

FINITE ELEM

E CONNECTION

UNDER SEISMIC LOADING

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ABSTRAK

Sambungan "bolted end-plate" telah dipilih sebagai salah satu jenis sambungan yang penting dalam penyambungan "rasuk-tiang" dari segi aspek "kerumitan dalam analisis dan tingkah laku disebabkan oleh jumlah kesambungan elemen dan tingkah laku memindahkan bukan linear. Banyak eksperimen telah dijalankan dan bencana gempa bumi yang berlaku telah membuktikan bahawa sambungan separa tegar komposit menunjukkan prestasi cemerlang dalam aspek mekanikal dan seismik. Dalam bidang kejuruteraan, penggunaan sambungan separa tegar komposit boleh mengurangkan penggunaan keluli dan mempercepatkan proses pembinaan. Kajian ini bertujuan untuk menganalisis tingkah laku sambungan "bolted end-plate" di bawah pengaruh seismik dengan menggunakan kaedah "finite element method". Perbandingan antara sambungan "flush" dan "extended end-plate" dengan pengukuh atau tanpa pengukuh telah dikaji untuk meramalkan kekuatan dan kestabilan sambungan "end-plate". Kajian terdahulu menunjukkan bahawa penyambungan "end plate" yang dipanjangkan di kedua-dua belah dapat memberikan kekukuhan putaran, kemuluran dan keupayaan pelesapan tenaga yang diperlukan dalam penggunaan kerangka keluli seismic. Program LUSAS 14.0 telah digunakan untuk menganalisa lapan model dalam projek ini. Dimensi tiang adalah 250x300x2200 mm; rasuk 200x300x1200 mm; "end-plate" pengukuh 100x100x10 mm dan rasuk pengukuh 120x300x12 mm. Analisis linear telah dilakukan untuk menentukan bentuk berubahan, maksimum tekanan dan terikan. Di samping itu, dua analisis juga telah dijalankan untuk mendapatkan bentuk berudahan dengan analisis "eigenvalue" frekuensi dan parameter gerakan dalam analisis seismik. Berdasarkan keputusan ujian dan analisis, "end-plate" sambungan yang ada pengukuh pada tiang atau "end-plate" menunjukkan kestabilan yang tinggi dan rintangan kepada seismik berbanding dengan tanpa pengukuh. Ia menyimpulkan bahawa sambungan "endplate" boleh memberikan kekuatan yang lebih baik dan pelesapan tenaga dibandingkan dengan penyambungan "flush end plate" memerlukan sambunagan "moment resistant" di rantau gempa bumi. Pelbagai parameter geometri sambungan "end-plate" akan disyorkan untuk penyiasatan selanjutnya.

ABSTRACT

The bolted end-plate connection was chosen as an important type of beamcolumn connections for its complexity in the analysis and behaviour due to the amount of connection element and their inheritable non-linear behaviour. A great deal of experiments and earthquake disasters have shown that semi-rigid composite connections have excellent mechanical and seismic performance. In practical engineering field, the using of semi-rigid composite connections can reduce steel consumption and accelerate construction process. This study attempts to analyze the behavior of the bolt end-plate connection under seismic loading by using finite element method. Comparison between the flush and extended end-plate connections with or without stiffeners is made to predict the strength and stability of the end-plate connection. Previous research shows that typical end-plate connection extended on both sides can provide the strength, joint rotational stiffness, ductility and energy dissipation capacity required for use in seismic moment steel frames. LUSAS 14.0 program used to analysis the eight models in this project. The dimensions of columns are 250x300x2200 mm; beam is 200x300x1200 mm; end-plate stiffener 100x100x10 mm and column flange stiffener 120x300x12 mm respectively. Linear analysis is performed to determine the deformed, maximum stress and strain under the cyclic load condition. In addition, another two analyses are also conducted to obtain deformed mesh by eigenvalue frequency analysis and motion parameter by seismic analysis. Based on the test results and analysis, end-plate connection was stiffened eithers at column flange or end plate shows higher stability and resistance to seismic loading compare to without stiffeners. It concludes that end-plate connection can provide better strength and energy dissipation compare to flush end-plate connection which required for moment resistant connection in earthquake region. Various parameter geometric of end-plate connection is recommended for further investigate.

