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Prediction modelling of power and torque in end-milling

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Abstract
This paper presents the development of mathematical models for torque and power in milling 618 stainless steel using coated carbides cutting tool. Response surface method was use to predict the effect of power and torque in the end-milling. From the model, the relationship between the manufacturing process factors including the cutting speed, feed rate, axial depth and radial depth with the responses such as torque and power can be developed. Beside the relationship, the effect of the factors can be investigated from the equation developed. It can seen that the torque increases with decreases of cutting speed while increase of the feed rate, axial depth and radial depth. The acquired results also shown that the power increases with the increases of cutting speed, feed rate, axial depth and radial depth .It can be found that the second order is more accurate based on the variance analysis and the predicted value is closely match with the experimental result. Third- and fourth- order model generated for both response to investigate the 3- and 4-way interaction between the factors. The third and fourth order model shows that 3- and 4-way interaction found less significant for the variables.
Keywords: Torque, power, end-milling, response surface method

1 Introduction

In this work, experimental results were used for modeling using response surface roughness methodology (RSM) [1]. The RSM is practical, economical and relatively easy for use and it was used by lot of researchers for modeling machining processes [2,,3,4]. Mead and Pike [5] and Hill and Hunter [6] reviewed the earliest work on response surface methodology. Response surface methodology (RSM) is a combination of experimental and regression analysis and statistical inferences. The concept of a response surface involves a dependent variable y called the response variable and several independent variables x1, x2. . . xk [7]. The main aim of the paper is to investigate the effect of variables towards the responses and investigate the 3- and 4-way interaction between the factors.

2 Torque and Power model

The proposed relationship between the responses (torque and power) and machining independent variables can be represented by the following:

$$\tau = C\,(V^m F^n A_x^y A_r^z)\varepsilon' \quad (1)$$

$$P = C\,(V^m F^n A_x^y A_r^z)\varepsilon' \quad (2)$$

Where τ is the torque in Nm, P is the power in watt, V, F, A_x, and A_r are the cutting speed (m/s), feed rate (mm/rev), axial depth (mm) and radial depth (mm). C, m, n, y and z are the constants. Equation (1) and (2) can be written in the following logarithmic form:

$$\ln \tau = \ln C + m \ln V + n \ln F + y \ln A_x + z \ln A_r + \ln \varepsilon' \quad (3)$$

$$\ln P = \ln C + m \ln V + n \ln F + y \ln A_x + z \ln A_r + \ln \varepsilon' \quad (4)$$

Equation (3) and (4) can be written as a linear form:

$$\tau = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon \quad (5)$$

$$P = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon \quad (6)$$

Where τ is the torque in Nm, P is the power in watt, x_0 = 1(dummy variables), $x_1 = \ln V$, $x_2 = \ln F$, $x_3 = \ln A_x$, $x_4 = \ln A_r$ and $\varepsilon = \ln \varepsilon$,where ε is assumed to be normally-distributed uncorrelated random error with zero mean and constant variance, $\beta_0 = \ln C$ and β_1, β_2, β_3, and β_4 are the model parameters. The second model can be expressed as:

$$y'' = \beta_0 x_o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{11} x^2{}_1 + \beta_{22} x^2{}_2 + \beta_{33} x^2{}_3 + \beta_{44} x^2{}_4 + \beta_{11} x_1 x_2 + \beta_{12} x_1 x_3 + \beta_{13} x_1 x_4 + \beta_{14} x_2 x_3 + \beta_{15} x_3 x_4 \quad (7)$$

The values of β_1, β_2, β_3, and β_4 are to be estimated by the method of least squares. The basic formula is:

$$\beta = (x^T x)^{-1} x^T y \quad (8)$$

where x^T is the transpose of the matrix x and $(x^T x)^{-1}$ is the inverse of the matrix $(x^T x)$ and y is the value from experiment. The details of the solution by this matrix approach are explained in [1]. The parameters have been estimated by the method of least-square using a Matlab computer package.

