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Topological Properties on the Space of Diffeomorphism of Dynamical Systems of Flat Electroencephalography

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ABSTRACT: Electroencephalograph is one of the useful and favoured instruments in diagnosing various brain disorders especially in epilepsy due to its non-invasive characteristic and ability in providing wealthy information about brain functions. To study epilepsy more effectively, a flattening method called Flat Electroencephalography was invented to view EEG signals on the real plane for further analysis. This novel method is well known for its ability to preserves the orientation and magnitude of EEG sensors and signals. As such, it certainly contains affluent information about seizure process. Since both events of epileptic seizure and Flat EEG are continuous processes, its states connectivity will be explored topologically. Generally, this paper study the topological properties of Flat Electroencephalography. In addition, the topology on the space of diffeomorphism of the dynamical systems of Flat Electroencephalography will also be studied.

KEYWORDS: flat electroencephalography, diffeomorphism, dynamical system, topology

1. INTRODUCTION

Epilepsy is a general term referring to a type of brain disorder that is characterized by seizures. Formally, it is defined by the Commission on Epidemiology and Prognosis and

International League Against Epilepsy 1993 as "the occurrence of at least two unprovoked seizure" [1], that is, it usually occur more than once and is not prompted. This brain disorder is not contagious i.e., cannot be transmit or spread from one person to another. However, it can happen to anyone in the world at any age and regardless of gender. Statistically, the number of people with epilepsy in the world is at least 50 million in total which is roughly 1.5% of the world populations [2].

During epilepsy, a miniature brainstorm take place within human brain by a group of brain cells called neurons. The electrical potential produce by these neurons are recordable non-invasively via electroencephalogram. In other words, recorded electrical potentials on EEG are the reflection of neuronal activity inside the brain [3]. This harmless and painless electrophysiological process are referred as electroencephalography and recorded signals are often portrayed on an electroencephalograph (EEG signal) for further analysis.

EEG is used extensively to diagnose epilepsies, classify the type of seizure occurring and locate the source of electrical activity [4] (Figure 1). According to [5, 6 and 7], this is one of the most important laboratory tests in identifying epilepsies. Perhaps the best reason for its wide acceptance is that EEG allows neurologists to analyze and locate damaged brain tissue and also to make planning prior to surgery to avoid or lessen the risk of injury on important parts of the brain. Recently, obtaining the graphic electrical activity inside the brain has in general become a necessary part of surgical [8].

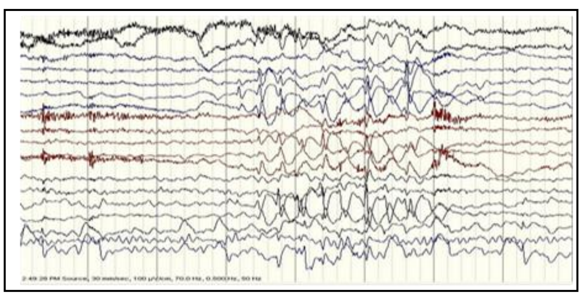

Figure 1 EEG signal

2. LITERATURE REVIEW

Two of the methods of controlling or stopping seizures are by taking anticonvulsant medications and undergo surgery. The former method is challenging in terms of finding the

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exact combinations of medications and dosage. Besides, this method is not promising because some seizures can still failed to be controlled in spite with the best available medication. Moreover, anticonvulsant medications carry undesired side effect [9, 10]. Frequently, it adversely affects the cognitive ability of patients (Bennett, 1992). Consequently, some patients choose to undergo epilepsy surgery. As a matter of fact, neurologists often suggest surgery as the best solution. The target of this method is to remove problematic epileptogenic tissue while sparing essential brain areas to avoid neurologic deficits. Therefore, knowing the exact location of problematic cells i.e., epileptic foci is crucial.

Various research using different concepts and techniques to identify epileptic foci has been established in the interest of creating better life for epileptic patients. For examples, via multimodality approach [12], by using large-area magnetometer and functional brain anatomy [13], examining correlations among electrodes captured by linear, nonlinear and multi linear data analysis technique [14], 3-D source localization of epileptic foci by integrating EEG and MRI data [15] and even approaches that are based on statistical tools such as Bayesian method [16] and maximum likelihood estimation approach by Jan et al. [17]. Each of the methods has their own advantages and weaknesses.

