

ANALYSIS OF FATIGUE SURFACE CRACK USING THE PROBABILISTIC
S-VERSION FINITE ELEMENT MODEL

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TESIS YANG DIKEMUKAKAN UNTUK MEMPEROLEHI IJAZAH
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DECLARATION

I hereby declare that the work in this thesis is my own except for quotations and summaries, which have been duly acknowledged.

29 June 2016

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LIST OF SYMBOLS

a	Crack depth
a_i	Initial crack depth
b, b_G^e, b_L^e	Body force, for global, local element
B, B^G, B^L	Deformation matrix, for global, local
c	Crack length
c_i	Initial crack length
C	Paris coefficient
da	Crack growth increment
$da_{a/c}$	Crack length or depth increment
da_{max}	Maximum crack growth increment
dN_i	Fatigue life cycle range
D	Material properties matrix
D_n	Maximum difference of two cumulative distribution functions
D_n^α	Standard difference values at various significance level α
E	Young's modulus
$E[\cdot]$	Mean operator
f	Nodal force
f_i	Joint reaction at node
F, F_G^e, F_L^e	Load vector at global, local element
F_X	Cumulative distribution function
$F_X(x_i)$	Theoretical cumulative distribution function
$g(X)$	Performance function

G, G_I, G_{II}, G_{III}	Energy release rate, for failure mode I, II, III
h	Width of specimen
i	Node number
k	Stiffness matrix
K	Stress intensity factor
K_{Ic}	Fracture toughness
K_{eq}	Equivalent stress intensity factor
$K_{GG, GL, LG, LL}$	Stiffness matrix at global, global-local, local-global, local
ΔK_{th}	Threshold value
max	Maximum
n	Paris coefficient
N	Shape function
N_t	Fatigue life cycle
N_t	Total number of sample
N_f	Total number of failure sample
p	Portion number
P_f	Probability of failure
PR	Poisson ratio
r	Width of element parallel to the crack front
\bar{r}	Distance from the crack front
r_P	Pearson product moment correlation coefficient
R	Crack front radius
R^2	Coefficient of determination

S_1, S_2	Area before, after crack front
$S_n(x_i)$	Empirical cumulative distribution function
t	Vector of boundary tractions
Q	Shape factor
q_i	Random number
u, u^G, u^L, u^e	Displacement, for global, local, node
u_{iU}, u_{iL}	Displacement for upper, lower surface crack
$Var(\cdot)$	Variance operator
x_i	Random parameter
x, y, z	Physical nodal coordinate
$\bar{x}, \bar{y}, \bar{z}$	Estimation coordinate
X_i^G, Y_i^G, Z_i^G	Nodal coordinate for global mesh
X_i^L, Y_i^L, Z_i^L	Nodal coordinate for local mesh
ξ, η, ζ	Gaussian coordinate
ξ^G, η^G, ζ^G	Gaussian coordinate for global element
ξ^L, η^L, ζ^L	Gaussian coordinate for local element
$\bar{\xi}^G, \bar{\eta}^G, \bar{\zeta}^G$	Vector for trial value coordinate
$\bar{\xi}_{next}^G, \bar{\eta}_{next}^G, \bar{\zeta}_{next}^G$	Vector for next trial value coordinate
σ	Stress
σ_{33}	Cohesive stress
σ_X	Standard deviation
$\bar{\sigma}_X$	Sample standard deviation
$\varepsilon, \varepsilon^G, \varepsilon^L$	Strain, for global, local
$\Omega, \Omega^G, \Omega^L$	Domain, for global, local

$\Gamma, \Gamma^t, \Gamma^u, \Gamma^{GL}$	Boundary, of displacement, force, global region, overlay region
α	Significance level
τ	Shear stress
γ	Shear strain
ν	Poisson's ratio
ν_i	Crack opening displacement function
\wedge	Length of element perpendicular to the crack front
ϕ	Angle between the direction of r and normal direction of the crack front
θ	Angle
φ_0	Crack growth angle
μ	Shear modulus
μ_X	Mean
$\bar{\mu}_X$	Sample mean
Φ^{-1}	Inverse of cumulative distribution function

LIST OF ABBREVIATIONS

ASTM	American Society for Testing and Materials
CDF	Cumulative Distribution Function
Det. S-FEM	Deterministic S-version Finite Element Model
ECDF	Empirical Cumulative Distribution Function
EPFM	Elastic Plastic Fracture Mechanics
FEA	Finite Element Analysis
FEM	Finite Element Model
LEFM	Linear Elastic Fracture Mechanics
LHS	Latin Hypercube Sampling
MC	Monte Carlo
MCS	Monte Carlo Simulation
PDF	Probability Density Function
ProbS-FEM	Probabilistic S-version Finite Element Model
POF	Probability of Failure
SIF	Stress Intensity Factor
STD	Standard Deviation
S-FEM	S-version Finite Element Model
VCCM	Virtual Crack Closure Method