2.1 Experimental design

To develop the first-order, a design consisting 27 experiments was conducted. Box-Behnken Design is normally used when performing non-sequential experiments. That is, performing the experiment only once. These designs allow efficient estimation of the first and second –order coefficients. Because Box-Behnken Design has fewer design points, they are less expensive to run than central composite designs with the same number of factors. Box-Behnken Design do not have axial points, thus can be sure that all design points fall within the safe operating. Box-Behnken Design also ensures that all factors are never set at their high levels simultaneously [8,9,10].Preliminary tests were carried out to find the suitable cutting speed, federate, axial depth and radial depth as shown in table 1.

Table 1 Levels of independent variables

Factors \ Coding of Levels	-1	0	1
Speed, V_c (m/s)	100	140	180
Feed, f (mm/rev)	0.1	0.2	0.3
Axial depth of cut, a_a, (mm)	1	1.5	2
Radial depth of cut, a_r, (mm)	2	3.5	5

2.1.1 Experimental details

The 618 stainless steel workpieces were provided in fully annealed condition in sizes of 65x170 mm. The tools used in this study are carbide inserts PVD coated with one layer of TiN. The inserts are manufactured by Kennametal with ISO designation of KC 735M.They are specially developed for milling applications where stainless steel is the major machined material. The end-milling tests were conducted on Okuma CNC machining centre MX-45VA. Every one passes (one pass is equal to 85mm), the cutting test was stopped. The same experiment has been repeated for 3 times to get more accurate result.

3 Results and discussion

3.1 First-order model for the torque and power model

The machining power is the product of cutting speed, υ and the cutting force, F_c. Thus the equation for the power is:

$$P = F_c\, \upsilon \tag{9}$$

Where P is the power in watt, υ is the cutting speed in m/min and F_c is the cutting force from the experiment in N. From the equation (9), the power can be calculated. The first order model from matlab for power and torque are:

$$P' = 6.1993 + 0.1633x_1 + 0.3025x_2 + 0.26x_3 + 0.2592x_4 \tag{10}$$

$$T' = 2.6215 - 0.1308x_1 + 0.2292x_2 + 0.1408x_3 + 0.2142x_4 \tag{11}$$

The predicted result from the first order model for power and torque shows in Figure 1 a and 1 b. Table 2 and 3 shows the 95% confidence interval for the experiments and analysis of variance. For the linear model, the p-value for lack of fit are 0.196 and 0.123.Therefore, the model is adequate. The transforming equations for each of the independent variables are:

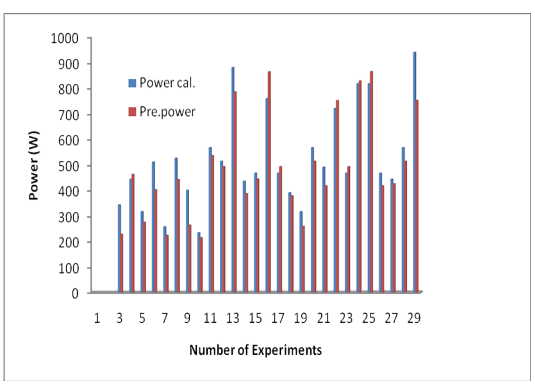
$$x_1 = \frac{\ln(V) - \ln(v)centre}{\ln(v)high - \ln(v)centre} \quad x_2 = \frac{\ln(F) - \ln(f)centre}{\ln(f)high - \ln(f)centre}$$

$$x_3 = \frac{\ln(A_x) - \ln(a_x)centre}{\ln(a_x)high - \ln(a_x)centre} \quad x_4 = \frac{\ln(A_r) - \ln(a_r)centre}{\ln(a_r)high - \ln(a_r)centre} \tag{12}$$

Equation (9) describing the torque and power model can be transformed using Equation (12) into the following form:

$$T' = 315.23(V^{-0.5204} F^{0.796719} A_x^{0.489432} A_r^{0.60055}) \tag{13}$$

$$P' = 3.7065(V^{0.6498} F^{1.0515} A_x^{0.9037} A_r^{0.7267}) \tag{14}$$



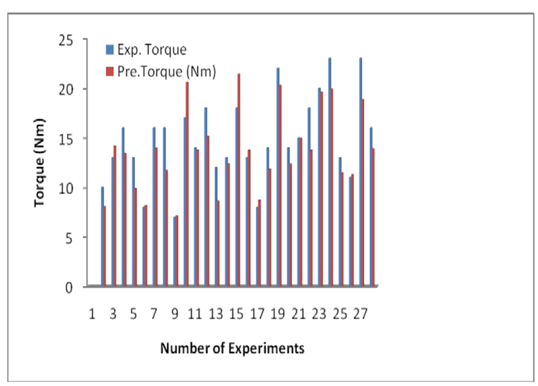


Figure 1: Comparison between predicted value and experimental for: (a) power, (b) torque