Fuzzy Topographic Topological Mapping (FTTM) is a fuzzy and topological based model for solving neuromagnetic inverse problem (Figure 2). Consisting of four components i.e., Magnetic Contour Plane (MC), Base Magnetic Plane (BM), Fuzzy Magnetic Field (FM) and Topographic Magnetic Field (TM), each of these components are homeomorphic to one another [18]. For a recorded data, the model is capable of portraying current sources topographically in three dimensions space. The advantage of this method is that, it does not need priori information and it is not time consuming [19].

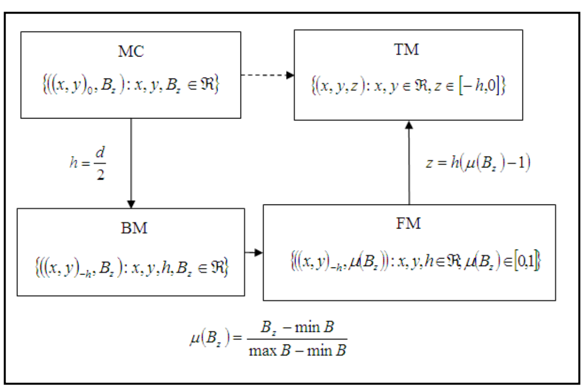

Figure 2 FTTM

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To study the brain disorder more effectively, a novel method called Flat electroencephalography (Flat EEG) (Figure 3) was invented to view EEG signals on the first component of Fuzzy Topographic Topological Mapping (FTTM). Thus, theoretically, by FTTM model, EEG signals can be portrayed in 3-dimension space. Built in [20] this method consists of a flattening procedure (a stereographic projection) which serves as the transformation from EEG to MC (Figure 4). The main scientific value of this method lies in its ability to preserve the orientation and magnitude of EEG signals to MC, allowing it to be compressed and analyzed.

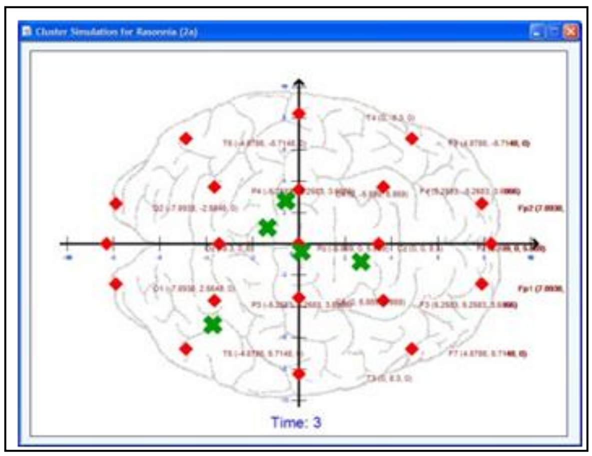

Figure 3 A random Flat EEG

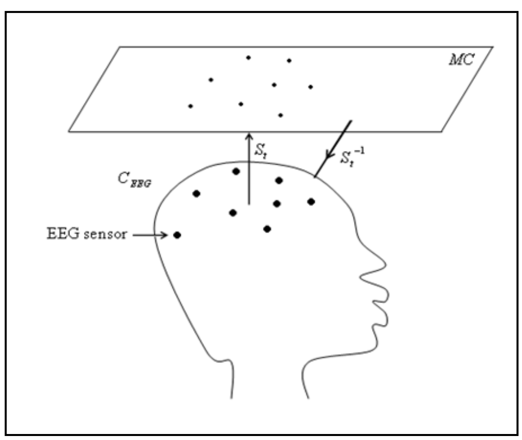

Figure 4 Stereographic projection

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Basically, Flat EEG is a platform which enables EEG signals be studied on the Cartesian plane mathematically, hence allowing the extraction of "hidden" information within EEG signals that cannot be obtained by traditional visual inspection. Furthermore, characteristics and properties obtained from this platform can be used to describe epileptic seizure since epileptic seizure and Flat EEG are topologically conjugated [21] (Figure 5).

