Suction/Injection Effect on a Boundary Layer Flow Past a Stretching Cylinder with Slip Condition

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Abstract—In this study, the mathematical modeling for boundary layer flow and heat transfer on a stretching cylinder with suction/blowing effect and velocity slip condition is considered. The transformed boundary layer equations are solved numerically using the Runge-Kutta-Fehlberg method. The numerical solutions are obtained for the reduced Nusselt number and skin friction coefficient as well as the velocity and the temperature profiles. The features of the flow and heat transfer characteristics for various values of the Prandtl number, Reynolds number, stretching parameter, suction/injection parameter and velocity slip parameter are analyzed and discussed.

Keywords—slip condition; stretching cylinder; suction/injection effect

1. INTRODUCTION

Convection boundary layer flow and heat transfer is an important topic considered in industrial and engineering activities nowadays. In considering the convection flow on circular cylinder, this configurations applied widely for examples to handle hot wire, steam pipe, petroleum reservoirs, coal combustors, ground water pollution and filtration processes, underground disposal of nuclear or non-nuclear waste and stretching metal wire production and cylinder rod in nuclear reactor during emergency shutdown [1, 2]. Because of the large contributions and issues, this topic has attracted many researchers to study and expand the knowledge so that it could be applied in order to handle the thermal problems produced by these industrial outputs.

Present paper discussed the mathematical model of boundary layer flow and heat transfer on a stretching cylinder with suction/injection effect and velocity slip condition. The early studies on this topic is by Wang [3] before being updated by Ishak et al. [4, 5] who extended with the MHD and suction/injection effects. He concluded that the suction may increase the skin friction coefficient while injection does oppositely. Further, the Nusselt number is decreasing with the increase of magnetic parameter. Unsteady case of shrinking cylinder is investigated by Wan Zaimi et al. [6]. Next, research trends are extended to other types of industrial fluid like the viscoelastic fluid and the nanofluid [7, 8]. Recent studies regarding this topic is from the works by Sulochana and Sandeep [9] who considered the Cu-water nanofluid flow on stagnation point flow towards horizontal and exponential stretching /shrinking cylinders.

The investigations mentioned above obey the no-slip condition. It is worth mentioning that the no-slip conditions is not applicable for all cases in fluid flow [10]. This configuration might be replaced with the presence of slip or partial slip velocity conditions. Das [11] and Yusoff et al. [12] studied the slip effects on non-Newtonian power-law nanofluid over a stretching sheet and radiating moving plate, respectively. Recently, Mohamed et al. [13, 14] observed the stagnation point flow on a stretching sheet in a viscoelastic fluid and nanofluid with slip and partial slip velocity conditions. The Walter's-B viscoelastic model is considered. It is found that the presence of slip and partial slip parameter has reduced the stretching and viscoelastic parameter effect on the skin friction coefficient, respectively.

Motivated from the above literature, it is hope that the results obtained from this paper will contribute to explain and verify the experimental results in future. It is further mentioning that the results in this study is new since problem discussed in this paper have never been considered before.

2. MATHEMATICAL FORMULATION

Consider a steady axis symmetric boundary layer flow of a viscous and incompressible fluid along a continuously stretching cylinder of radius a as shown in Fig. 1, where (z,r) are cylindrical coordinates measured in the radial and axial directions, respectively. It is assumed that the velocity distribution of the stretching surface of the cylinder is $\lambda w_w(z)$, where λ is the constant stretching parameter with $\lambda > 0$, and we assume that $w_w(z) = 2cz$ where c is a positive constant. Under these assumptions and based on the axisymmetric flow assumptions, and that there is no azimuthal velocity component, the equations which model the problem under consideration are

$$\frac{\partial}{\partial r}(r\,u)+\frac{\partial}{\partial z}(r\,w)=0 \tag{1}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right) \tag{2}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right) \tag{3}$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{4}$$

