

## Identification of Nonlinearities in Joints of a Wing Structure

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**Abstract.** Nonlinear structural identification is essential in engineering. As new materials are being used and structures become slender and lighter, nonlinear behaviour of structures becomes more important. There have been many studies into the development and application of system identification methods for structural nonlinearity based on changes in natural frequencies, mode shapes and damping ratios. A great challenge is to identify nonlinearity in large structural systems. Much work has been undertaken in the development of nonlinear system identification methods (e.g. Hilbert Transform, NARMAX, and Proper Orthogonal Decomposition), however, it is arguable that most of these methods are cumbersome when applied to realistic large structures that contain mostly linear modes with some local nonlinearity (e.g. aircraft engine pylon attachment to a wing). In this paper, a multi-shaker force appropriation method is developed to determine the underlying linear and nonlinear structural properties through the use of the measurement and generation of restoring force surfaces. One undamped mode is excited in each multi-shaker test. Essentially, this technique is a derivative of the restoring surface method and involves a non-linear curve fitting performed in modal space. A reduced finite element model is established and its effectiveness in revealing the nonlinear characteristics of the system is discussed. The method is demonstrated through both numerical simulations and experiments on a simple jointed laboratory structure with seeded faults, which represents an engine pylon structure that consists of a rectangular wing with two stores suspended underneath.

### 1 Introduction

Linear identification in structural dynamics is matured and well established [1-3]. Nowadays, nonlinear identification is become a very popular area in structural dynamics. Most of engineering structures exhibit some degree of nonlinearity characteristics especially when the deformations are large [4-5]. Nonlinear systems have a range of behaviour not seen in linear vibrating systems. Furthermore, nonlinear dynamic analysis becomes very important for the identification of damage in structures. Detection, localisation and quantification of nonlinearity are very common in nonlinear structural dynamics area [6-7]. Five typical sources of nonlinearities in structural dynamics were as follows: geometric nonlinearity, inertia nonlinearity, material nonlinearity, damping dissipation and boundary conditions.

There have been many studies on the use of system identification methods to identify structural nonlinearity, which include changes in natural frequencies, mode shapes and damping ratios. Masri and Caughey [8] introduced the restoring forces surface (RFS) to identify

nonlinearity in single-degree of freedom (SDOF) systems by exploiting Newton's 2nd law to directly measure restoring and dissipative forces in the system. RFS was extended by [9] to identify multi-degree of freedom (MDOF) systems by transforming the equations of motion from physical to modal coordinate space. RFS is more efficient than the Weiner-kernel approach in identifying nonlinear dynamic systems of the types considered. The parametric identification method by force state mapping technique was developed [10], which is similar to RFS. Dimitriadis and Cooper [11] attempted to identify MDOF systems using a variant of RFS method, which considers time response at similar amplitudes, and subsequently, constant nonlinear restoring forces could be achieved. Kerschen et al. [12] applied RFS method for two different cases: a symmetrical nonlinear beam with piecewise linear stiffness and an asymmetrical nonlinear beam with bilinear stiffness. The nonlinear identification method recently proposed [13-14] is based on the measured linear and nonlinear Frequency Response Functions (FRFs). The method is easy to implement and requires standard

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testing methods. The data required is limited with measured linear and nonlinear FRFs. A method or procedure for the identification of non-linear single and multi-degree of freedom using restoring forces method with three types of nonlinearity was simulated by [15]. Noel et al. [16] demonstrated that the Restoring Force Surface (RFS) method can provide a reliable identification of a nonlinear spacecraft structure. The nonlinear component comprises an inertia wheel mounted on a support, the motion of which is constrained by eight elastomer plots and mechanical stops. Several adaptations to the RFS method are proposed, which include the elimination of kinematic constraints and the regularization of ill-conditioned inverse problems.

This paper presents the identification of nonlinear wing structure using combination force appropriation and restoring force method. Test structure in this paper represent the configuration wing structure with two stores which nonlinear pylon connection between them.