$$\begin{tikzcd}
\cdots & a_0 \in A \arrow[r, "\varphi_t"] \arrow[d, leftrightarrow, "h"] & a_1 \in A \arrow[r, "\varphi_t"] \arrow[d, leftrightarrow, "h"] & a_2 \in A \arrow[r, "\varphi_t"] \arrow[d, leftrightarrow, "h"] & a_3 \in A \arrow[d, leftrightarrow, "h"] & \cdots \\
\cdots & b_0 \in B \arrow[r, "\psi_t"'] & b_0 \in B \arrow[r, "\psi_t"'] & b_0 \in B \arrow[r, "\psi_t"'] & b_0 \in B & \cdots
\end{tikzcd}$$

Figure 5 Pictorial representation of topological conjugacy.

There have been numerous researches utilizing various mathematical tools to visualize and extract "hidden" information within EEG signals via Flat EEG on a particular frame (or time) with promising outcome. For instance, implementation of Fuzzy C-Means (FCM) on Flat EEG enables one to compute the number of cluster centers along with its locations, hence made tracking brainstorm during epileptic seizure possible [20].

Apart from that, the algebraic study on Flat EEG demonstrates not only the possibility of transforming Flat EEG from one mathematical structure to another i.e., from topological to algebraic but also shows that Flat EEG can be decomposed into semigroup of upper triangular matrices under matrix multiplication and hence revealing that patterns exist in epileptic seizure process rather than chaotic [22].

Furthermore, study on the dynamic structure of Flat EEG, in particular, its structural stability from topological viewpoint, proves that Flat EEG in the presence of artifacts could still offer significant descriptions of electrical activities in the brain during seizure attack [23]. All this signifies that Flat EEG is a worthy platform to study epileptic seizure (Figure 6).

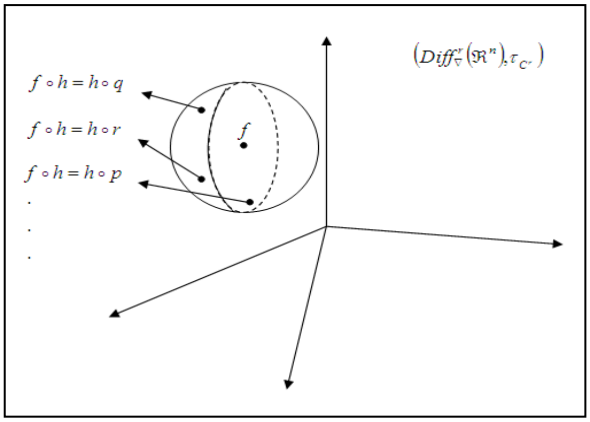

Figure 6 A structurally stable diffeomorphism f in $\left(Diff_{\nabla}^{r}(\Re^{n}), \tau_{C^{r}}\right)$.

The event of epileptic seizure and Flat EEG are continuous processes. This is evident from the embedment of time parameter in the processes. As a matter of fact, they can be modelled generally as dynamical systems [21]. To date, the connectivity of the states in the events has not been studied. Furthermore, compactness is one of the topological properties that are preserved by homeomorphism. This notion is defined via open covering. Metrizable spaces and compact Hausdorff spaces are two of the well-behaved classes of spaces to deal in mathematics. This is due to the many useful properties they display, which can be used in proving theorems. Thusly, in this study, the events will be explored topologically, anticipating several characteristic of the events to be revealed.

3. MATERIAL AND METHODS

Consider the flow of Flat EEG modelled in [21]:

$\psi_t(y)$ where $\psi : \Re \times Y \to Y$ such that the following two properties are fulfilled:

i. $\psi_0(y) = y \quad \forall y \in Y = \Re^n$, and

ii. for all t and $s \in \Re$

$\psi_t \circ \psi_s = \psi_{t+s}$

Here, for any $y_k \in Y = \Re^n$, $\psi_t(y)$ is generally defined as $\psi_i(y_k) = y_i$ i.e., the state of the system which initiate from y_k at time i is y_i.

Basically, for each $y_k \in Y = \Re^n$, the flow of Flat EEG defines an event of Flat EEG (EoFE). Before going any further, we introduce the following notations

$\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ - An EoFE with order topology

Theorem 1 (Davis, 2005): *Every metrizable space is Hausdorff* [25].

