Adaptive Beamforming Algorithm based on Generalized Opposition-based Simulated Kalman Filter

Kelvin Lazarus[1], Nurul Hazlina Noordin[1], Kamil Zakwan Mohd Azmi[1], Nor Hidayati Abdul Aziz[1,2], and Zuwairie Ibrahim[1]

[1]Faculty of Electrical and Electronics Engineering
Universiti Malaysia Pahang
26600 Pahang, Malaysia

[2]Faculty of Engineering and Technology
Multimedia University
Jalan Ayer Keroh Lama
75450 Melaka, Malaysia

Abstract— In this paper, a new population-based metaheuristic optimization algorithm named Generalized Opposition-based Simulated Kalman Filter (GOBSKF) is proposed as adaptive beamforming algorithm. GOBSKF is an improved version of Simulated Kalman Filter (SKF). Adaptive beamforming algorithm based on GOBSKF is compared with previously published work which is Adaptive Mutated Boolean PSO (AMBPSO) and Minimum Variance Distortionless Response (MVDR) for different noise level. The results show that GOBSKF is proven to be better than AMBPSO and MVDR.

Keywords— Adaptive Beamforming; Generalized Opposition-based Simulated Kalman Filter

1. Introduction

Wireless communication system involves time-varying signal propagation environment where the user and interferers move around with time. Adaptive beamforming is used to adapt continuously to the changing electromagnetic environment by continuously adjusting the weights of individual elements in an array. In adaptive beamforming techniques, the main beam must be pointed towards the direction of the desired signal and nulling the interference at the same time.

Since adaptive beamforming is considered as an optimization problem, there are a number of optimization algorithms that were applied in adaptive beamforming application [1]–[22]. These algorithms are used to find the optimum weights so as to steer the main beam towards the signal of interest (SOI) and null the interference to maximize the signal to interference plus noise ratio (SINR) value.

In this paper, a new metaheuristic optimization technique named Generalized Opposition-based Simulated Kalman Filter (GOBSKF) is proposed for adaptive beamforming application. GOBSKF is an extended version of Simulated Kalman Filter (SKF). SKF is introduced by Ibrahim et. al. [23] and has been applied to solve various optimization problems [24]–[29]. Generalized Opposition-based Learning is one of many Opposition-based Learning method used to improve optimization algorithm [30]. The main idea of Opposition-based Learning is to check the current solution continuously with the opposite solution within the search space in order to get a better approximation of current solution [31]. GOBSKF is used to estimate weights of individual elements in an array which gives the maximum signal to interference plus noise ratio (SINR) value.

2. SYSTEM MODEL

Assuming an array antenna of M elements and N number of interfering signal with signal of interest (SOI) of kth time sample, $s(k)$, arriving at angle θ_0, and signal not of interest (SNOI), $i_1(k)$, $i_2(k)$, $i_3(k)$, …, $i_{N-1}(k)$, $i_N(k)$, arriving at angle θ_1, θ_2, θ_3, …, θ_{N-1}, θ_N, as shown in Fig. 1 [32].

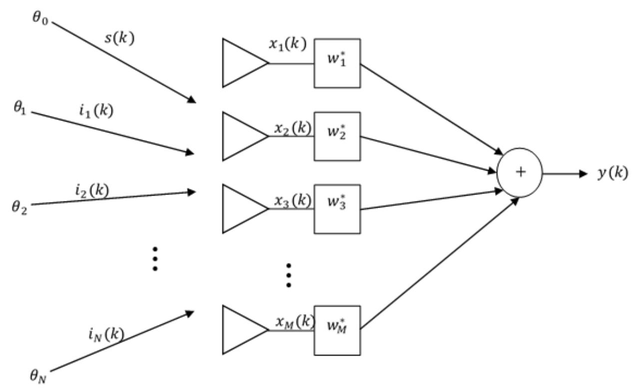


Figure 1: Array Model

The array output, $y(k)$ can be represented by

$$y(k) = \bar{w}^H \cdot \bar{x}(k) \tag{1}$$

where w stands for weights for individual elements, H for Hermitian transpose and $\bar{x}(k)$ is the signal vector. The signal vector $\bar{x}(k)$ can be further expanded as

