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NUMERICAL STUDY OF MIXED CONVECTION BOUNDARY LAYER FLOW NEAR THE LOWER STAGNATION POINT OF A HORIZONTAL CIRCULAR CYLINDER IN NANOFLUIDS

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ABSTRACT

The steady mixed convection boundary layer flow over a horizontal circular cylinder filled with nanofluids has been investigated numerically using different types of nanoparticles. These nanoparticles presences in the fluids increase the thermal conductivity up to approximately two times and thus enhancing the performance of the heat transfer. Enhancement of heat transfer is essential subject from an energy saving perspective, therefore the past years has witnessed extensive research on the convective heat transfer in nanofluids. In this paper, the nanofluid model proposed by Tiwari and Das has been used as this model is successfully applied in several papers. The problem is then being extended for the case of convective boundary conditions where the bottom surface of the cylinder is heated by convection from hot fluids. The resulting partial differential equations are solved numerically using implicit finite-difference scheme via Keller-box method. The effects of mixed convection λ, nanoparticle volume fraction, ϕ and conjugate parameters γ on the temperature and velocity profiles near the lower stagnation point of the cylinder $x \approx 0$ are examined. Detailed results are presented through figures for the temperature and velocity profiles. It is found that as the conjugate γ and mixed convection parameter λ increase, the temperature and velocity profile increases, while an increase in the nanoparticle volume fraction ϕ led to the increment of temperature profile and velocity profiles at $\lambda = -1$.

Keywords: mixed convection, nanofluids, convective boundary conditions, horizontal circular cylinder, numerical methods.

1. INTRDUCTION

Conventional fluids such as oil, ethylene glycol mixture, and water have low thermal conductivity and thus resulting the limitation in enhancing the performance of many engineering devices. An innovative way of improving the thermal conductivities of a fluid is to suspend metallic nanoparticle within it [1] . The process of suspended the metallic, non-metallic or polymeric nano-sized particle in a fluids are called nanofluids. This term was first introduced by in 1995 by Choi [2]. The nanofluid possesses a significantly higher thermal conductivity and single-phase heat transfer coefficients than the respective base fluids.

In conventional case, the suspended particles are of micrometer or even millimetre dimensions. This large dimension cause severe problem such as abrasion and clogging. Therefore, suspended large particles in fluids are not helping in increasing the thermal conductivity. Therefore, the discovery of nanofluid has boost the work in improving thermal conductivity. Since the size of nanoparticles is in nanometre-sized, they have the ability to flow smoothly through the micro channel easily [3], hence improve the thermal conductivity. The enhance thermal behaviour of nanofluids could provide a basis for an enormous innovation for heat transfer intensification, which is a major importance to a number of industrial sector including transportation, chemical and metallurgical sector, power generation, micro-manufacturing, as well as heating, cooling and air-conditioning. Because of that, there are plethora studies on the mechanism behind the enhanced heat transfer using nanofluids. The collection of papers on this topic is included in the review papers by [4-6].

Currently, there are several nanofluids models available in open literature. Buongiorno and Tiwari and Das are among the popular models for examining the convective transport of nanofluids. Tiwari and Das model analyzes the behaviour of nanofluids taking into account the solid volume fraction of the nanofluid. In this analysis, we use nanofluid model by Tiwari and Das as this model has been successfully applied in several papers such as [7-9]. Recently, [10] studied stagnation point flow over permeable stretching/shrinking sheet. They focused on Copper nanoparticles diluted in water base fluid. They concluded in their study that inclusion of nanoparticles into the base fluid produced an increase in the skin friction coefficient and the local Nusselt number. More recently, [11] also published their work in nanofluid focusing on square filled porous medium.

Above literature shows that many researchers has already applied the mathematical model proposed by Tiwari and Das in boundary layer flow problems. However, most of them incorporated constant wall temperature and constant heat flux at the boundary conditions. Therefore, this study aims to look at another type of boundary conditions namely convective boundary condition. We extended the problem solved by [12] for the case of convective boundary condition where the bottom surface of the cylinder is heated by convection from hot fluids. Three different nanoparticles namely Cu (copper), Al_2O_3 (aluminium) and TiO_2 (titanium) is immersed in water-based fluid to form a nanofluid. These nanoparticles presences in the fluids increase the thermal conductivity up to approximately two times and thus enhancing the performance of the heat transfer [13]. The resulting layer boundary conditions are solved numerically using an implicit finite-difference scheme, known as Keller-box method. Particular efforts have been focused of, temperature and velocity profiles near the stagnation point of the cylinder $x \approx 0$. The effects of these parameters are presented and illustrated through figures.