Theorem 3 (Davis, 2005): *Every metrizable space is paracompact* [25].

Theorem 3 (Davis, 2005): *Every metrizable space is normal* [25].

Theorem 4 (Smirnov Metrization Theorem): *A space X is metrizable if and only if it is a paracompact Hausdorff space that is locally metrizable.*

Theorem 5 (Nagata-Smirnov Metrization Theorem): *A space X is locally metrizable if and only if X is regular and has a basis that is countable locally finite.*

Theorem 6 (Kelly, 1975): *Every locally compact Hausdorff space is a Tychonoff space.* [26]

Theorem 7: $\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ is Hausdorff.

Proof: Every metrizable space is Hausdorff (Theorem 1). Since $\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ is metrizable, it is Hausdorff. ■

Theorem 8: $\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ is paracompact.

Proof: Every metrizable space is paracompact (Theorem 2). Since $\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ is metrizable, it is paracompact. ■

Theorem 9: $\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ is normal.

Proof: Every metrizable space is normal (Theorem 3). Since $\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ is metrizable, it is normal. ■

Theorem 10: $\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ is locally metrizable.

Proof: By Smirnov Metrization Theorem, Theorem 8 and Theorem 9. ■

Theorem 11: $\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ is regular and has a basis that is countable locally finite.

Proof: By Nagata-Smirnov Metrization Theorem. ■

Theorem 12: $\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ is locally compact.

Proof: For each point $x_i \in O_{\psi_t(y_k)}$, there exist points $x_i, x_j \in O_{\varphi_t(x_k)}$ where $j < i < k$, such that x_i lies in the open interval (x_j, x_k) which in turn is contained in the closed and bounded (hence compact) neighborhood $[x_j, x_k]$.■

Theorem 13: $\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ is a Tychonoff space.

Proof: Since $\left(O_{\psi_t(y_k)}, \tau_{O_{\psi_t(y_k)}}\right)$ is a locally compact Hausdorff space (by Theorem 6, 7, and 12). ■

In the following, the coincidence of the topology defined by using weak subbasis and subspace topology on the set of diffeomorphisms $Diff_{\nabla}^{r}\left(\Re^{m+1}\right)$ i.e., the space of all dynamical systems of Flat EEG will be showed. Firstly, consider the definition of weak topology or C^r compact-open topology

Definition 1 (Morris, 1994): *Let $g \in C^r\left(\Re^{m+1}, \Re^{m+1}\right)$; (φ, U) and (ψ, V) be charts on $\Re^{m+1}$ and $\Re^{m+1}$; $K \subset U$ be a compact set such that $g(K) \subset V$; and $0 < \varepsilon \le \infty$. Define the weak subbasis neighborhood* [27]

$$N^r(g; (\varphi, U), (\psi, V), K, \varepsilon)$$

to be the set of C^r *maps* $k:\Re^{m+1} \to \Re^{m+1}$ *such that* $k(K) \subset V$ *and*

$$\left\| D^k\left(\psi g \varphi^{-1}\right)(x) - D^k\left(\psi k \varphi^{-1}\right)(x)\right\| < \varepsilon$$

for all $x \in \varphi(K)$, $k = 0,1,2,3,\ldots,r$

Then the weak (or C^r *compact-open) topology on the set* $C^r\left(\Re^{m+1}, \Re^{m+1}\right)$ *is generated by the weak subbasis neighborhood.*

Let this topology be denoted as $\tau_{C^r\left(\Re^{m+1},\Re^{m+1}\right)}$, and an element of it as β, then

$$\tau_{C^r\left(\Re^{m+1},\Re^{m+1}\right)} = \left\{ \beta \mid \beta = \bigcup_{i=1} \left(\bigcap_{j=1}^{n} \left\{ N^r\left(g;(\varphi,U),(\psi,V),K,\varepsilon\right)\right\}_j \right)\right\} \tag{1}$$

where $g \in C^r\left(\Re^{m+1}, \Re^{m+1}\right)$; (φ,U) and (ψ,V) be charts on $\Re^{m+1}$ and $\Re^{m+1}$; $K \subset U$ be a compact set such that $g(K) \subset V$; and $0 < \varepsilon \le \infty$

In other words, elements in $\tau_{C^r\left(\Re^{m+1},\Re^{m+1}\right)}$ is the unions of finite intersections of the weak subbasis $N^r\left(g;(\varphi,U),(\psi,V),K,\varepsilon\right)$.