$$\bar{x}(k) = \bar{a}_0 s(k) + [\bar{a}_1\ \bar{a}_2\ \cdots\ \bar{a}_N] \cdot \begin{bmatrix} i_1(k) \\ i_2(k) \\ \vdots \\ i_N(k) \end{bmatrix} + n(k) = \bar{x}_s(k) + \bar{x}_i(k) + \bar{n}(k) \tag{2}$$

where $\bar{a}_i$ stands for M-element array steering vector for θ_i direction of arrival; $\bar{x}_s(k)$ is the desired signal vector, $\bar{x}_s(k)$ the interference signal vector and with noise, $\bar{n}(k)$.

The total array output, $y(k)$ is expanded as

$$y(k) = \bar{w}^H \cdot [\bar{x}_s(k) + \bar{x}_i(k) + \bar{n}(k)] = \bar{w}^H \cdot [\bar{x}_s + \bar{u}(k)] \tag{3}$$

where the undesired signal, $\bar{u}(k)$ is formulated as

$$\bar{u}(k) = \bar{x}_i(k) + \bar{n}(k) \tag{4}$$

Next, the array correlation matrices are calculated for both desired signal, $\bar{R}_{ss}$ and undesired signal, $\bar{R}_{uu}$. The weighted array output power for desired signal, σ_s^2 is given as below

$$\sigma_s^2 = E[|\bar{w}^H \cdot \bar{x}_s|^2] = \bar{w}^H \cdot \bar{R}_{ss} \cdot \bar{w} \tag{5}$$

where the signal correlation matrix, $\bar{R}_{ss}$, can be formulated as

$$\bar{R}_{ss} = E[\bar{x}_s \bar{x}_s^H] \tag{6}$$

For undesired signal, the weighted array output power, σ_u^2 is

$$\sigma_u^2 = E[|\bar{w}^H \cdot \bar{u}|^2] = \bar{w}^H \cdot \bar{R}_{uu} \cdot \bar{w} \tag{7}$$

where the undesired correlation matrix, $\bar{R}_{uu}$ is formulated as

$$\bar{R}_{uu} = \bar{R}_{ii} + \bar{R}_{nn} \tag{8}$$

with $\bar{R}_{ii}$ denotes as the interference correlation matrix and $\bar{R}_{nn}$ as noise correlation matrix.

The fitness function is the signal to interference plus noise ratio, SINR, can be formulated as [31]. GOBSKF will find the optimum weights, w which give maximum SINR value.

$$SINR = \frac{\sigma_s^2}{\sigma_u^2} = \frac{\bar{w}^H \cdot \bar{R}_{ss} \cdot \bar{w}}{\bar{w}^H \cdot \bar{R}_{uu} \cdot \bar{w}} \tag{9}$$

3. GENERALIZED OPPOSITION-BASED SIMULATED KALMAN FILTER

Generalized Opposition-based Simulated Kalman Filter (GOBSKF) is extended from the existing Simulated Kalman Filter (SKF). SKF is inspired by the estimation capabilities of Kalman Filter [23]. In SKF, each agent acts as an individual Kalman Filter. Consider t as the number of iteration and N number of agents, estimated solution of optimization problem of the i^{th} agent at a time t, $\boldsymbol{X_i(t)}$ is defined as

$$\boldsymbol{X_i}(t) = \boldsymbol{x_i^1}(t), \boldsymbol{x_i^2}(t), \dots, \boldsymbol{x_i^d}(t), \dots, \boldsymbol{x_i^D}(t) \tag{10}$$

where $\boldsymbol{x_i^d}(t)$ is the estimated state of i^{th} agent in d^{th} dimension and D is the maximum number of dimensions. In an iteration, t, all the agents are involved in the calculation of fitness, and the agent with best fitness is identified as $\boldsymbol{X_{best}(t)}$. SKF performs a simulated measurement process to get to the true value, $\boldsymbol{X_{true}}$. The $\boldsymbol{X_{true}}$ will update when better $\boldsymbol{X_{true}}$ solution is found.