2. MATHEMATICAL FORMULATION

Consider the steady two-dimensional mixed convection flow of a nanofluids past a circular cylinder of radius a and wall temperature T_w. It is assumed that the free stream velocity is in the form of $u(x)$ and ambient temperature is T_∞. The coordinates x and y are measured along the surface of the cylinder, starting with the lower stagnation point and normal to it, respectively. The basic steady mixed convection boundary layer flow for a nanofluid in Cartesian coordinates after transformation are (see [12]).

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\frac{1}{(1-\phi)^{2.5}\left[1-\phi+(\phi\rho_s/\rho_f)\right]}\frac{\partial^2 u_e}{\partial x^2}+\left[\frac{\phi\rho_s}{(1-\phi)\rho_f+\phi\rho_s}\left(\frac{\beta_s}{\beta_f}\right)+\frac{(1-\phi)\rho_f}{(1-\phi)\rho_f+\phi\rho_s}\right]\lambda\sin x \tag{2}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{k_{nf}/k_f}{(1-\phi)+\phi(\rho C_p)}\frac{\partial^2\theta}{\partial y^2} \tag{3}$$

with the boundary conditions

$$\begin{aligned} &u=0,\ \ v=0,\ \ \frac{\partial\theta}{\partial y}=-\gamma(1-\theta) \quad \text{at} \quad y=0 \\ &u\to u_e(x),\quad \theta\to 0 \quad \text{as} \quad y\to\infty \end{aligned} \tag{4}$$

The solutions of Equations (1) to (3) are in the form of

$$\psi = xf(x,y),\quad \theta=\theta(x,y) \tag{5}$$

where ψ is the stream function which is defined in the usual way as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ automatically satisfies Equation (1). By substituting Equation (5) into Equations (2) and (3) and boundary conditions (4), and taking account that $u_e(x)=\sin x$, we obtain

$$\frac{1}{(1-\phi)^{2.5}\left[1-\phi+(\phi\rho_s/\rho_f)\right]}\frac{\partial^3 f}{\partial y^3}+f\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial f}{\partial y}\right)^2+\frac{\sin x\cos x}{x}+\frac{1}{(1-\phi)^{2.5}\left[1-\phi+(\phi\rho_s/\rho_f)\right]}\frac{\lambda\theta\sin x}{x}=x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial xdy}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right) \tag{6}$$

$$\frac{1}{\Pr}\frac{k_{nf}/k_f}{(1-\phi)+\phi(\rho C_p)}\theta''+f\theta'=x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right) \tag{7}$$

along with the boundary conditions:

$$\begin{aligned} &f=0,\ \ \frac{\partial f}{\partial y}=0,\ \ \frac{\partial\theta}{\partial y}=-\gamma(1-\theta) \quad \text{at} \quad y=0 \\ &\frac{\partial f}{\partial y}\to\frac{\sin x}{x},\quad \theta\to 0 \quad \text{as} \quad y\to\infty \end{aligned} \tag{8}$$

where k_{nf} is effective thermal conductivity of the nanofluids, k_f is the thermal conductivity of the fluid, k_s is the thermal conductivity of the solid, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid, ρ_f is the density of the fluid fraction, ρ_s is the density of the solid fraction and μ_f is the viscosity of the fluid fraction. ϕ is the nanoparticles volume fraction or solid volume fraction of the nanofluid, and β_f is the thermal coefficient expansion coefficient of the fluid fraction, and β_s is the thermal expansion coefficient of solid fraction, which are given by [7]. Near the lower stagnation point of the cylinder $x \approx 0$, Equations (6) and (7) reduce to the ordinary differential equations

$$\frac{1}{(1-\phi)^{2.5}\left[1-\phi+(\phi\rho_s/\rho_f)\right]}f'''+ff''-"(f')^2+1+\frac{1}{(1-\phi)^{2.5}\left[1-\phi+(\phi\rho_s/\rho_f)\right]}\lambda\theta=0 \tag{9}$$

$$\frac{1}{\Pr}\frac{k_{nf}/k_f}{(1-\phi)+\phi(\rho C_p)}\theta''+f\theta'=0 \tag{10}$$

subject to the boundary conditions

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$$f(0) = f'(0) = 0, \quad \theta'(0) = -\gamma\left(1-\theta(0)\right)$$
$$f(\infty) \to 1, \quad \theta(\infty) \to 0 \qquad (11)$$