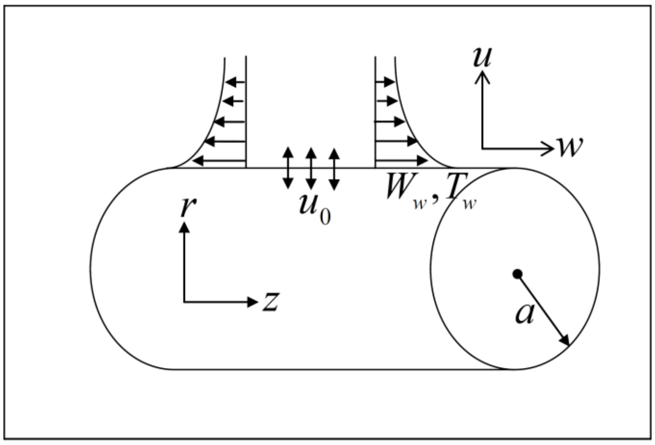

Figure 1: Physical model and the coordinate system

subject to the boundary conditions

$$\begin{gathered} w=\lambda\,w(z)+\gamma^{*}\mu\frac{\partial w}{\partial r},\quad u=u_0\quad T=T_w\quad \text{at}\quad r=a \\ w\to 0,\quad T\to T_\infty\quad \text{as}\quad r\to\infty \end{gathered} \tag{5}$$

where u and w are the velocity components along the axes r and z, respectively, ν is the kinematic viscosity, ρ is the fluid density, u_o is mass flux velocity with $u_o < 0$ for suction and $u_o > 0$ for injection or withdrawal of the fluid and p is the pressure which is constant p_∞ at infinity. Further, γ^* and $\mu = \rho\nu$ are the velocity slip factor and dynamic viscosity, respectively.

Equations (1) to (4) can be transformed into the corresponding ordinary differential equations by using the similarity transformations (see [4])

$$w = 2cz\, f'(\eta), \quad u = -ca\frac{f(\eta)}{\sqrt{\eta}}, \quad \eta = \left(\frac{r}{a}\right)^2 \tag{6}$$

where prime denotes differentiation with respect to η. The pressure term can be obtained from (2) is given by

$$p = p_\infty - \frac{\rho c^2 a^2}{2\eta} f^2(\eta) - 2c\rho\nu\, f'(\eta) \tag{7}$$

Using (6) and taking into account that there is no longitudinal pressure gradient, (3) and (4) reduces to the following ordinary differential equation

$$\eta f''' + f'' + \mathrm{Re}(f f'' - f'^2) = 0 \tag{8}$$

$$\eta\theta'' + (1 + \mathrm{Re}\Pr f)\theta' = 0 \tag{9}$$

and the boundary conditions (5) becomes

$$\begin{gathered} f(1) = S, \quad f'(1) = \lambda + \gamma f''(1), \quad \theta(1) = 1, \\ f'(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \end{gathered} \tag{10}$$

where $\mathrm{Re} = a^2c/2\nu$ is the Reynolds number, $S = -u_0/(ac)$ is the constant parameter of suction $(S > 0)$ or injection $(S < 0)$ $\gamma = \frac{2\gamma^*\mu}{a}$ is the velocity slip parameter.

The physical quantity of interest is the skin friction coefficient and Nusselt number, which is defined as

$$C_f = \frac{\tau_w}{\rho w_w^2(z)/2}, \quad Nu = \frac{aq_w}{k(T_w - T_\infty)}, \tag{11}$$

where the shear stress τ_w and the heat transfer at the tube surface q_w is given by

$$\tau_w = \mu\left(\frac{\partial w}{\partial r}\right)_{r=a}, \quad q_w = -k\left(\frac{\partial T}{\partial r}\right)_{r=a} \tag{12}$$

Using the similarity variables (6) and (12), then (11) is reduced to

$$C_f \,\mathrm{Re}\, z/a = f''(1), \quad Nu/2 = -\theta'(1), \tag{13}$$

3. RESULT AND DISCUSSION

Equations (8) to (10) were solved numerically using the Runge-Kutta-Fehlberg method which encoded in Maple software. Five parameters are considered, namely as the Prandtl number Pr, the Reynolds number Re, the stretching parameter λ, the suction/injection parameter S and the velocity slip parameter γ. In order to validate the efficiency of the method used, the comparison with the previous published result has been made. Table 1 presents the comparison values of $-\theta'(1)$ with various values of Pr and S. It is concluded that the numerical method used in this study is accurate thus, promoted confidence with the results obtained.