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LIST OF SYMBOLS

Øs	Shearing Rotation
Ø _{ep}	Gap Rotation
θ	Plane Angle
Δ	Change / Difference
a	Acceleration
d	Displacement
v	Velocity
Ν	Newton
KN	Kilo Newton
MPa	Mega Pascal
ms ⁻¹	Velocity
ms ⁻²	Acceleration

LIST OF ABBREVIATIONS

FEM	Finite Element Method
IMD	Interactive Modal Dynamics
LUSAS	London University Structural Analysis Software
JSH4	3D Joint Elements 4 Node
QSL8	QuadrilareralSemiloof with 8 Node
E _{max}	Maximum Strain
N _{max}	Maximum Stress
S	Specimen

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Connections are more complex than members to analyses. Therefore, connections are designed more conventionally than member elementdue to its essential part of any steel structure. The bolted end-plate connection was chosen as an important type of beam–column connections for its complexity in the analysis and behavior due to the amount of connection element and their inheritable non-linear behavior.

Welded connections have the advantage that member no necessary to drilled any holes and consequently have higher efficiencies. Conversely, welding in the field may be expensive, challenging, and time consuming. Welded connections are also vulnerable to failure by cracking under repeated cyclic loading due to fatigue which due to working loads such as earthquakes (seismic load or low-cycle fatigue). Therefore, the choice of using a particular type of connection is entirely to resistant working loads such as earthquakes (seismic load or low-cycle fatigue) due to it semi rigid behavior. A great deal of experiments and earthquake disasters have shown that semi-rigid composite connections have excellent mechanical and seismic performance. In practical engineering field, the using of semi-rigid composite connections can reduce steel consumption and accelerate construction process.

Kovacset al. (2008) stated that the steel framed structures used in earthquake regions can be categorized either as dissipative structures, where the energy of the

seismic load is absorbed by ductile plastic hysteretic behavior, oras non-dissipative structures, for structures where the energy input is sustained by elastic action. The global ductility in these structures consequences from the local ductility of the dissipative zones. The local ductile behavior of these dissipative regions could be quantified by the hysteretic moment–rotation response.

1.2 PROBLEM STATEMENT

The Northridge (USA) and Hyogoken-Nanbu (Japan) earthquakes, the confidence of structural engineers in welded moment resisting steel connections was strongly compromised due to the widespread brittle damage discovered in numerous frames. As a consequence of these observations, starting great deal of theoretical, investigation and experimental research activity is presently being developed in the number of country such as Europe, Japan and USA on the cyclic behavior of both welded and alternative configurations of beam-to-column connections.

At the ground's surface, earthquakes manifest themselves by shaking and displacement of the ground.Principally will resulting in more or less severe destruction to inflexible structures and otherbuildings. Consequently the behavior of the bolt end-plate connection under seismic effect due to bolted connections has partial strength and semi rigidity characteristic is very significant to analyses.

1.3 OBJECTIVES OF STUDY

In order to explore the finite element analyses of end-plate connection under seismic loading, this study is guided by the following research objectives:

- i. To investigate the behavior of the bolt end-plate connection under seismic loading.
- ii. To analyze the stress and strain behavior of end-plate connection subjected to seismic loads.
- iii. To study the comparison between the flush end-plate connections and extended endplate connections with or without stiffeners.

1.4 SCOPE OF STUDY

The scope of this study is to simulate a finite element analysis on the behavior of the end-plate bolt connection under seismic loading. Comparison will be made between the flush end-plate connections and extended end-plate connections with or without stiffeners. Besides, linear analysis, eigenvalue frequency analysis and seismic analysis will be conducted to investigate the end-plate connection subjected to seismic loads.

1.5 SIGNIFICANT OF STUDY

The priority to be obtained from this research is to analysis of end-plate joints under seismic loading conditions by analytical studies through simulation finite element analysis. The experimental programs are performed on bolted end-plate type joints of composite members under cyclic loading conditions with the purpose to study the cyclic loading response of the considered connection type.

The parameters investigated were, type of flush and extended end-plate, column stiffener and end-plate stiffener. The modeling results by finite element analysis are presented in terms of deformed mesh, stress and strain contour plot, eigenvalue frequencies and motion parameter. The results indicate that extended endplate connections have adequate strength, joint rotational stiffness, ductility and energy dissipation capacity required for use in seismic moment frames. Based on the test results and analysis, details on end-plate moment connections for seismic steel frames has provide enough rotation capacity and energy dissipation capacity under earthquake loading and its ultimate failure mode is ductile.

The finite element model is improved to enhance the defects of the finite element model used. These defects are; the long time need for the analysis and the inability of the contact element type to follow the behavior of moment-rotation curve under cyclic loading. As a contact element, the surface-to-surface element is used instead of node-to-node element to enhance the model. The FE results show good correlation with the experimental one. It is important to compare the moment rotation curve of the result from the LUSAS finite element analysis and experimental testing.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, seismic behavior of steel beam-column connections is discussed focusing on end-plate bolt Connections, seismic performances and analytical modeling approaches. Accurate modeling of the cyclic behavior of connections is very important in evaluation of seismic performances and design of steel moment-frame buildings.