## 2 Finite Element (FE) Analysis of Wing Structure

Two engine pylons are designed and attached to a rectangular wing structure. The FE model of the wing structure is shown in Fig.1 which consists of 852 solid elements (CHEXA) and 40 plate elements (CQUAD4). There are 1824 nodes and free-free boundary condition is applied to this model.

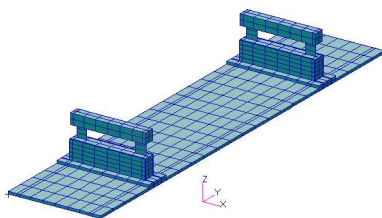


Fig. 1. FE Model of Wing Structure

NASTRAN code for normal mode analysis (SOL 103) is developed and used to compute natural frequencies and mode shape of wing structure. The results are shown in Table 1 and Fig. 2 respectively.

Table 1. FE Natural Frequencies of Wing Structure

Mode	Natural Frequencies (Hz)
1	15.25
2	18.40
3	33.10
4	78.10
5	134.80

**M** and **K** matrices are extracted from FE model by using matlab code of pch output file. However, **C** matrices can be developed using proportional damping (Rayleigh Method) is commonly used in nonlinear dynamics analysis. **M**, **K** and **C** matrices will be used in nonlinear modal model.

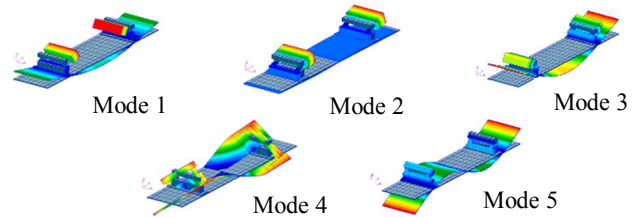


Fig. 2. FE Mode Shape of Wing Structure

## 3 Experimental Modal Analysis (EMA)

The overall test structure used in this project is intended to represent roughly the configuration of an aircraft wing having two under-wing stores with nonlinear pylon connection between them. Fig. 3 shows the arrangement of the wing model, supported from a frame through bungee cords in a free-free boundary configuration. Two Data Physics V4 shakers were attached to the center line of the wing via force transducers and driven by Data Physics power amplifiers. The natural frequencies, damping ratios, modal masses for the first five modes, extracted from FRF matrix using PolyMAX parameter estimation method are shown in Table 2.

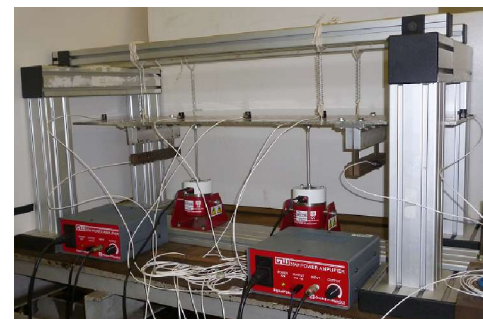


Fig. 3. Physical Wing Structure Arrangement

Table 2. Experimental frequencies, damping ratio and modal mass of overall structure of wing model by performing double shaker test.

Mode	Experimental Natural Frequencies (Hz)	Damping Ratio (%)	Modal Mass (kg)
1	15.09	0.76	2.15
2	18.40	0.91	2.44
3	32.33	0.32	1.66
4	78.01	0.09	0.99
5	135.08	0.27	1.17

## 4 Nonlinear Modal Model

The equations of motion of discretised structures in the physical space can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{g}_{nl}(\dot{\mathbf{x}}, \mathbf{x}) = \mathbf{f}(t) \quad (1)$$

where **M**, **C** and **K** are  $n \times n$  mass, damping and stiffness matrices;  $\mathbf{g}_{nl}$  is an  $n \times n$  nonlinear stiffness matrix,  $\mathbf{f}(t)$  is applied nodal force vector and  $\mathbf{x}(t)$  is the vector of

physical displacements. The equations can be obtained for example, from finite element modelling of a structure. Transformation by  $\mathbf{x} = \Phi \mathbf{p}$  leads to