Next, Table 2 and 3 shows the values of $-\theta'(1)$ and $f''(1)$ with various values of Re and S, respectively. From both tables, it is found that the increase of Re and $S > 0$ result to the increase of $-\theta'(1)$ while $f''(1)$ decreases. This is physically mean that the increase in Re and the presence of suction effect has promoted the convection heat transfer capability. The situation goes contrary for injection effect $(S < 0)$ where the values of $-\theta'(1)$ reduced to 0 which denoted to pure conduction process occur. It is suggested that, the strong injection is sufficient to completely stop a convective heat transfer process in a boundary layer [4].

The values of $-\theta'(1)$ and $f''(1)$ for various values of $\Pr$, λ and γ are tabulated in Tables 4 and 5, respectively. It is seen that the increase of $\Pr$ gives a drastically increases in $-\theta'(1)$. Basically, small number of $\Pr$ usually is a liquid metal. Conduction $\left(-\theta'(1) \square 0\right)$ is dominant for metal and this is a reason why the small $\Pr$ results less in $-\theta'(1)$. Further, from both tables, it is suggested that the presence of stretching effect λ also enhanced the $-\theta'(1)$. Further, the increase of λ has enhanced the temperature gradient which refers to the skin friction scalar. However, since the gradient is in the opposite direction, hence it is said that the values of $f''(1)$ decreases. In considering the effect of γ, it is observed that the present of γ reduced the values of $-\theta'(1)$ while $f''(1)$ increase drastically. The γ enhanced directly onto $f''(1)$ while diminished the stretching effect on the values of $-\theta'(1)$ and $f''(1)$ made λ less influence in a boundary layer.

Next, Figs. 2 and 3 present the temperature profiles $\theta(\eta)$ for various values of Re and $\Pr$, respectively. From both tables, it is found that the increase of results to a decrease in thermal boundary layer thickness. The reduction in thicknesses is more significant for $\Pr$ where the large $\Pr$ consumes much thinner boundary layer thickness compared to the small $\Pr$. This is because the smaller $\Pr$ usually is highly in thermal diffusivity, therefore the energy ability to heat transfer is high thus enlarged the boundary layer thickness.

Temperature profiles $\theta(\eta)$ for various values of S are plotted in Fig. 4. It is suggested that for case of suction $(S > 0)$, the increase of S results to the boundary layer thinning while in the case of injection $(S < 0)$, the presence of injection effect has promoted to the drastically increases in the boundary layer thickness.

Lastly, Figs. 5 and 6 illustrates the velocity profiles $f'(\eta)$ for various values of λ and γ, respectively. In considering the stretching effect λ, as λ increases, the velocity profiles and its gradient also increases. Further, the velocity boundary layer thickness increases with λ. The increase in λ has speed up the velocity distribution at the cylinder surface which provided high velocity difference with the ambient flow. This situation expanded the boundary layer thickness. For the velocity slip parameter γ, the increase of γ reduced the velocity profiles and its gradient which results to a decreasing of scalar in skin friction coefficient. In considering the boundary layer thicknesses, it is found that the velocity boundary layer thickness decreases as γ increases.

Table 1: Comparison values of $-\theta'(1)$ with previous published result for $\Pr$ and S when $\mathrm{Re} = 10$ and $\lambda = \gamma = 1$.

Pr	$S = 0$			$S = -0.5$		$S = 0.5$	
	Ishak et al. [4]	Wang [3]	Present	Ishak et al. [4]	Present	Ishak et al. [4]	Present
2	3.0360	3.035	3.0351	0.0631	0.0631	11.1517	11.1516
7	6.1592	6.160	6.1558	0	0.0000	36.6120	36.6115
10	7.4668	10.77	7.4624	0	0.0000	51.7048	51.7039

Table 2: Values of $-\theta'(1)$ and $f''(1)$ with various values of Re when $\Pr = 7, \lambda = 0.5, S = 0$ and $\gamma = 1$.