Where primes denote the differentiation with respect to y, λ and γ is the mixed and conjugate parameter, respectively. It is worth to mention here that when $\phi = 0$ this is the case for regular fluid and Equations (9) and (10) reduced to those derived by [13]. Meanwhile, for the case when $\gamma \to \infty$ this case reduced to constant wall temperature as solve by [12].

3. RESULTS AND DISCUSSION

Equations (9) to (10) subject to boundary conditions (11) are solved numerically using a scheme based on efficient implicit finite difference method known as the Keller-box scheme. Three different types of nanoparticles have been considered in this study and representative results for temperature and velocity profile at lower stagnation point of cylinder $x \approx 0$ for the case of convective boundary conditions are presented here. The results have been obtained for the following range of nanoparticles volume fraction ϕ : $0 \le \phi \le 0.2$.

Figures-1 and 2 illustrate the velocity and temperature profiles near the lower stagnation point $x \approx 0$ for various value of mixed convection parameter λ ,respectively. It is seen from Figure-1 that, the velocity boundary layer thickness decreases from Cu to Al_2O_3 and TiO_2, which in turn increases the velocity gradient at the surface. Meanwhile for temperature profile in Figure-2, the thermal boundary layer thickness is not significantly sensitive to the mixed convection parameter for all type of nanoparticles. However, the thermal boundary layer thickness increases with an increase in parameter λ .

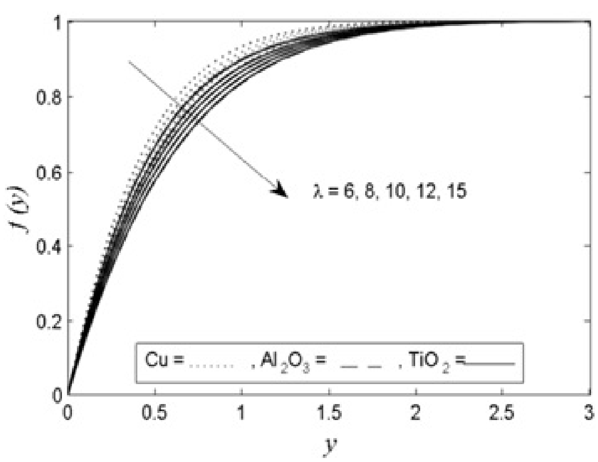

Figure-1. Velocity profiles at $x \approx 0$ using various nanoparticles with $\gamma = 0.3$, $\Pr = 6.2$, $\phi = 0.1$ and various values of λ

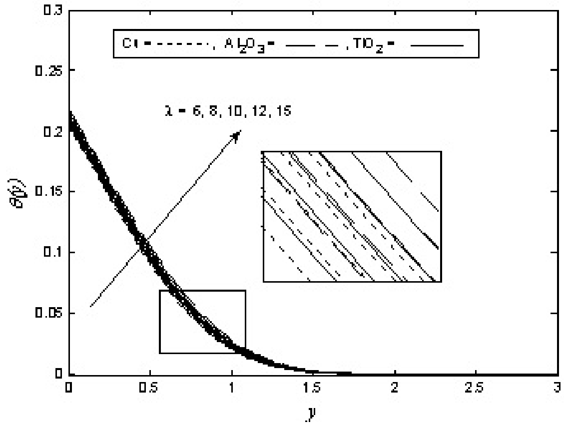

Figure-2. Temperature profiles at $x \approx 0$ using various nanoparticles with $\gamma = 0.3$, $\Pr = 6.2$, $\phi = 0.1$ and various values of λ .

Figures-3 and 4 display the effect of conjugate parameter γ on the velocity and temperature profiles. It is observed that increasing γ leads to the increase of the temperature and velocity profiles. The reason behind this, as the γ increases, the convective heat transfer from the hot fluid on the surface of the cylinder to the cold side increase leading to increase in both velocity and temperature profiles.

