentration parameter may be obtained by minimizing the negative value of the log-likelihood function. It is found that the new approximation solutions are more efficient compared to the other existing approximation solutions especially for large κ.

Keywords: mean resultant length, concentration parameter, modified Bessel function, von Mises distribution, maximum likelihood estimator

1. INTRODUCTION

Circular statistics is a branch of statistics that involve circular data which may deal with direction or cyclic time and measured in degrees $(0,2\pi]$ or radian $(0°, 360°]$. For example, statistics that involve temporal period such as days in a week, Monday is said to be closer to Sunday than Wednesday. Similarly for data that measures rotation, 350° closer to 1° than 300°. The applications of circular statistics may be found in various areas such as biology, geology, geography and medical. For instance, biologist used circular statistics to study the orientation of an animal while meteorologist obtained circular data in the study of wind direction.

A circular distribution is a probability distribution of a circular random variable θ. The von Mises distribution is the most common continuous probability distribution on the circle and known as the circular normal distribution because of its close relationship to the normal distribution in real line and is a close approximation to the wrapped normal distribution. The von Mises