GOBSKF begins with random initialization of agents, $\boldsymbol{X(0)}$, within the search space. The initial value for error covariance estimate, $P(0)$, process noise, Q and the measurement noise, R, are defined at initial stage.

Iteration begins with fitness calculation of i^{th} agent, $fit_i(\boldsymbol{X(t)})$. $\boldsymbol{X_{best}(t)}$ is updated based on the type of problem. For minimization problem,

$$\boldsymbol{X_{best}}(t) = min_{i \in 1,2,\dots,N} fit_i(\boldsymbol{X(t)}) \tag{11}$$

whereas, in maximization problem,

$$\boldsymbol{X_{best}}(t) = max_{i \in 1,2,\dots,N} fit_i(\boldsymbol{X(t)}) \tag{12}$$

Later, $\boldsymbol{X_{true}}$ will be updated when better solution is found ($\boldsymbol{X_{best}}(t) < \boldsymbol{X_{true}}$ for minimization problem or $\boldsymbol{X_{best}}(t) > \boldsymbol{X_{true}}$ for maximization problem).

In prediction process, the following time-update equations

$$x_i(t|t+1) = x_i(t) \tag{13}$$

$$P(t|t+1) = P(t) + Q \tag{14}$$

are used to make a prediction for the state and error covariance estimates given the prior estimates. The next step is measurement which acts as a feedback to the estimation process. The following equation simulates the measurement for each agent:

$$z_i(t) = x_i(t|t+1) + sin(rand \times 2\pi) \times |x_i(t|t+1) - x_{true}| \tag{15}$$

The final process is estimation. In this process, Kalman gain, $K(t)$, is computed as follows:

$$K(t) = \frac{P(t|t+1)}{P(t|t+1) + R} \tag{16}$$

After that, the measurement-update equations are used to improve the *a posteriori* estimates from the *a priori* estimates by making use of the measurement.

$$x_i(t+1) = x_i(t|t+1) + K(t) \times z_i(t) - x_i(t|t+1) \tag{17}$$

$$P(t+1) = (1 - K(t)) \times P(t|t+1) \tag{18}$$

Using the measured position as feedback and influenced by the Kalman gain value, $K(t)$, each agent updates an estimate of the optimum for that corresponding iteration.

Fig. 2 shows the illustration of Opposition-based Learning where x stands for current position and ox stands for opposite position in domain [a,b].

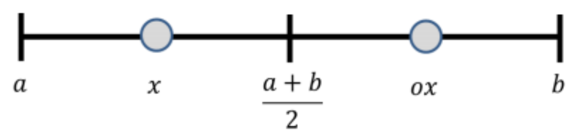


Figure 2: Opposite point, ox, defined in domain [a,b]

The opposite population, $x\_ob_{i,j}$, is generated using equation 19, where $x\_ob_{i,j}$ is the opposite solution for current solution, $x_{i,j}$, in domain [a,b], the $rand$ is random numbers between -1 and 1, N represents the number of agents, and D is the number of dimensions.

$$x\_ob_{i,j} = rand \times (a_j + b_j) - x_{i,j} \tag{19}$$
$$i = 1,2, \ldots N \; ; j = 1,2, \ldots D$$

In GOBSKF, the opposite population is generated within the current population's range as in equation 20 where MIN_j^x and MAX_j^x represent the lowest and highest agent in the current population, respectively. The execution of opposite population generation depends on the value of jumping rate, *Jr*.