2.2 BOLT CONNECTIONS

Bolt end-plate connection has the advantages of fast erection and easy fabrication compared to welded connection. However, details of bolted end-plate connections very enormously as well as their erection and fabrication. Many variables parameters will affect their behavior such asbolt spacing, bolt grad, bolt diameter, stiffness, numbers of bolt row and columns, end plate dimension, column and beam size, bolt pre-tension force, yield strength of steel and slip coefficient of contact surfaces.

Several studies and experimental have been conducted to investigate the behavior of the bolts end-plate moment connections. The principal focuses of the studies were topredict and measure the possible prying forces within bolted end-plate connections. The common of the bolt force prediction methods were developed using an analogy between the end-plate connection and an equivalent tee-stub in tension.

Several studies (Srouji et al., 1983a, b;Hendrick et al.,1984, 1985; and Borgsmilleret al.,1995) use a modified Kennedy approach to predict the bolt forces in extended, flush, stiffened, and unstiffened configurations of end-plate moment connection. The primary variation to the Kennedy method is modification of the distribution of the flange force to the particular bolt rows and an adjustment to the location of prying force.

Murray et al. (1992) investigated the behavior of end-plate moment connections with snug-tight bolts subject to cyclic loading. Eleven tests representing 6 different configurations of connection and the results were outstanding and reliable with the analytical predictions. It was concluded that end-plate moment connections 17 with snug-tight bolts provide slightly reduced stiffness when compared to fullytightened end-plate connections.

According toMurray et al. (2002)bolt connection can be classified in the following ways. Bolted beam-column connections can be sorted to groups by different circumstances, for example by beam and column profiles. The groups are by bolts working condition, forces to be transferred, types of connectionmember, and methods of connecting.

The bolted connections are referred to concentric connections which is force transfer in compression and tension structure show in Figure 2.1(a); eccentric connections which reaction transferring brackets or moment resisting beam to column connectionsshow inFigure 2.1(b). Shear connection the load spread at joint will ultimately be through shearing forces in the bolts show in Figure 2.1(c);tension connection load transmission is by pure tension in the bolts show in Figure 2.1(d).



Figure 2.1: Example type of bolt connections:(a) Concentric connections; (b)Moment connections; (c) Shear connections; (d) Tension connections

Source: Murray et al. (2002)

2.3 ENDS-PLATE CONNECTIONS

Sumner et al. (2000) has been conducted experimental and analytical research to develop unified design procedures for eight end-plate moment connection configurations subject to cyclic loading. In addition the appropriateness of end-plate moment connections for use in seismic force resisting moment frames was investigated.

End-plate connections consist of a plate that is shop-welded to the end of a beam that is then field bolted to the connecting member using rows of highstrength

bolts. The connections are primarily used to connect a beam to a column or to splice two beams together. There are two major types of end-plate connections: flush endplate, as shown in Figure 2.2 and extended end-plate, as shown in Figure 2.3.



Figure 2.2: Example of flush end plate connections

Source: Sumner et.al (2000)



(a) Unstiffened



(b) Stiffened



Source: Sumner et al. (2000)

A flush end-plate connection has an end-plate that all of the bolt rows are positioned inside the flanges and have not extend part beyond the outside of the connecting beam flanges. However, Flush end-plate connections can be stiffened or unstiffened. The stiffened configurations have gusset plates (stiffeners) welded to the beam web and to the end-plate on both sides of the web, as shown in Figure 2.2(b). The stiffeners can be positioned between the bolt rows or outside the bolt rows.

The extended end-plate connection is termed "extended" because that extends beyond the outside of the connecting beam flanges and at least one row of bolts is positioned outside the flanges on the extended part of the end-plate. In the case of extended end-plates used for seismic design, the end-plate is extended above and below both beam flanges that will be in tension under load. Extended end-plate connections also can be stiffened or unstiffened. The stiffened configurations have a gusset plate (stiffener) welded to the outside of the beam flange and to the end-plate, as shown in Figure 2.3(b). The stiffener is aligned with the web of the connecting beam to strengthen the extended portion of the end-plate.

Moment end-plate connections are further described by the number of bolts at the tension flange and the configuration of the bolt rows. For gravity and/or wind load applications, the end-plate connection is often designed to carry tension only at one flange. For seismic/cyclic loading, where the connection may experience load reversals, the end-plate is designed to carry tension at both flanges.

2.4 JOINT ROTATION OF END-PLATE CONNECTION

In this study by Shi et al. (2007), the joint rotation ø of the beam-to-column end-plate connection is defined as the anglechange or relative rotation of the centerlines of the beam flanges at the beam end from its original configuration.

The joint rotation includes two measures: the shearing rotation \emptyset_s , contributed by the panel zone of the column, and the gap rotation \emptyset_{ep} , caused by the relative