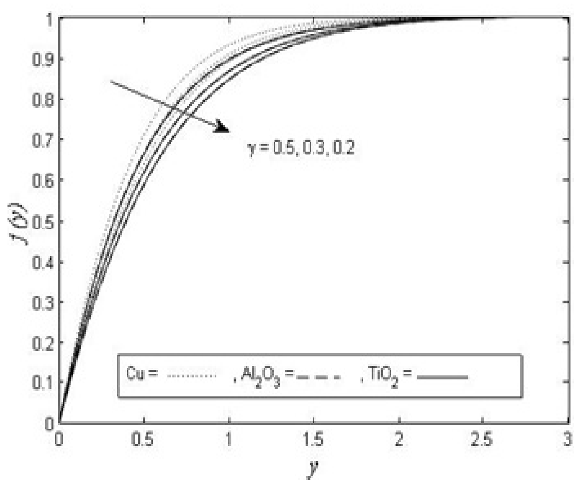

Figure-3. Velocity profiles at $x \approx 0$ using various nanoparticles with $\lambda = 0.5$, $\Pr = 6.2$, $\phi = 0.1$ and various values of γ .

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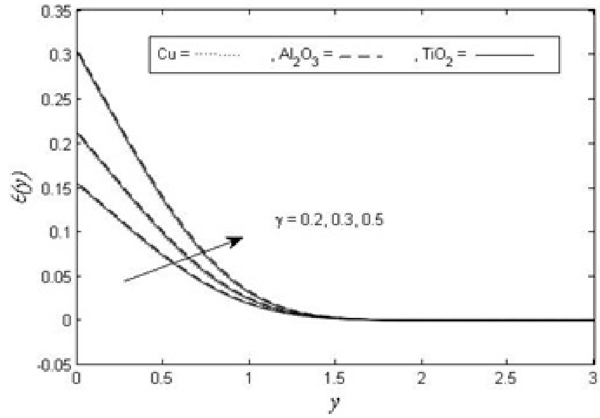

Figure-4. Temperature profiles at $x \approx 0$ using various nanoparticles with $\lambda = 0.5$, $\Pr = 6.2$, $\phi = 0.1$ and various values of γ

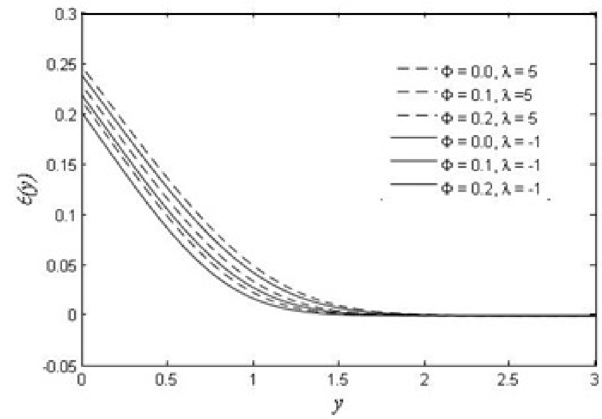

Figure-6. Temperature profiles at $x \approx 0$ using Cu nanoparticles with $\gamma = 0.3$, $\Pr = 6.2$, $\phi = 0, 0.1$, and 0.2 and $\lambda = -1$ and 5.

Figures-5 show the velocity profiles meanwhile Figure-6 show the temperature profile of Cu nanoparticle when $\lambda = -1$ and $\lambda = 5$ with the nanoparticle fraction $\phi := 0,\ 0.1,\ 0.2$ respectively. It can be seen that velocity boundary layer thickness decreases with the increment of ϕ at $\lambda = 5$. Result at $\lambda = -1$ are obtained otherwise. Meanwhile, the thermal boundary layer thickness increase as ϕ increases. Among these three nanoparticles, it can be concluded that Cu (nanoparticles with high density) has highest thermal conductivity followed by Al_2O_3 and TiO_2. From these figures, the profiles satisfy the far field boundary conditions asymptotically, and this support the validity of the numerical result obtained.