Next, denote the flow of Flat EEG as f, then the set $Diff_{\nabla}^r\left(\Re^{m+1}\right)$ such that $f \in Diff_{\nabla}^r\left(\Re^{m+1}\right) \subset Diff^r\left(\Re^{m+1}\right) \subset C^r\left(\Re^{m+1}, \Re^{m+1}\right)$ can be regarded as the set of all C^r-diffeomorphism where all these diffeomorphism are assumed to have no periodic trajectory, have the same number of trajectories and equilibrium points.

Since the interest is on the set of diffeomorphisms i.e., $Diff_{\nabla}^r\left(\Re^{m+1}\right)$, a new topology must be defined. This topology can obtained by using subspace topology which can be defined as follow

$$\tau_{Diff_{\nabla}^r\left(\Re^{m+1}\right)} = \left\{ \alpha \cap \beta : \alpha \subseteq Diff_{\nabla}^r\left(\Re^{m+1}\right), \beta \in \tau_{C^r\left(\Re^{m+1},\Re^{m+1}\right)}\right\} \tag{2}$$

Notice that the topology which starts from defining weak subbasis on the set $Diff_{\nabla}^{r}\left(\Re^{m+1}\right)$ will be the same as the subspace topology. Formally, the weak subbasis on the set $Diff_{\nabla}^{r}\left(\Re^{m+1}\right)$ can be formulated as follow

Let $g \in Diff_{\nabla}^{r}\left(\Re^{m+1}\right)$; (φ,U) and (ψ,V) be charts on $\Re^{m+1}$ and $\Re^{m+1}$; $K \subset U$ be a compact set such that $g(K) \subset V$; and $0 < \varepsilon \leq \infty$. Define the weak subbasis neighborhood

$$N^{r}\left(g;(\varphi,U),(\psi,V),K,\varepsilon\right)$$

to be the set of C^{r} maps $k:\Re^{m+1} \to \Re^{m+1}$ such that $k(K) \subset V$ and

$$\left\| D^{k}\left(\psi g \varphi^{-1}\right)(x) - D^{k}\left(\psi k \varphi^{-1}\right)(x) \right\| < \varepsilon$$

for all $x \in \varphi(K)$, $k = 0,1,2,3,\ldots,r$

Then the weak (or C^{r} compact-open) topology on the set $Diff_{\nabla}^{r}\left(\Re^{m+1}\right)$ is generated by the weak subbasis neighborhood.

Briefly, this topology can be defined as

$$\tau_{Diff_{\nabla}^{r}\left(\Re^{m+1}\right)} = \left\{ \gamma \mid \gamma = \bigcup_{i=1} \left(\bigcap_{j=1}^{n} \left\{ N^{r}\left(g;(\varphi,U),(\psi,V),K,\varepsilon\right) \right\}_{j} \right) \right\} \tag{3}$$

where $g \in Diff_{\nabla}^{r}\left(\Re^{m+1}\right)$; (φ,U) and (ψ,V) be charts on $\Re^{m+1}$ and $\Re^{m+1}$; $K \subset U$ be a compact set such that $g(K) \subset V$; and $0 < \varepsilon \leq \infty$

The following theorem proves our claim that topology which starts from defining weak subbasis on the set $Diff_{\nabla}^{r}\left(\Re^{m+1}\right)$ will be the same as the subspace topology

Theorem 1: $\left\{ \alpha \cap \beta : \alpha \subseteq Diff_{\nabla}^{r}\left(\Re^{m+1}\right), \beta \in \tau_{C^{r}\left(\Re^{m+1},\Re^{m+1}\right)} \right\} = \left\{ \gamma \mid \gamma = \bigcup_{i=1} \left(\bigcap_{j=1}^{n} \left\{ N^{r}\left(g;(\varphi,U),(\psi,V),K,\varepsilon\right) \right\}_{j} \right) \right\}$