Table 2: ANOVA analysis for power

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	4	434.746	434.746	108.687	186.37	0.000
Linear	4	434.746	434.746	108.687	186.37	0.000
Residual Error	22	12.830	12.830	0.583		
Lack-of-Fit	20	12.830	12.830	0.642	5.1033	0.196
Pure Error	2	0.000	0.000	0.1258		
Total	26	447.576				

Table 3: ANOVA analysis for torque

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	4	802491	802491	200623	39.69	0.000
Linear	4	802491	802491	200623	39.69	0.000
Residual Error	22	111191	111191	5054		
Lack-of-Fit	20	109740	109740	5487	7.56	0.123
Pure Error	2	1451	1451	726		
Total	26	913682				

This result shows that feed rate has the most significant effect on the torque, follow by axial depth, radial depth and cutting speed. The equation shows that the torque increase with reducing the cutting speeds. Equation (9) is utilized to develop torque contour at the selected cutting speed, and feed rate. Figure 2(a) to 2(c) show the torque contour with selected cutting speed and feed rate. These contours help to predict the torque at any zone of experimental zone. From the contour, the torque reach the highest value at figure 2(c) where the value of cutting speed at its lower value ,feed rate, axial depth and radial depth at their maximum value. The torque can reach more than 25Nm in figure 2(c) .The lowest torque is in figure 1(a) when the cutting speed at its maximum value and the other factors at its maximum value. From this contour plot, the safety zone of torque can be selected for any experiment. This result shows that feed rate has the most significant effect on the power, follow by axial depth, radial depth and cutting speed. The equation shows that the power increasing with increasing feed rate, axial depth and radial depth. Equation (14) is utilized to develop power surface plot at the selected axial depth, radial depth. Figure 3(a) to 3(c) shows the cutting force plot with selected axial and radial depth.

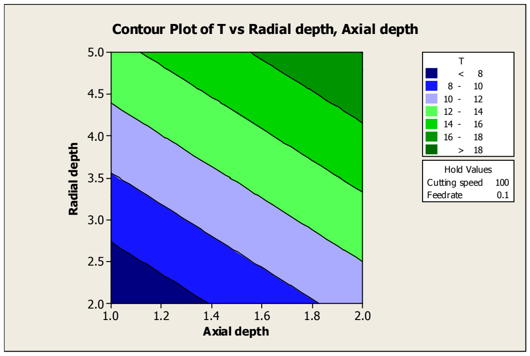


Figure 2 (a) Torque contours in the Axial depth-radial depth plane for cutting speed 100m/s and feed rate 0.1mm/rev

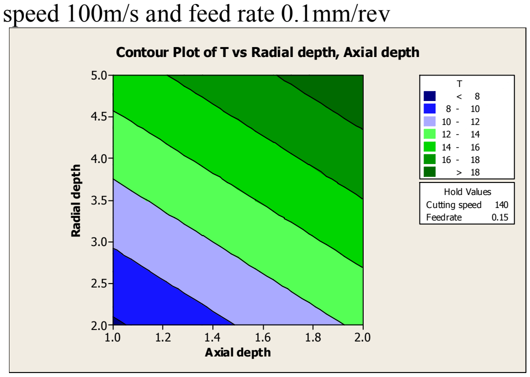


Figure 2 (b) Torque contours in the Axial depth-radial depth plane for cutting speed 140m/s and feed rate 0.15mm/rev