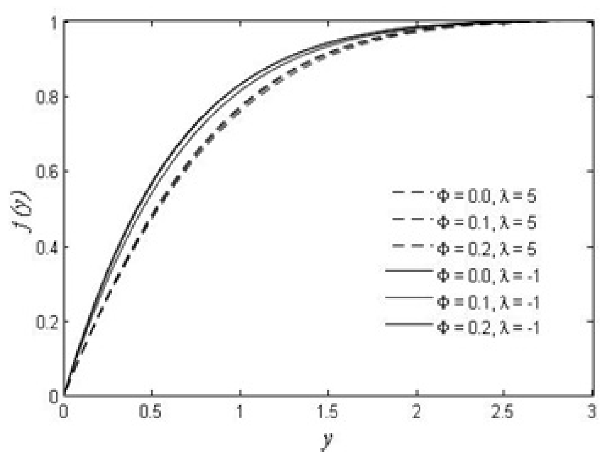

Figure-5. Velocity profiles at $x \approx 0$ using Cu nanoparticles with $\gamma = 0.3$, $\Pr = 6.2$, $\phi = 0, 0.1$, and 0.2 and $\lambda = -1$ and 5.

4. CONCLUSIONS

In this paper, we have investigated numerically the problem of the mixed boundary layer flow and heat transfer over a horizontal circular cylinder immersed in nanofluid with a convective boundary condition where the bottom surface of the cylinder is heated by convection from the hot fluids. The numerical solution is obtained using the Keller box method. We have focused on the effects of conjugate parameters γ, mixed convection parameter λ, the type of nanoparticles (Cu, Al_2O_3, TiO_2), and nanoparticle fraction ϕ on the flow and heat transfer characteristics. The following conclusion can be drawn from this study

- the thermal boundary layer thickness depends strongly on the mixed and the conjugate parameter. It is found that an increase in mixed and conjugate parameter results in a increase of the temperature profiles. Similar result were found for the velocity profile, increase in both mixed and conjugate parameter lead to increase in velocity profile.
- An increase in the nanoparticle volume fraction ϕ led to the increment of temperature profile and velocity profiles at $\lambda = -1$. For $\lambda = 5$, that velocity boundary layer thickness decreases with the increment of ϕ.
- Cu has the highest thermal conductivity compared than Al_2O_3 and TiO_2 nanoparticles.

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REFERENCES

[1] R. K. Tiwari and M. N. Das. 2007. Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. International Journal of Heat and Mass Transfer. 50: 2002-2018.

[2] Choi S. 1995. Enhancing thermal conductivity of fluids with nanoparticles. ASME International Mechanical Engineering Congress and Esposition, San Francisco, CA USA, ASME, FED 231/MD. pp. 99-105.

[3] K. Khanafer, K. Vafai and M. Lighstone. 2003. Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. International Journal of Heat and Mass Transfer. 46: 3639-363.

[4] J. Buongiorno. 2006. Convective Transport in Nanofluids. Journal of Heat Transfer. 128: 240-250.

[5] W. Daungthongsuk and S. Wongwises. 2007. A critical review of convective heat transfer nanofluids. Renew Sustainable Energy Reviews.11: 797–817.

[6] S. Kakaç and A. Pramuanjaroenkij. 2009. Review of convective heat transfer enhancement with nanofluids. International Journal of Heat and Mass Transfer. 52: 3187-3196.

[7] H. F. Oztop and E. Abu-Nada. 2008. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. International Journal of Heat and Fluid Flow. 29: 1326-1336.

[8] S. Ahmad, A. Mohd Rohni and I. Pop. 2011. Blasius and Sakiadis problems in nanofluids. Acta Mechanicca. 218: 195–204.

[9] N. A. Yacob, A. Ishak, I. Pop and K. Vajravelu. 2011. Boundary layer flow past a stretching/shrinking surface beneath an external uniform shear flow with a convective surface boundary condition in a nanofluid. Nanoscale Research Letters. 6: 314-320.

[10] N. Bachok, A. Ishak , R. Nazar and N. Senu. 2013. Stagnation-point flow over a permeable stretching/shrinking sheet in a copper-water nanofluid. Boundary Value Problems. 2013: 39-48.

[11] M. A. Sheremet, T. Grosan and I. Pop. 2015. Free Convection in a Square Cavity Filled with a Porous Medium Saturated by Nanofluid Using Tiwari and Das' Nanofluid Model. Transp Porous Med. 106: 595–610.

[12] L. Tham, R. Nazar and I. Pop. 2010. Mixed convection boundary layer flow from a horizontal circular cylinder in a nanofluid. International Journal of Numerical Method & Fluid Flow. 22: 576-606.

[13] J. H. Merkin. 1977. Mixed convection from a horizontal circular cylinder. International Journal of Heat and Mass Transfer. 20: 73-77.