Proof:

$$\left\{\alpha\cap\beta:\alpha\subseteq Diff_{\nabla}^{r}\left(\Re^{m+1}\right),\beta\in\tau_{C^{r}\left(\Re^{m+1},\Re^{m+1}\right)}\right\}$$

$$=\left\{\alpha\cap\bigcup_{i=1}\left(\bigcap_{j=1}^{n}\left\{N^{r}\left(g;(\varphi,U),(\psi,V),K,\varepsilon\right)\right\}_{j}\right):\alpha\subseteq Diff_{\nabla}^{r}\left(\Re^{m+1}\right),\beta\in\tau_{C^{r}\left(\Re^{m+1},\Re^{m+1}\right)}\right\} \qquad \text{by (1)}$$

$$=\left\{\lambda:\text{ for some }\lambda\subseteq Diff_{\nabla}^{r}\left(\Re^{m+1}\right)\right\} \qquad \text{since } \alpha\cap\bigcup_{i=1}\left(\bigcap_{j=1}^{n}\left\{N^{r}\left(g;(\varphi,U),(\psi,V),K,\varepsilon\right)\right\}_{j}\right)\subseteq\alpha$$

$$=\left\{\bigcup_{i=1}\left(\bigcap_{j=1}^{n}\left\{N^{r}\left(g;(\varphi,U),(\psi,V),K,\varepsilon\right)\right\}_{j}\right):\text{ for some }\left\{N^{r}\left(g;(\varphi,U),(\psi,V),K,\varepsilon\right)\right\}_{j}\subseteq Diff_{\nabla}^{r}\left(\Re^{m+1}\right)\right\}$$

$$=\left\{\gamma\mid\gamma=\bigcup_{i=1}\left(\bigcap_{j=1}^{n}\left\{N^{r}\left(g;(\varphi,U),(\psi,V),K,\varepsilon\right)\right\}_{j}\right)\right\}.\blacksquare$$

6. CONCLUSION

In this paper, several topological properties on the event of Flat EEG has been established. Generally, the event of Flat EEG are well behaved. Futhermore, the topology defined via the weak subbasis on the set of all diffeomorphism containing the dynamical system of Flat EEG will be the same as its subspace topology.