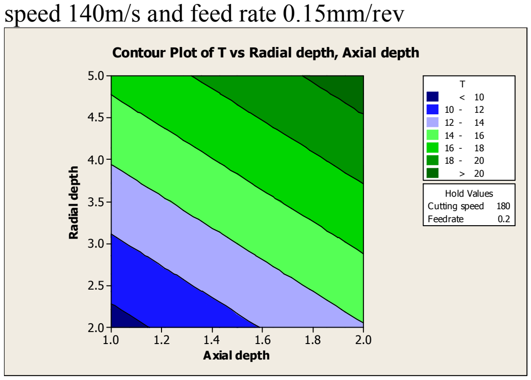


Figure 2(c) Torque contours in the Axial depth-radial depth plane for cutting speed 180m/s and feed rate 0.2mm/rev

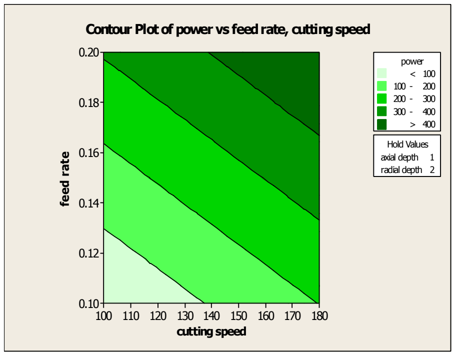


Figure 3 (a) Power surface plot in the Cutting speed-feed rate plane for axial depth 1 mm and radial depth 2mm

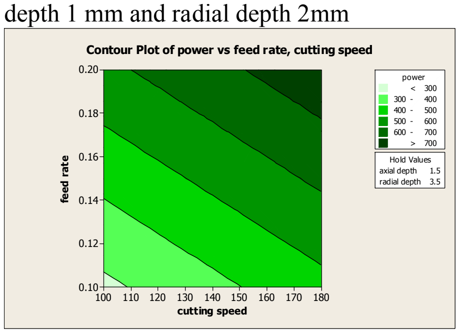


Figure 3 (b) Power surface plot in the Cutting speed-feed rate plane for axial depth 1.5 mm and radial depth 3mm

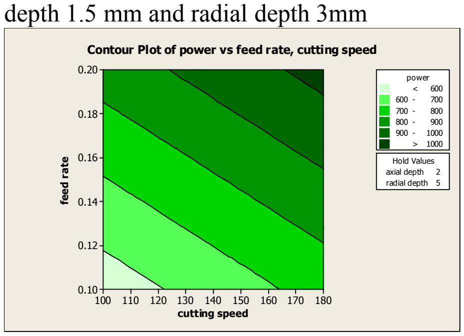


Figure 3 (c) Power surface plot in the Cutting speed-feed rate plane for axial depth 2 mm and radial depth 5mm

3.2 Second order and Third-order model for Torque and Power

The second-order model was postulated in obtaining the relationship between the responses and the machine independent variables. The model equations are:

$$P'' = 2.05074 - 0.031x_1 + 47.37x_2 + 2.97x_3 + 1.60x_4 + 0.00029x^2{}_1 - 50.17x^2{}_2 - 0.78x^2{}_3 - 0.14x^2{}_4$$
$$- 0.29x_1x_2 - 0.018x_1x_3 - 0.0094x_1x_4 + 24.3x_2x_3 + 12.8x_2x_4 + 0.80x_3x_4 \qquad (15)$$

$$T'' = -2080 - 17.22x_1 - 3099.72x_2 - 945.20x_3 - 113.22x_4 + 0.036x^2{}_1 - 3315.17x^2{}_2 + 146.30x^2{}_3 + 2.55x^2{}_4$$
$$+ 30x_1x_2 + 2.25x_1x_3 + 0.49x_1x_4 + 1633.40x_2x_3 + 116.63x_2x_4 + 63.52x_3x_4 \qquad (16)$$

The third-order model obtained to investigate the 3-way interaction between the variables. The third-order model as shown below

$$y''' = c + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_5x^2{}_1\beta 6x^2{}_2 + \beta_7x^2{}_3 + \beta_8x^2{}_4 + \beta_9x^3{}_1 + \beta_{10}x^3{}_2 + \beta_{11}x^3{}_3$$
$$\beta_{12}x^3{}_4 + \beta_{13}x_1x_2 + \beta_{14}x_1x_3 + \beta_{15}x_1x_4 + \beta_{16}x_2x_3 + \beta_{17}x_2x_4 + \beta_{18}x_3x_4 + \beta_{19}x_1x_2x_3 + \beta_{20}x_1x_2x_4 +$$
$$\beta_{21}x_1x_3x_4 + \beta_{22}x_2x_3x_4 \qquad (17)$$