Re	$-\theta'(1)$	$f''(1)$
1	1.1640	-0.2132
5	2.1624	-0.2769
10	2.7891	-0.3070
50	5.0487	-0.3671
100	6.4733	-0.3892

Table 3: Values of $-\theta'(1)$ and $f''(1)$ with various values of S when $\Pr = 7$, $\mathrm{Re} = 10$ and $\lambda = \gamma = 1$.

S	$-\theta'(1)$	$f''(1)$
-1	0	-0.38712
-0.5	0	-0.49405
-0.1	0.7048	-0.62714
0	3.6192	-0.66747
0.1	8.7819	-0.70899
0.5	35.2772	-0.84225
1	70.0796	-0.90986
2	140.0209	-0.95244

Table 4: Values of $-\theta'(1)$ with various values of Pr and λ when $\mathrm{Re}=10$, $S=0.5$ and $\gamma=1$.

Pr	$-\theta'(1)$		
	$\lambda=0.0$	$\lambda=0.5$	$\lambda=1.0$
0.7	3.5066	3.5783	3.6468
1	5.0064	5.0909	5.1666
5	25.0000	25.1372	25.2646
7	35.0000	35.1407	35.2772
10	50.0000	50.1452	50.2875

Table 5: Values of $-\theta'(1)$ and $f''(1)$ with various values of λ and γ when $\mathrm{Re}=10, \mathrm{Pr}=7$ and $S=0.5$.

λ	$\gamma=0$		$\gamma=1$	
	$-\theta'(1)$	$f''(1)$	$-\theta'(1)$	$f''(1)$
0	35.0000	0.0000	35.0000	0.0000
0.5	35.8448	-2.9620	35.1433	-0.4192
1	36.6114	-6.6247	35.2772	-0.8423
1.5	37.3220	-10.8264	35.4039	-1.2681
3	39.2245	-25.9524	35.7478	-2.5603
10	45.8825	-129.6486	36.9860	-8.7409

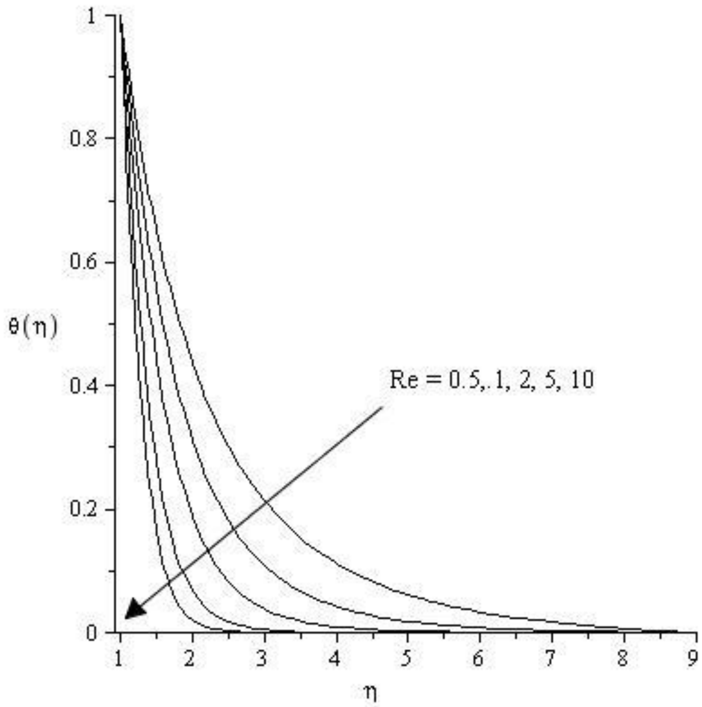

Figure 2: Temperature profiles $\theta(\eta)$ for various values of Re when $\mathrm{Pr}=7, \lambda=0.5, S=0$ and $\gamma=1$.