$$x\_ob_{i,j} = rand \times (MIN_j^x + MAX_j^x) - x_{i,j} \tag{20}$$
$$i = 1,2, \ldots N \; ; j = 1,2, \ldots D$$

If the generated opposite solution exceeds the converged search space, the opposite solution is reinitialized using equation 21.

$$x\_ob_{i,j} = MIN_j^x + rand \times (MIN_j^x - MAX_j^x) \tag{21}$$
$$i = 1,2, \ldots N \; ; j = 1,2, \ldots D$$

Figure 3 shows the flowchart for GOBSKF. The step will be repeated until the maximum number of iterations, $tMax$, is reached.

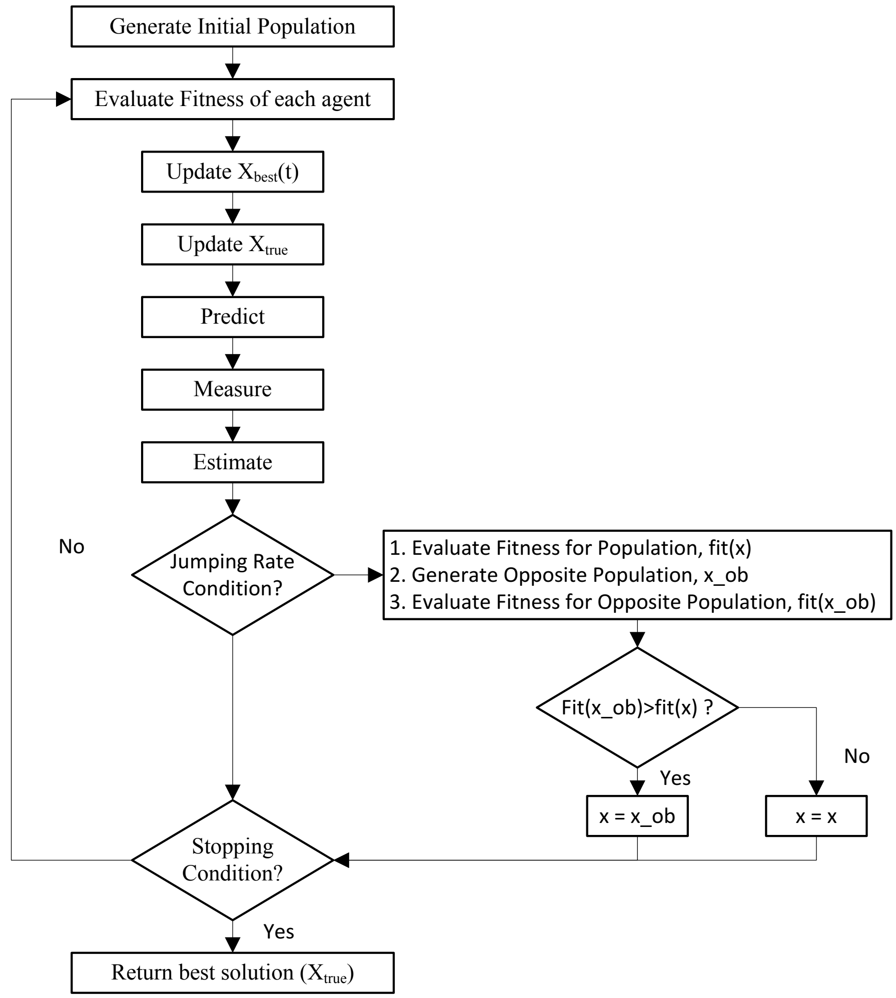


Figure 3: Flowchart of GOBSKF

4. EXPERIMENTAL SETUP

GOBSKF is applied to 10 elements uniform linear array antenna in the simulation. The distance between elements is set to 0.5 λ as its most commonly used distance. Table 1 shows the parameters used in GOBSKF. The desired angle is set to $30°$ and the interference angle is set to $-70°, -40°, -30°, -10°, 0°, 10°, 50°$, and $70°$ similar to the parameters established by Z.D. Zaharis and T.V. Yioultsis [1].

