Mathematical formulation to study the thermal post buckling of orthotropic circular plates

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Abstract— A simple mathematical formulation to evaluate the post buckling load of orthotropic circular plates with both simply supported and clamped boundary conditions are presented in this paper. The formulation is on the basis of radial edge tensile load developed in the plate because of the large deflection of the plate. The numerical results achieved from the present formulation in terms of linear buckling load parameters for different orthotropic parameter values are compared with the results from the literature.

Keywords— circular plates, post buckling, linear buckling load

1. INTRODUCTION

Nonlinear analysis of structural members with effects of geometric and material nonlinearity is of research importance, as the results obtained by the analysis leads to meaningful yet accurate results in lesser times. Most of the earlier studies have been using the classical methods/energy methods/ variational methods and the versatile FE method. The commonly used structural members such as circular plates are of great importance in many real life applications. The simply supported and clamped boundary conditions are considered by many researchers in the study of structures.

Thompson and Hunt [1] and Dym [2] investigated the post buckling behavior of uniform, isotropic circular and rectangular plates under mechanical loads analytically. Raju and Rao [3] presented the post buckling behavior of cylindrically orthotropic circular plates through the finite element formulation. Raju and Rao [4] discussed the thermal post buckling behavior of isotropic plates and comparison of the numerical results using Finite element formulation and Rayleigh – Ritz analysis. Rao and Varma [5] investigated a simple formulation to predict the thermal post buckling load of circular plates and the method of evaluating the radial edge tensile load developed due to the large lateral displacement. Meanwhile, the thermal post buckling of circular plates using Berger's approximation [6, 7] are presented. . A modified Berger's approximation to study the large amplitude vibration behavior of circular plates has been discussed by Rao *et al.* [8].

The post buckling behavior of moderately thick circular plates with cylindrically orthotropic material properties has been mentioned in [9] by using finite element formulation approach. In all these studies, the given methods take more computational efforts to obtain the approximate solutions because of the nonlinear nature of the problem.

The aim of this paper is to reduce the complexity of formulation and to find the linear buckling load parameters and thermal post buckling load orthotropic plates for different orthotropic parameters by assuming suitable admissible function for lateral displacement *w*.

2. FORMULATION

A mathematical formulation for the thermal post buckling of orthotropic circular plates by evaluating linear buckling load parameters due to the large lateral displacements are presented in the following session.

By considering a circular plate of radius *'a'* and of uniform thickness *'t'* subjected to a radial uniform compressive load 'N_r' per unit length at the boundary, then the von Karman type strain – displacement relations can be written as

$$\varepsilon_r = \frac{du}{dr} + \frac{1}{2}\left(\frac{dw}{dr}\right)^2 \tag{1}$$

$$\varepsilon_\theta = \frac{u}{r} \tag{2}$$

$$\chi_r = -\frac{d^2 w}{dr^2} \tag{3}$$

$$\chi_\theta = -\frac{1}{r}\left(\frac{dw}{dr}\right) \tag{4}$$

where r and θ are the radial and circumferential coordinates, $\varepsilon_r, \varepsilon_\theta$ are the strains and χ_r, χ_θ are the curvatures.

As the strain – displacement relations stated above, the strain energy U of the plate with orthotropic material properties [3], is given by

$$U = \frac{1}{2}\int_0^{2\pi}\int_{r_1}^{r_2}\left[C_1 \varepsilon_r{}^2 + C_2 \varepsilon_\theta{}^2 + C_{12}\,\varepsilon_r \varepsilon_\theta + D_1 \chi_r{}^2 + D_2 \chi_\theta{}^2 + D_{12}\,\chi_r \chi_\theta\right] r\,dr\,d\theta, \tag{5}$$

where $C_1 = \frac{E_r h}{1-\nu^2}$, $C_2 = \frac{E_\theta h}{1-\nu^2}$, $C_{12} = \nu_r C_2 = \nu_\theta C_1$, $D_1 = \frac{E_r h^3}{12\left(1-\nu^2\right)}$, $D_2 = \frac{E_\theta h^3}{12\left(1-\nu^2\right)}$, and $D_{12} = \nu_r D_2 = \nu_\theta D_1$.

