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COVARIANCE MATRIX ANALYSIS IN SIMULTANEOUS LOCALIZATION AND MAPPING

NUR AQILAH BINTI OTHMAN

Thesis submitted in fulfillment of the requirements
for the award of the degree of
Doctor of Philosophy (Instrumentation Engineering)

Faculty of Electrical & Electronics Engineering
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APRIL 2016

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I hereby declare that the work in this thesis is my own except for quotations and summaries which have been duly acknowledged. The thesis has not been accepted for any degree and is not concurrently submitted for award of other degree.

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His command is only when He intends a thing that He says to it, "Be," and it is.
(Al-Qur’an, Ya-Sin 36:82)

To Hasnun Arif, Hana Ayesha, Haifa Aaleya, beloved parents, and family.

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ABSTRACT

Estimation at a specific time or also known as the filtering technique in estimation and control theory is a method to estimate the desired parameters from indirect and uncertain observations, taking into account the system and measurement errors. One of the applications that implements estimation technique is the simultaneous localization and mapping (SLAM) of a mobile robot. SLAM is one of the navigation techniques which enables the mobile robot to move autonomously and observes its surrounding in an unknown environment. SLAM does not require a priori map, but with the aid of sensors on board, the mobile robot incrementally builds a map of the environment and use this map to localize its position. Therefore, an estimation technique is used to provide the approximate location of mobile robot and landmarks at any time based on the measurement data that are recursively recorded by the sensors. In mobile robot SLAM, extended Kalman filter (EKF) has been one of the most preferable estimators due to its relatively simple algorithm and efficiency of the estimation through the representation of the belief by a multivariate Gaussian distribution; unimodal distribution, with a single mean annotated with a corresponding covariance uncertainty. Nonetheless, EKF-based SLAM suffers from high computational cost due to the update process of covariance matrix. One of the objectives of this thesis is to propose an alternative method to simplify the structure of covariance matrix by means of matrix diagonalization method using eigenvalues. The non-diagonal parts of the covariance matrix are cross-correlation elements, which represent the correlation between the position of the robot and the landmarks. In diagonalizing the covariance matrix, these terms would be eliminated. However, this thesis has proven that the cross-correlation elements are important to ensure the accuracy of the estimation; hence it should be integrated in the diagonalization process. In EKF-based SLAM, measurement data are required at each time step to complete the estimation process. Sudden absence of these data might affect the state estimation and the covariance value. Hence, this thesis also investigates the impact of this phenomenon, which is addressed as intermittent measurement condition. The requirement of Gaussian distribution behavior of measurement and system noises has limited the use of extended Kalman filter in all conditions. In mobile robot SLAM, the noise characteristics might be unknown. Therefore, H_∞ filter is used instead of EKF under this condition. One of the biggest challenges in implementing H_∞ filter-based SLAM is the manual parameter tuning. Thus, this thesis also proposes a sufficient condition for the estimation of H_∞ filter-based SLAM by providing a lower boundary of the selection for the parameter gamma under specific assumptions and environmental conditions. All of these issues are examined and investigated from an estimation-theoretic perspective through mathematical analysis. Theorems, lemmas and propositions are proposed to represent the findings of the analysis. The results obtained were validated through simulation analysis.