GOBSKF is used to find the optimum weights by maximizing SINR value. Fig. 4 shows the flowchart of GOBSKF in adaptive beamforming application. The performance of GOBSKF is compared with AMBPSO and MVDR [1]. The test was executed 100 times and a statistical analysis was performed.

5. SIMULATION RESULTS AND DISCUSSION

Fig. 5 shows the radiation pattern produced GOBSKF with a signal to noise ratio, SNR = 30 dB and is compared with radiation pattern obtained by Z.D. Zaharis and T.V. Yioultsis using conventional MVDR and AMBPSO [1]. The desired signal at 30^0 and the interference angle is set to $-70°, -40°, -30°, -10°, 0°, 10°, 50°$ and $70°$ just like the condition set in [1]. From Fig. 5, AMBPSO is able to produce much lower maximum sidelobe level compared with MVDR and GOBSKF, but GOBSKF is able to produce much deeper nulls compared to MVDR and AMBPSO. Statistical analysis is performed to confirm the results of GOBSKF is significant.

Table 1: Parameters for GOBSKF

Iteration	10000
Agents	100
P	1000
Q	0.5
R	0.5
Jumping Rate, Jr	0.1

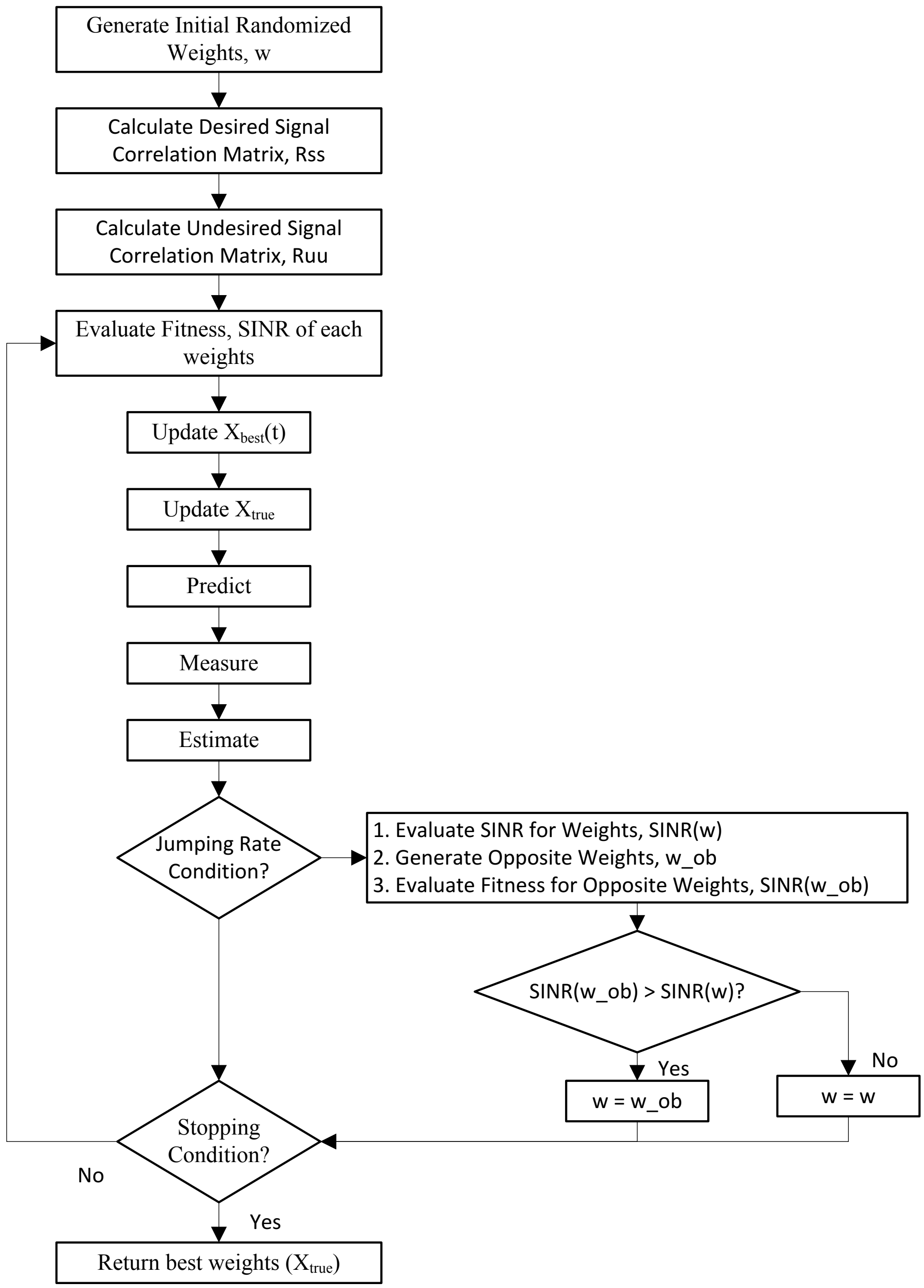
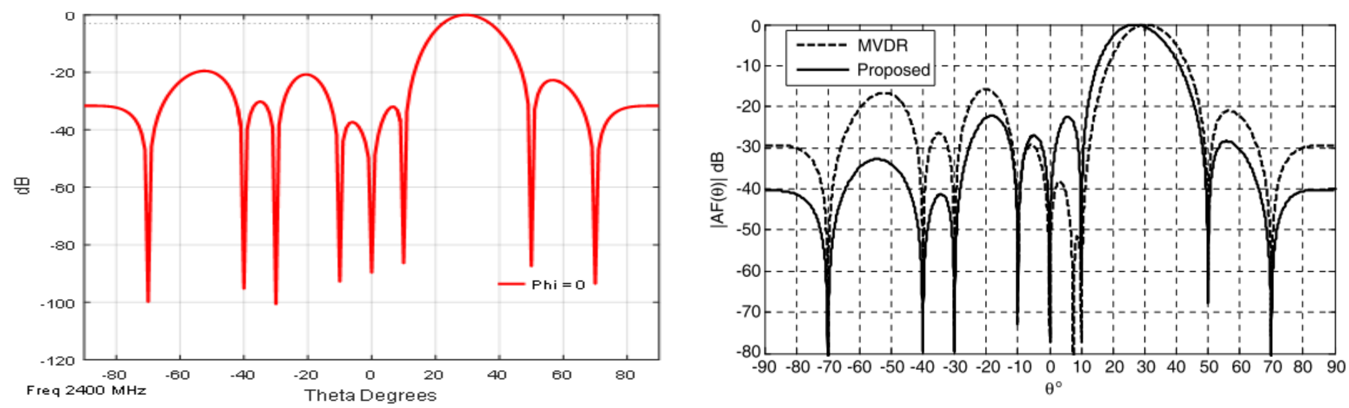


Figure 4: Flowchart of GOBSKF as Adaptive Beamforming Algorithm

Table 2 lists the results for GOBSKF compared with AMBPSO and also with conventional MVDR adaptive beamforming algorithm with various signal to noise ratio (SNR) values. GOBSKF is mostly able to produce much higher mean SINR values compared with the mean SINR of AMBPSO and also the SINR of MVDR. The SINR values for GOBSKF does not fluctuate much compared to AMBPSO because of its lower standard deviation, STD values.

Statistical analysis is also performed between MVDR and mean results for AMBPSO and GOBSKF using Friedman rank test. Table 3 shows the Friedman rank for GOBSKF, AMBPSO, and MVDR.