From this model the most important points are the main effect, 2-way interaction and 3-way interaction. So the third order model can be reduced as below:

$$y''' = c + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_{13}x_1x_2 + \beta_{14}x_1x_3 + \beta_{15}x_1x_4 + \beta_{16}x_2x_3 + \beta_{17}x_2x_4 + \beta_{18}x_3x_4 \qquad (18)$$
$$+ \beta_{19}x_1x_2x_3 + \beta_{20}x_1x_2x_4 + \beta_{21}x_1x_3x_4 + \beta_{22}x_2x_3x_4$$

This model parameters can be solved using least squares method. β are the model parameters,x_1= cutting speed,x_2=feedrate,x_3=axial depth and x_4=radial depth. The third order model for torque and power are:

$$T''' = -176.95 + 1.3922x_1 + 1103.97x_2 - 7.6632x_3 + 56.7540x_4 - 7.8022x_1x_2 - 0.05x_1x_3 \qquad (19)$$
$$- 0.4237x_1x_4 + 50x_2x_3 - 353.753x_2x_4 + 3.4863x_3x_4 + 2.4792x_1x_2x_3$$

$$P''' = -9728 + 70x_1 + 1024x_2 - 2048x_3 + 4096x_4 - 19x_1x_3 - 31x_1x_4 + 23.552x_2x_3 - 10496x_2x_4 - 1408x_3x_4$$
$$- 168x_1x_2x_3 + 82x_1x_2x_4 + 14x_1x_3x_4 + 512x_2x_3x_4 \qquad (20)$$

The variance analysis for the torque and power carried out to determine the model adequate and significant of 3-way interaction for both model are shown in table 4 and 5. From the variance analysis both model not significant to the 3-way interaction since the p value>0.05.The third-order model adequate for torque and power since the p-value for lack of fit for torque is 0.818 and for power is 0.135. F-static for torque and power are 0.52 and 6.77.

Table 4 Variance analysis for third-order torque model

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	294.98	311.38	77.845	11.80	0.000
2-Way Interactions	6	51.30	73.36	12.226	1.85	0.156
3-Way Interactions	1	26.13	26.13	26.131	3.96	0.065
Residual Error	15	99.00	99.00	6.600		
Lack of Fit	12	67.00	67.00	5.583	0.52	0.818
Pure Error	3	32.00	32.00	10.667		
Total	26	471.41				

Table 5 Variance analysis for third –order power model

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	802491	600701	150175	35.60	0.000
2-Way Interactions	6	42054	39187	6531	1.55	0.244
3-Way Interactions	4	18520	18520	4630	1.10	0.402
Residual Error	12	50618	50618	4218		
Lack of Fit	10	49166	49166	4917	6.77	0.135
Pure Error	2	1451	1451	726		
Total	26	913682				

3.3 Fourth –order model for Torque and Power

The fourth-order model obtained to investigate the 4-way interaction between the variables. The fourth-order model as shown below:

$$y'''' = c + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x^2{}_1 \beta_6 x^2{}_2 + \beta_7 x^2{}_3 + \beta_8 x^2{}_4 + \beta_9 x^3{}_1 + \beta_{10} x^3{}_2 + \beta_{11} x^3{}_3$$
$$+ \beta_{12} x^3{}_4 + \beta_{13} x^4{}_1 + \beta_{14} x^4{}_2 + \beta_{15} x^4{}_3 + \beta_{16} x^4{}_4 + \beta_{17} x_1 x_2 + \beta_{18} x_1 x_3 + \beta_{19} x_1 x_4 + \beta_{20} x_2 x_3 + \beta_{21} x_2 x_4 + \beta_{22} x_3 x_4$$
$$+ \beta_{23} x_1 x_2 x_3 + \beta_{24} x_1 x_2 x_4 + \beta_{25} x_1 x_3 x_4 + \beta_{26} x_2 x_3 x_4 + \beta_{27} x_1 x_2 x_3 x_4 \qquad (21)$$