The work done, W by the external load N_r per unit length at the boundary of the element [3] is given by

$$W = \frac{1}{2}\int_0^{2\pi}\int_{r_1}^{r_2}\overline{N}_r\left(\frac{dw}{dr}\right)^2 r\,dr\,d\theta. \tag{6}$$

By taking $\beta = E_\theta / E_r$ as orthotropic parameter where E_θ and E_r are the Young's moduli in radial and circumferential directions respectively and applying the substitutions for C_1, C_2, C_{12}, D_1, D_2, D_{12} and $\varepsilon_r, \varepsilon_\theta, \chi_r, \chi_\theta$ in equation (5), we can write the strain energy equation as

$$U = \frac{1}{2}\int_0^1\left(\frac{d^2 w}{dr^2}\right)^2 + \beta\frac{1}{r^2}\left(\frac{dw}{dr}\right)^2 + 2\nu\frac{\beta}{r^2}\left(\frac{d^2 w}{dr^2}\right)\left(\frac{dw}{dr}\right)dr, \tag{7}$$

Also by making use of substitutions, equation (6) can be written in terms of β as,

$$W = \frac{\lambda\beta}{2}\int_0^1 \frac{d^2 w}{dr^2}\,dr. \tag{8}$$

The total potential energy of the plate can be defined as

$$\Pi = U - W. \tag{9}$$

The linear buckling load λ for various values of β can be attained by assuming suitable admissible functions for the lateral displacement *w*. The value of Poisson's ratio in the radial direction, ν is taken as 0.3. Both simply supported and clamped boundary conditions are considered in this study.

The admissible function for the lateral displacement *w* considered in the study is of the form given by Yamaki [10] which satisfies both the geometric and natural boundary conditions. The algebraic function for the lateral displacement is given as

$$F_3 = b_0\left[1+\alpha_1\left(\frac{r}{a}\right)^2+\alpha_2\left(\frac{r}{a}\right)^4\right]+b_1\left[\alpha_3\left(\frac{r}{a}\right)^2+\alpha_4\left(\frac{r}{a}\right)^4+\left(\frac{r}{a}\right)^6\right]+b_2\left[\alpha_5\left(\frac{r}{a}\right)^4+\alpha_6\left(\frac{r}{a}\right)^6+\left(\frac{r}{a}\right)^8\right] \tag{10}$$

where the values of α_1, α_2, α_3, α_4, α_5 and α_6, are known in terms of *v* and β as

$$\alpha_1 = -\frac{12+4\nu+4\beta}{10+2\nu+2\beta},\ \alpha_2 = \frac{2+2\nu+2\beta}{10+2\nu+2\beta},\ \alpha_3 = \frac{18+2\nu+2\beta}{10+2\nu+2\beta},$$

$$\alpha_4 = -\frac{28+4\nu+4\beta}{10+2\nu+2\beta},\ \alpha_5 = \frac{26+2\nu+2\beta}{18+2\nu+2\beta}\ \text{ and } \alpha_6 = -\frac{44+4\nu+4\beta}{18+2\nu+2\beta} \tag{11}$$

Using the eigen vector the nonlinear terms in the stiffness matrix are obtained through numerical integration. The values of linear buckling load parameters λ can be evaluated by solving equation (9) with appropriate boundary conditions and by extracting the eigen values.

An equivalent uniform compressive edge load N_r is established when the plate is heated to a temperature ΔT from the initial stress free state. As the temperature is increased to critical temperature (ΔT_r), buckling of the plate occurs owing to the development of critical compressive uniform radial edge load $N_{r_{cr}}$. The total equivalent compressive uniform radial edge load carrying capacity of the circular plate ($N_{r_{NL}}$) or post buckling load can be expresses in non-dimensional mathematical form as

$$\overline{N}_{r_{NL}} = \overline{N}_{r_{Cr}} + \overline{N}_{r_T}, \tag{12}$$

where each term in equation (10) is non – dimensionalised as

$$\overline{N}_r = \frac{N_r a^2}{D} \tag{13}$$

and *D* is the plate flexural rigidity.

The thermal post buckling load carrying capacity (γ), is obtained from equation (12) using the values of radial edge tensile load parameters evaluated from the assumed admissible functions for the lateral displacement *w* and the linear buckling load parameters as

$$\gamma = \frac{N_{r_{NL}}}{N_{r_{cr}}} = 1 + c\left(\frac{b_0}{t}\right)^2. \tag{14}$$

The results in the form of linear buckling load parameters λ are evaluated for different values of orthotropic parameter β ranging from 1.2 to 2.0.

3. NUMERICAL RESULTS AND DISCUSSION

A numerical method for calculating thermal post buckling load of orthotropic circular plates having simply supported and clamped boundary conditions are presented by assuming suitable admissible functions for the lateral displacement *w*. The numerical results of linear buckling load are obtained for various values of β ranging from 1.2 to 2.0 in steps of 0.2.