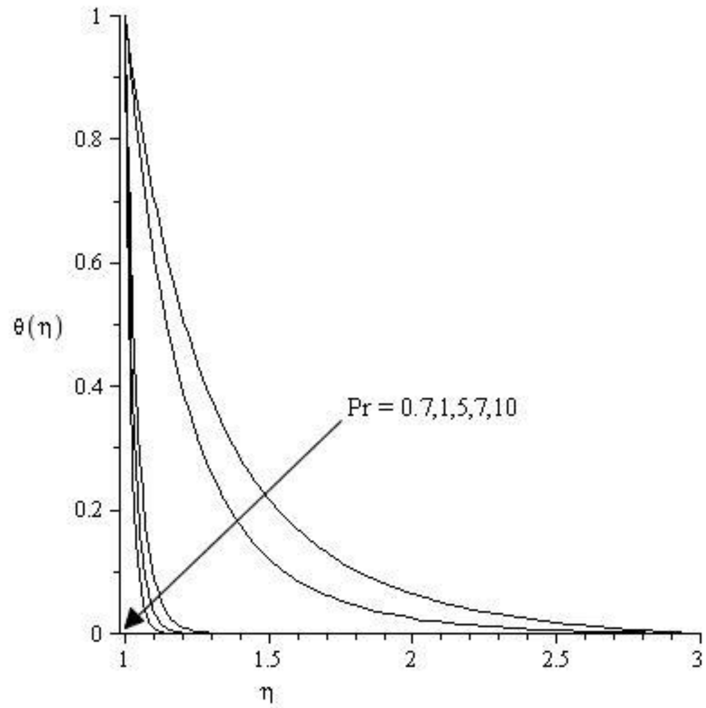

Figure 3: Temperature profiles $\theta(\eta)$ for various values of Pr when $\mathrm{Re}=10$, $S=\lambda=0.5$ and $\gamma=1$.

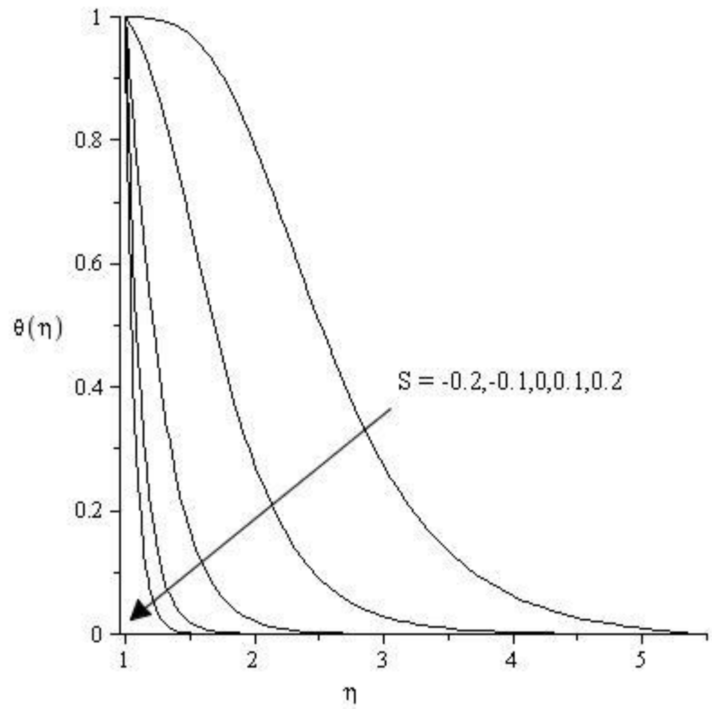

Figure 4: Temperature profiles $\theta(\eta)$ for various values of S when $\mathrm{Pr}=7, \lambda=0.5, \mathrm{Re}=10$ and $\gamma=1$.