After that, Friedman PostHoc analysis is performed using Holm's procedure. Table 4 shows the results obtained after performing Friedman PostHoc analysis. Holm`s procedure rejects those hypotheses that have a p-value ≤ 0.05. Therefore, there is a significant difference between GOBSKF and AMBPSO and also with MVDR.



(a) (b)

Figure 5: Radiation Pattern for SNR = 30dB: (a) GOBSKF; (b) MVDR and AMBPSO by Z.D. Zaharis and T.V. Yioultsis [1]

Table 2: Comparison of SINR (dB) Values of MVDR, AMBPSO, and GOBSKF

SNR(dB)	MVDR	AMBPSO				GOBSKF (Proposed Method)			
		Best	Worst	Mean	STD	Best	Worst	Mean	STD
-20	-10.0540	-10.0522	-10.0548	-10.0523	0.0004	-10.0522	-10.0522	***-10.0522***	***0.0000***
-15	-5.1601	-5.1395	-5.1512	-5.1399	0.0020	-5.1395	-5.1395	***-5.1395***	***0.0000***
-10	-0.3701	-0.2975	-0.3692	-0.2998	0.0098	-0.2975	-0.2976	***-0.2975***	***0.0000***
-5	4.3345	4.5321	4.3422	4.5269	0.0218	4.5321	4.5321	***4.5321***	***0.0000***
0	8.8967	9.4241	8.5481	9.3749	0.1643	9.4241	9.3888	***9.4233***	***0.0050***
5	13.4522	14.3768	12.0647	14.2676	0.3628	14.3768	14.2140	***14.3708***	***0.0194***
10	18.2011	19.3598	15.1371	***19.2810***	0.4463	19.3573	18.5395	19.2749	***0.1142***
15	22.6889	24.3542	16.5370	24.1008	1.0643	24.3466	23.4416	***24.1327***	***0.1956***
20	27.3035	29.3509	17.2416	***29.0332***	1.2290	29.3390	27.3695	28.9813	***0.3147***
25	31.7012	34.3515	22.4314	33.6680	1.4163	34.3468	32.8502	***33.9586***	***0.3633***
30	36.4811	39.3341	30.2715	38.7648	0.8722	39.3483	37.0333	***38.8979***	***0.4563***
35	40.4633	44.3440	32.6393	43.1564	1.5287	44.3508	42.0255	***43.8573***	***0.4774***
40	45.2813	49.3461	36.6898	48.1781	1.2409	49.3417	46.2094	***48.8883***	***0.5595***
45	49.3217	54.3358	43.2386	52.5134	1.5788	54.3477	51.7545	***53.8767***	***0.4519***
50	54.8269	59.3317	47.6499	58.3221	1.7439	59.3469	57.5601	***58.9174***	***0.4195***
55	59.1356	64.3458	52.7630	63.0660	1.6119	64.3500	61.0678	***63.9052***	***0.5396***
60	63.4345	69.3468	56.3379	67.5858	1.8369	69.3510	67.2908	***68.8662***	***0.4599***

Table 3: Friedman Ranking

GOBSKF	1.1176
AMBPSO	1.8824
MVDR	3.0000

Table 4: Friedman PostHoc Analysis

Data Set	p	z	Holm
MVDR vs GOBSKF	0.0000	5.4880	0.0167
MVDR vs AMBPSO	0.0011	3.2585	0.0250
AMBPSO vs GOBSKF	0.0258	2.2295	0.0500

6. CONCLUSION

The main purpose of any adaptive beamforming algorithm is to improve the signal to interference plus noise ratio, SINR by steering the main beam towards the desired signal and null the undesired signal at the same time. With the ever changing electromagnetic environment, it also important that the adaptive beamforming algorithm efficiently adapt and maintain its maximum SINR value every time. Therefore, the proposed method, GOBSKF, is proven better compared to AMBPSO and also conventional MVDR. GOBSKF provided better mean SINR values and is also consistent. Results obtained by GOBSKF is also proven to be significant.

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