From this model the most important points are the main effect, 2-way interaction, 3-way interaction and 4-way interaction. So the fourth order model can be reduced as below:

$$y'''' = c + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_6 x_1 x_2 + \beta_7 x_1 x_3 + \beta_8 x_1 x_4 + \beta_9 x_2 x_3 + \beta_{10} x_2 x_4 + \beta_{11} x_3 x_4$$
$$+ \beta_{12} x_1 x_2 x_3 + \beta_{13} x_1 x_2 x_4 + \beta_{14} x_1 x_3 x_4 + \beta_{15} x_2 x_3 x_4 + \beta_{16} x_1 x_2 x_3 x_4 \qquad (22)$$

This model parameters can be solved using least squares method. β are the model parameters,x_1= cutting speed,x_2=feedrate,x_3=axial depth and x_4=radial depth. The fourth order model for torque and power are:

$$T'''' = -216 - 0.0625\ x_1 + 512\ x_2 + 30 x_3 + 63.5 x_4 + 5.5 x_1 x_2 + 0.7031 x_1 x_3 - 0.0625 x_1 x_4 + 444 x_2 x_3 - 155 x_2 x_4 - 3.75\ x_3 x_4$$
$$- 10 x_1 x_2 x_3 - 1 x_1 x_2 x_4 - 0.2266 x_1 x_3 x_4 - 139 x_2 x_3 x_4 + 2.75\ x_1 x_2 x_3 x_4 \qquad (23)$$

$$P'''' = -6272 + 18176x_2 + 768x_3 + 1856x_4 + 144x_1x_2 + 18x_1x_3 - 4x_1x_4 + 12672x_2x_3 - 4608x_2x_4 - 72\ x_3x_4 - 272x_1x_2x_3 - 20x_1x_2x_4 - 5x_1x_3x_4 - 4736x_2x_3x_4 + 90x_1x_2x_3x_4 \quad (24)$$

The variance analysis for the torque and power carried out to determine the model adequate and significant of 4-way interaction for both model are shown in table 6 and 7. From the variance analysis both model not significant to the 4-way interaction since the p value>0.05.The fourth-order model adequate for torque and power since the p-value for lack of fit for torque is 0.599 and for power is 0.123. F-static for torque and power are 0.99 and 7.53.

Table 6 Variance analysis for fourth-order torque model

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	359.667	176.667	44.167	6.38	0.007
2-Way Interactions	6	28.000	22.827	3.804	0.55	0.761
3-Way Interactions	4	7.629	4.327	1.082	0.16	0.956
4-Way Interactions	1	0.000	0.000	0.000	0.00	1.000
Residual Error	11	76.112	76.112	6.919		
Lack of Fit	9	62.112	62.112	6.901	0.99	0.599
Pure Error	2	14.000	14.000	7.000		
Total	26	471.407				

Table 7 Variance analysis for fourth-order power model

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	802491	372145	93036.3	20.22	0.000
2-Way Interactions	6	42054	38881	6480.2	1.41	0.294
3-Way Interactions	4	18520	17812	4453.1	0.97	0.463
4-Way Interactions	1	0	0	0.0	0.00	1.000
Residual Error	11	50618	50618	4601.6		
Lack of Fit	9	49166	49166	5462.9	7.53	0.123
Pure Error	2	1451	1451	725.7		
Total	26	913682				

4. Conclusion

Reliable torque model have been developed and utilized to enhance the efficiency of the milling 618 stainless steel. The torque equation show that feed rate, cutting speed, axial depth and radial depth plays the major role to produce the torque. The higher the feed rate, axial depth and radial depth, the torque generates very high compare with low value of feed rate, axial depth and radial depth. Contours of the torque outputs were constructed in planes containing two of the independent variables. These contours were further developed to select the proper combination of cutting speed, feed, axial depth and radial depth to produce the optimum torque. The higher the feed rate, cutting speed, axial depth and radial depth, the power generates very high compare with low value of feed rate, cutting speed, axial depth and radial depth. Dual response contours of torque and power are very useful in assessing the maximum attainable torque. The third order model and fourth order model very important to investigate the 3-way interaction and 2-way interaction. The third order model and fourth order model, shows that the 3-way interaction and 4-way interaction not significant.

Acknowledgement

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