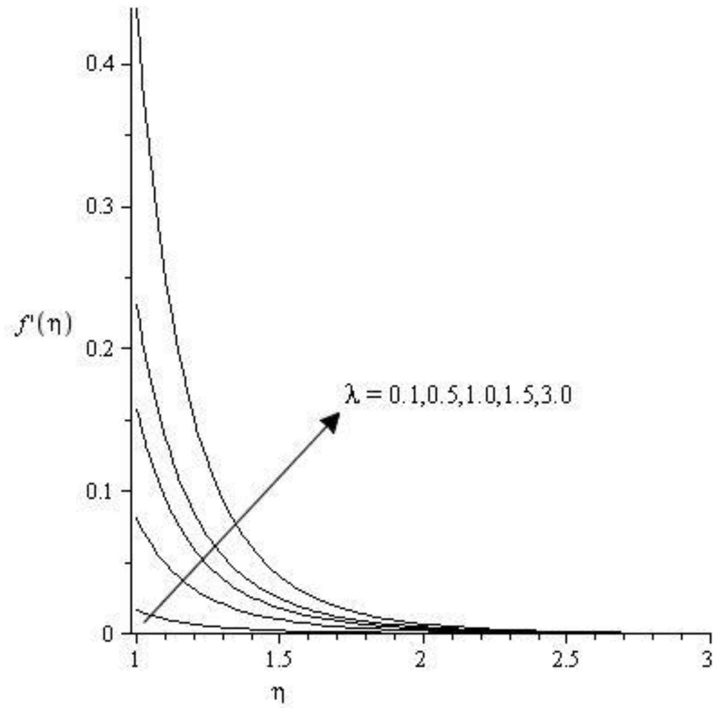

Figure 5: Velocity profiles $f'(\eta)$ for various values of λ when $\Pr = 7, S = 0.5, \mathrm{Re} = 10$ and $\gamma = 1$.

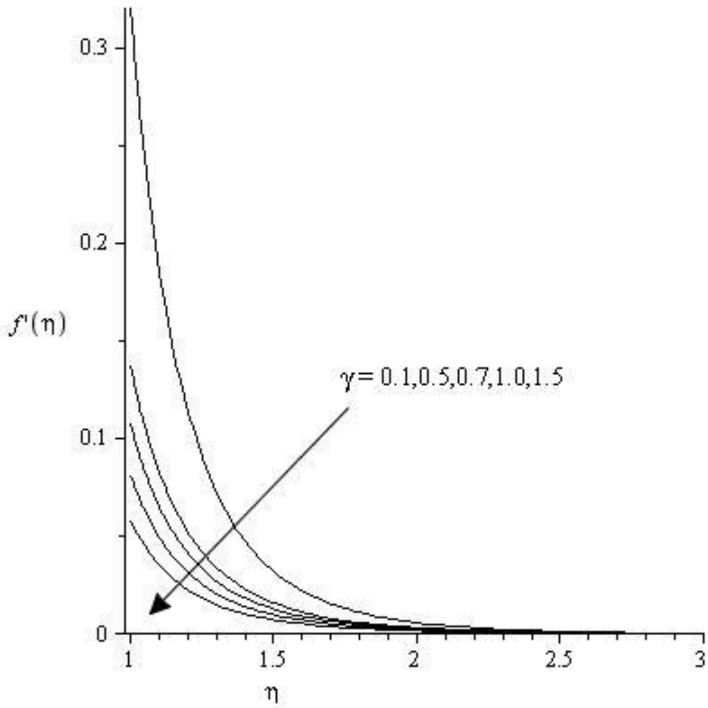

Figure 6: Velocity profiles $f'(\eta)$ for various values of γ when $\Pr = 7, \lambda = S = 0.5$ and $\mathrm{Re} = 10$.

4. CONCLUSION

The suction/injection effect on a boundary layer flow past a stretching cylinder with velocity slip condition is numerically studied. Pertinent parameters such a Prandtl number Pr, the Reynolds number Re, the stretching parameter λ, the suction/blowing parameter S and the velocity slip parameter γ show significant effects onto physical quantities interested, the boundary layer thicknesses as well as temperature and velocity profiles.

As a conclusions, it is found that the increase of $\Pr, \mathrm{Re}, \lambda$ and $S > 0$ for suction effect results to the increase in Nusselt number while γ does oppositely. It is suggested that, the strong injection $(S < 0)$ is sufficient to completely stop a convective heat transfer process in a boundary layer. Next, from temperature profiles, the thermal boundary layer thickness decreases as $\Pr, \mathrm{Re}$ and $S > 0$ increases meanwhile injection effect $(S < 0)$ has promoted to the drastically increases in the boundary layer thickness.

Lastly, the presence of λ enhanced the velocity profiles and its boundary layer thickness while γ does contrary. This is due to the increase in the velocity distribution at the cylinder surface as λ increases which provided high velocity difference with the ambient flow thus, expanded the boundary layer thickness.

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