$$\Phi^T \mathbf{M} \Phi \ddot{\mathbf{p}} + \Phi^T \mathbf{C} \Phi \dot{\mathbf{p}} + \Phi^T \mathbf{K} \Phi \mathbf{p} + \Phi^T \mathbf{g}_{nl}(\Phi \mathbf{p}) = \Phi^T \mathbf{f}(t) \quad (2)$$

where  $\Phi$  is the modal vector matrix. By using orthogonality of the modes, equation (2) become

$$\bar{\mathbf{M}} \ddot{\mathbf{p}} + \bar{\mathbf{C}} \dot{\mathbf{p}} + \bar{\mathbf{K}} \mathbf{p} + \bar{\mathbf{g}}_{nl} = \bar{\mathbf{f}} \quad (3)$$

where  $\bar{\mathbf{M}} = \Phi^T \mathbf{M} \Phi = [\bar{M}_{rr}]$  and  $\bar{\mathbf{K}} = \Phi^T \mathbf{K} \Phi = [\bar{K}_{rr}]$  are diagonal matrices, and  $\bar{\mathbf{g}}_{nl} = \Phi^T \mathbf{g}_{nl}$ . If the structure has proportional damping,  $\bar{\mathbf{C}} = \Phi^T \mathbf{C} \Phi = [\bar{C}_{rr}]$  is also a diagonal matrix, and equation (3) reduces to

$$\bar{M}_{rr} \ddot{p}_r + \bar{C}_{rr} \dot{p}_r + \bar{K}_{rr} p_r + \bar{g}_{nl,r} = \bar{f}_r \quad (4)$$

where  $p_r$  is the  $r$ th modal displacement and other parameters in modal expression. Nonlinear terms,  $\bar{g}_{nl,r}$  refer to  $r$ th mode nonlinear restoring force and others mode allow for nonlinear cross-coupling terms.

## 5 Nonlinear Identification of Wing Structure

Polynomial basis functions of order 3 were used for the nonlinear models to retain the order of the model as low as possible. Static load test on two pylons and wing were performed. Equation (5) and (6) below show the nonlinearity in the pylons and wing structure contains cubic and quadratic terms. The curve for each pylon was fitted using a cubic polynomial and the equations were:

$$F_1 = (2.65 \times 10^8)x_1^3 + (2.76 \times 10^5)x_1^2 + (6.65 \times 10^3)x_1 - 0.05 \quad (5)$$

$$F_2 = (2.93 \times 10^8)x_2^3 + (3.87 \times 10^5)x_2^2 + (6.35 \times 10^3)x_2 + 0.45 \quad (6)$$

where  $F_1$  is the load on the first pylon,  $F_2$  is the load on the second pylon,  $x_1$  is the deflection of first load and  $x_2$  is the deflection of second load. From equation (5) and (6) it is shows that both assemblies pylon characteristic a slight symmetry, as the polynomial fits contain constant and second order terms.

Further investigation is dynamic test on overall wing structure using MIMO Normal mode test. This test was carried out on the unit to generate a freely decaying response. The existence of stiffness nonlinearity would be observed by a variation in natural frequency with amplitude and of a damping nonlinearity by changing damping ratio with amplitude. Damping effect on this study is very small and not affects the identification so much. No nonlinear damping was considered as the random excitation test showed little evidence of nonlinearity in the system. The finalized nonlinear curve fit was therefore of the form:

$$g_{nl,r} = f_r - m_r \ddot{p}_r - c_r \dot{p}_r - k_r p_r - A_{1,r} p_1^3 - B_{1,r} p_1^2 p_2 - C_{1,r} p_1^2 p_3 - D_{1,r} p_1 p_2^2 - E_{1,r} p_1 p_2 p_3 - F_{1,r} p_1 p_3^2 - G_{1,r} p_2^3 - H_{1,r} p_2^2 p_3 - I_{1,r} p_2 p_3^2 - J_{1,r} p_3^3 \quad (7)$$