Table 1 and 2 represent the values of λ for different values of β from 1.2 to 2.0 with simply supported and clamped boundary conditions, which shows a comparative study of the linear buckling load results. The numerical results obtained from the present formulation are compared with the results from reference [3] for the given β values. The highest error percentage of linear buckling load carrying capacity from reference is 4.54% for simply supported and 5.51% for clamped boundary conditions. The error occurred because of the admissible function we are taken and also depend on the boundary conditions.

A comparison between the numerical results from the present formulation and the results from the known literatures with both the boundary conditions are shown in Fig 1 and Fig 2. From the graphs it can be see that the results follows the same trend. Also the error is obtained due the large displacement and also depends on the edge boundary conditions.

Thermal post buckling load of circular plates are evaluated earlier using various mathematical analysis such as shooting method, finite element analysis and so on. But reducing the complexity of these mathematical methods, using a suitable substitution method to study the post buckling behavior of orthotropic circular plates.

Table 1: Represent the values of λ for orthotropic circular plates with simply supported boundary conditions

Linear buckling load (λ)			
β	**Present results**	**Reference [3]**	**Error percentage (%)**
1.2	4.9425	4.9798	0.74
1.4	5.6031	5.7650	2.81
1.6	6.2561	6.5538	4.54
1.8	7.1238	7.3468	3.04
2.0	7.9436	8.1438	2.46

Table 2: Represent the values of λ for orthotropic circular plates with simply supported boundary conditions.

Linear buckling load (λ)			
β	**Present results**	**Reference [3]**	**Error percentage (%)**
1.2	15.7199	16.6371	5.51
1.4	17.5998	18.5409	5.07
1.6	19.3710	20.4045	5.06
1.8	21.8672	22.2345	1.65
2.0	23.9865	24.0371	0.21

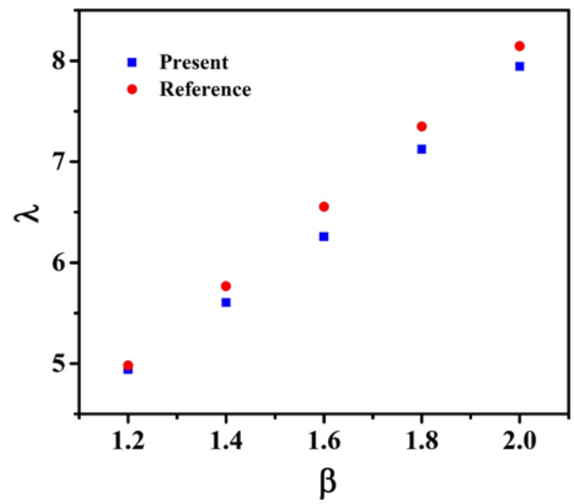

Fig 1: Comparison of the present numerical results with previous results for simply supported boundary condition

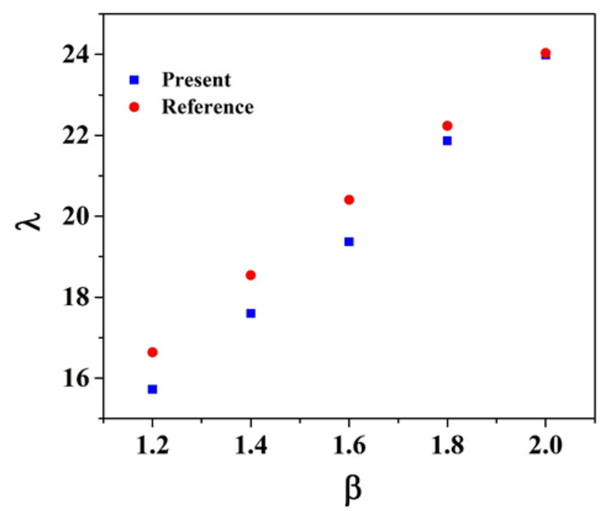

Fig 2: Comparison of the present numerical results with previous results for simply supported boundary condition

4. CONCLUSION

A simple mathematical formulation for the thermal post buckling behavior of orthotropic circular plates by evaluating the linear buckling load is presented in this paper. The values of λ are predicted for both simply supported and clamped boundary conditions. The error percentage is obtained in this study due to the admissible function we are considered and because of the edge boundary conditions.

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Nomenclature

a = radius of the circular plate

N_r = uniform radial edge compressive load per unit length

t = thickness of the plate

ΔT = temperature rise from the stress free temperature

ν = Poisson's ratio

U = strain energy

W = work done

α_1 - α_6 = generalized displacements

β = orthotropic parameter

E_r, E_θ = Young's moduli in radial and circumferential directions

λ = linear buckling load parameter

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