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REFERENCES

1. Panayiotopoulos, C. P. (2010). Atlas of Epilepsies 1st Edition. Springer-Verlag, London Limited.; Chapter 1, pp: 5.

2. Jahnecke, C. A. N., Schwarz, L., Sovierzoski, M. A., Azevedo, D. F. M. and Argoud, F. I. M. (2007). C++ Video-EEG Processing System With Sights to the Epileptic Seizure Detection, World Congress on Medical Physics and Biomedical Engineering. IFMBE Proceedings, 14(8), 1052-1055.
3. Bergey, G. K. and Franaszczuk, P. J. (2001). Epileptic seizure are characterized by changing signal complexity. Elsevier, Clinical Neurophysiology, 112(2), 241-249.
4. Sanei, S. and Chambers, J. (2007). EEG signals processing, 1st Edition,John Wiley & Sons, England, 52, 166.
5. Popp, A. J. and Deshaies, E. M. (2007). A Guide to the Primary Care of Neurological Disorders, 2nd Edition, American Associations of Neurosurgeons, Thieme, pp: 85.
6. Yudofsky S. C. and Hales, R. E. (2008). The American Psychiatric Publishing textbook of neuropsychiatry and behavioral neurosciences 5th Edition, American Psychiatric Publishing, USA, pp: 655.
7. Gilhus, N. E., Barnes, M. R., and Brainin, M. (2011). European Handbook of Neurological Management 2nd Edition, John Woley& Sons, Chapter 4, pp: 159.
8. Miller, J. W. and Cole, A. J. (2011). Is it necessary to define the ictal onset zone with EEG prior to performing respective epilepsy surgery? Elsevier, Epilepsy Behaviour. 20(2), 178-181.
9. Sharon, R., David, I. M. and Shlomo, Y. (2004). Anticonvulsant efficiency, behavioral performance and cortisol levels: a comparison of carbamazapine (CBZ) and a fatty acid compound (SR-3). Elsevier, Psychoneuroendocrinology. 29 (2): 113-124.
10. Stefan, H. and Feuerstein, T. J. (2007). Novel Anticonvulsant Drugs. Elsevier, Pharmacology & Therapeutics, 113(1), 165-183.
11. Bennett, T. L. (1992). Ciritical issue in Neuropsychology: The Newropsychology of Epilepsy 2nd Edition, Springer, pp: 88.
12. Desco, M., Pascau, J., Pozo, M. A., Santos, A., Reig, S., Gispert, J. and Garcia, B. P. (2001) Multimodality localization of epileptic foci. Physiology and Function from Multidimensional Images, Proceedings of SPIE, Medical Imaging, 4321(1), pp: 362-370.
13. Tiihonen, J., Hari, R., Kjola, M., Nousiainen, U. and Vapalahti, M. (2004). Localization of epileptic foci using large-area magnetometer and functional brain anatomy. Wiley, Annals, of Neurology, 27(3), 283-290.
14. Evim, A., Canan, A. B., Haluk, B. and Bulent, Y. (2006). Computational analysis of epileptic focus localization. *ACTA Press Anaheim, BioMed'06 Proceedings of the 24th IASTED international conference on Biomedical Engineering*, pp 317-322.
15. Natasa, M., Malek, D., Ilker, Y. and Prasanna, J. (2003). 3-D source Localization of Epileptic Foci Integrating EEG and MRI Data. *Springer-Verlag, Brain Topography*, 16(2), 111-119.
16. Toni, A., Aopa, N., Matti, S. H., Liro, P. J. Jouko, L., Aki, V. and Mikko, S. (2005). Bayesian analysis of the neuromagnetic inverse problem with l^p-norm priors. *Elsevier, Neuroimage*, 26(3), 870-884.
17. Jan, C. D. M., Fetsje, B., Pawel, G., Cezary, A. S., Maria, I. B. and Heethaar, R. M. (2004). A Maximum-Likelihood Estimator For Trial-to-Trial Variations in Noisy MEG/EEG Data Sets. *IEEE Transactions on Biomedical Engineering*, 51(2), 2123-2128.
18. Liau, L. Y. (2001). Homeomorfisma Antara S^2 Dan E^2MelaluiStrukturPermukaan Riemann Serta Deduksi Teknik Pembuktiannya Bagi Homeomorfisma Pemetaan Topologi Topografi Kabur (FTTM). *Master Thesis, Universiti Teknologi Malaysia, Skudai*.

19. Tahir, A., Rashdi, S. A., Liau, L. Y., Fauziah, Z. and Wan Eny Zarina, W. A. R. (2005). Homeomorphisms of Fuzzy Topographic Topological Mapping (FTTM). *UTM, Matematika*, 21(1): 35-42.
20. Fauziah, Z. (2008). Dynamic Profiling Of Electroencephalography Data During Seizure Using Fuzzy Information Space. *Universiti Teknologi Malaysia, PhD Thesis, Skudai*.
21. Tahir, A. and Tan, L. K. (2010). Topological Conjugacy Between Seizure and Flat Electroencephalography. *Science Publications*, *American Journal of Applied Sciences*, 7(3), 1470-1476.
22. Faisal, A. M. B. and Tahir, A. (2010). EEG Signals During Epileptic Seizure as a Semigroup of Upper Triangular Matrices. *Science Publications, American Journal of Applied Sciences*, 7(4), 540-544.
23. Tan, L. K. and Tahir, A. (2014). Structural Stability of Flat Electroencephalography. *Marsland Press, Life Science Journal-Acta Zhengzhou University Overseas Edition*, 11(8), 165-170.
24. Munkres, J. R. (2000). Topology 2nd Edition. *Pearson Prentice Halls*, Inc, USA, pp: 84.
25. Davis, W. S. (2005). Topology. (International Edition). *New York: McGraw-Hill*, pp: 160.
26. Kelley, J. L. (1975). General Topology. *Brikhauser*, pp: 147.
27. Morris, W. H. (1994). System of differential equations that are competitive or cooperative. IV: Structural stability in three-dimensional systems. *Siam Journal on Mathematical Analysis*, 21(5), 1225-1234.