for  $r=1,2,3$ , where the unknowns to be evaluated where  $c_r, k_r, A_{1,r}, B_{1,r}, C_{1,r}, D_{1,r}, E_{1,r}, F_{1,r}, G_{1,r}, H_{1,r}, I_{1,r}$  and  $J_{1,r}$ . As the number of coefficient in this model is relatively small, the best model structure was chosen by implementing exhaustive search. Model structure selection is important part of nonlinear system identification method, because not every coefficient term should be included in the final model. In addition, some of the  $A_{1,r}$  to  $J_{1,r}$  coefficients may have value of 0. The resulting model is the complete optimum best curve fit that given particular sets of candidate terms and data. Table 3 shows the estimates of the interest linear parameters,  $c_r$  and  $k_r$  determined from the nonlinear curve fits and compare with those obtained from the PolyMAX modal estimation model. It can be seen that the parameters are small differences between two set of estimation. Furthermore, Table 4 shows nonlinear term coefficients identified for first three modes of overall wing structure. A zero value denotes that the corresponding term was not included in the final model.

Table 3: Comparison linear direct term coefficients identified by combination force appropriation and restoring method with PolyMAX.

Mode	Force Appropriation and Restoring Forces Method		PolyMAX (MIMO Normal Mode Test)	
	$c_r$ , modal damping	$k_r$ , modal stiffness	$c_r$ , modal damping	$k_r$ , modal stiffness
1	2.47	1.79E4	2.35	1.71E4
2	3.24	3.13E4	3.17	3.34E4
3	4.14	6.35E4	4.06	6.46E4

Table 4: Nonlinear term coefficients identified for first three modes

Coefficient	Mode 1	Mode 2	Mode 3
$A_{1,r}$	3.76E8	0	-2.56E8
$B_{1,r}$	-1.66E9	6.06E9	0
$C_{1,r}$	-7.8E8	0	1.58E9
$D_{1,r}$	3.45E9	0	0
$E_{1,r}$	0	-1.53E10	6.37E10
$F_{1,r}$	0	0	-2.42E9
$G_{1,r}$	0	1.33E9	0
$H_{1,r}$	0	1.87E9	0
$I_{1,r}$	0	2.32E10	1.42E9
$J_{1,r}$	-9.72E7	0	0

The final model structure in Equation (7), identified equation of motion for the overall wing structure can be written as:

$$2.15 \ddot{p}_1 + 2.47 \dot{p}_1 + 1.79 \times 10^4 p_1 + 3.76 \times 10^8 p_1^3 - 1.66 \times 10^9 p_1^2 p_2 - 7.87 \times 10^8 p_1^2 p_3 + 3.45 \times 10^9 p_1 p_2^2 - 9.72 \times 10^7 p_3^3 = f_1(t) \quad (8)$$

$$2.44\ddot{p}_2 + 3.24\dot{p}_2 + 3.13 \times 10^4 p_2 + 6.06 \times 10^9 p_1^2 p_2 - 1.53 \times 10^{10} p_1 p_2 p_3 + 1.33 \times 10^9 p_2^3 + 1.87 \times 10^9 p_2^2 p_3 + 2.32 \times 10^{10} p_2 p_3^2 = f_2(t) \quad (9)$$

$$\ddot{p}_3 + 4.14\dot{p}_3 + 6.35 \times 10^4 p_3 - 2.56 \times 10^8 p_1^3 + 1.58 \times 10^9 p_1^2 p_3 + 6.37 \times 10^{10} p_1 p_2 p_3 - 2.42 \times 10^9 p_1 p_3^2 + 1.42 \times 10^9 p_2 p_3^2 = f_3(t) \quad (10)$$

$$0.99\ddot{p}_4 + 2.95\dot{p}_4 + 2.21 \times 10^5 p_4 = f_4(t) \quad (11)$$

$$1.17\ddot{p}_5 + 5.35\dot{p}_5 + 7.19 \times 10^5 p_5 = f_5(t) \quad (12)$$

It should be noted that under appropriated excitation, the amplitudes of the cross coupled modal responses were about an order of magnitude lower than direct modal responses. Thus, all the cross coupled nonlinear terms in Equation (8) to (12) are at least an order of magnitude smaller than the direct nonlinear terms.

## 6 Conclusions

A nonlinear wing experimental structure was identified using the force appropriation and restoring forces method. In the pylon rotation degrees of freedom, discrete hardening stiffness nonlinearity was located. The results presented evidence nonlinear stiffness at higher amplitudes for the first three modes. The combination nonlinear identification methods succeeded to estimate good quality nonlinear modal models for the mode interest. It can be conclude that mode 1 and 2 showed hardening stiffness nonlinearities, mode 3 just showed very weak nonlinearity because of small degree of rotation at engine pylon structure. However, mode 4 and 5 behaved linearly for the wing structure.

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## References

1. K. Worden, and G. R. Tomlinson, "Nonlinearity in structural dynamics: Detection, identification and modelling", University of Sheffield: Institute of Physics Publishing, (2001)
2. N. M. M. Maia, and J. M. M. Silva, "Theoretical and experimental modal analysis", Research Studies Press, Taunton, (1997)
3. D. J. Ewins, "Modal testing: theory and practice", Research Studies Press, Letchworth, (1995)

4. R. K. Dearson, "Discrete time dynamic model", Oxford University Press, (1994)
5. Ö. Arslan, M. Aykan, H. N. Özgüven, "Parametric identification of structural nonlinearities from measured frequency data", Mechanical System and Signal Processing, **vol. 25**, no. 11, pp 1112-1125, (2011)
6. G. Kerschen, K. Worden, A. F. Vakakis and J.C. Golinval, "Past, present and future of nonlinear system identification in structural dynamics", Mechanical System and Signal Processing, **vol. 20**, pp 505-592, (2006)
7. A. Hot, G. Kerschen, E. Foltête and S. Cogan, "Detection and quantification of non-linear structural behavior using principal component analysis", Mechanical System and Signal Processing, **vol. 26**, 104-116, (2012)
8. S. F. Masri, and T. K. Caughey, "A nonparametric identification technique for nonlinear dynamic problems", Journal of Applied Mechanics, **vol. 46**, pp. 433-447, (1979)
9. S. F. Masri, H. Sassi, and T. K. Caughey, "Nonparametric identification of nearly arbitrary nonlinear systems", Journal of Applied Mechanics, **vol. 49**, pp. 619-628, (1982)
10. E. F. Crawley, and A. C. Aubert, "Identification of nonlinear structural elements by force-state mapping", AIAA Journal, **vol. 24**, pp. 155-162, (1986)
11. G. Dimitriadis, and J. E. Cooper, "A method for the identification of non-linear multi-degree-of-freedom systems", Proceedings of the Institute of Mechanical Engineers, Part G 212, pp. 287-298, (1998)
12. G. Kerschen, J. C. Golinval and K. Worden, "Theoretical and experimental identification of a non-linear beam", Journal of Sound and Vibration, **vol. 244**, pp. 597-613, (2001)
13. M. Aykan, and H. N. Özgüven, "Parametric identification of nonlinearity from incomplete FRF data using describing function inversion", In: Proceedings of the SEM IMAC XXX conference, Jacksonville, **vol 3**, (2012)
14. M. Aykan, and H. N. Özgüven, "Identification of restoring force surfaces in Nonlinear MDOF systems from FRF data using nonlinearity matrix", Conference Proceedings of the Society for Experimental Mechanics Series, **vol. 35**, pp 65-76, (2013)
15. M.S.M. Sani, H. Ouyang, J.E. Cooper and C.K.E.N.C.K. Husin, "Smart Methodology of Stiffness Nonlinearity Identification Vibration System", 2<sup>nd</sup> International Conference on Mechanical Engineering Research 2013 (ICMER) (2013)
16. J. P. Noel, G. Kerschen, and A. Newerla, "Application of the Restoring Force Surface Method to a Real-life Spacecraft Structure", Conference Proceedings of the Society for Experimental Mechanics Series, pp 1-19, (2012).