ABSTRAK

Anggaran pada masa tertentu atau secara khususnya dalam teori anggaran dan kawalan dipanggil sebagai teknik penapisan, adalah salah satu kaedah untuk menganggarkan sesuatu parameter yang dikehendaki daripada hasil pemerhatian secara tidak langsung dan tidak menentu, dengan mengambil kira kemungkinan berlakunya kesilapan daripada sistem dan teknik pengukuran. Salah satu aplikasi yang menggunakan teknik anggaran ini adalah penyetempatan dan pemetaan serentak (SLAM) oleh robot mudah alih. SLAM adalah salah satu teknik navigasi yang membolehkan robot mudah alih untuk bergerak dengan sendirinya dan memerhatikan persekitaran sekelilingnya yang belum dikenali. SLAM tidak memerlukan peta yang telah tersedia, namun dengan bantuan penderia yang diletakkan diatas robot, robot mudah alih mampu untuk membina peta persekitarannya secara berperingkat dan seterusnya menggunakan peta ini untuk menyetempatkan kedudukannya. Oleh yang demikian, teknik anggaran ini akan digunakan untuk menganggarkan kedudukan robot mudah alih dan mercu tanda berdasarkan kepada data pengukuran yang direkodkan oleh penderia. Dalam proses SLAM oleh robot mudah alih, penapis Kalman lanjutan (EKF) adalah salah satu penganggar yang paling kerap digunakan kerana algoritmanya yang mudah dan kecekapan anggarannya, melalui perwakilan kepercayaan oleh taburan Gaussian dengan berbilang pembolehubah; agihan ekaragam, dengan min tunggal beranotasikan sebuah ketidakpastian kovarian sepadan. Walaubagaimanapun, SLAM berasaskan EKF memerlukan kos pengkomputeran yang tinggi disebabkan oleh proses mengemaskini matriks kovarian. Oleh yang demikian, salah satu objektif tesis ini adalah untuk mencadangkan satu kaedah alternatif untuk memudahkan struktur matriks kovarian melalui kaedah pepenjuruan matriks menggunakan nilai eigen. Bahagian-bahagian yang bukan pepenjuru matriks kovarian adalah dipanggil sebagai elemen sekaitan silang, yang mewakili hubungan antara kedudukan robot dan mercu tanda. Dalam teknik memenjurukan matriks kovarian, sebutan-sebutan ini akan dihapuskan. Walaubagaimanapun, tesis ini telah membuktikan bahawa unsur sekaitan silang ini adalah penting untuk memastikan ketepatan anggaran; oleh itu, ia perlu disepadukan dalam proses pepenjuruan tersebut. Dalam proses SLAM berasaskan EKF, pengukuran data diperlukan pada setiap langkah masa untuk menjayakan proses anggaran. Kehilangan data ini secara tiba-tiba mungkin boleh memberi kesan kepada keadaan anggaran dan nilai kovarian. Oleh itu, satu lagi objektif tesis ini adalah untuk mengkaji kesan fenomena ini, yang dinyatakan sebagai syarat ukuran terputus-putus. Keperluan akan sifat taburan tingkah laku dan kehinggaran sebagai Gaussian telah menghadkan penggunaan penapis Kalman lanjutan untuk digunakan pada semua keadaan. Dalam proses SLAM oleh robot mudah alih, ciri-ciri kehinggaran mungkin tidak diketahui. Oleh yang demikian, penapis H_∞ adalah lebih tepat untuk digunakan berbanding EKF dalam keadaan ini. Salah satu cabaran yang paling besar dalam melaksanakan SLAM berasaskan penapis H_∞ adalah parameter yang digunakan perlu ditentukan secara manual oleh pengguna. Oleh itu, tesis ini juga mencadangkan satu syarat yang mencukupi bagi penganggaran SLAM berasaskan penapis H_∞ dengan mencadangkan nilai minima bagi pemilihan parameter gamma dibawah andaian dan keadaan persekitaran yang tertentu. Kesemua isu-isu ini diteliti dan dikaji dari perspektif teori anggaran melalui analisis matematik. Teorem, lema dan usul dicadangkan untuk mewakili hasil analisis. Seterusnya, keputusan yang diperolehi disahkan melalui analisis simulasi.

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LIST OF SYMBOLS

B	eigenvector
C	invertible matrix
D	diagonalized matrix
D_k	user-defined matrix
F_k	state transition function
$F_{r(k)}$	state transition function of robot position
∇f_r	Jacobians transformations with respect to robot position
∇F_k	Jacobians transformations of parameter F_k in nonlinear H_∞ filter
∇F_X	Jacobian of f with respect to X_k evaluated at $\hat{X}_k$
∇F_w	Jacobian with respect to w_k
G_k	control matrix
$\nabla g_{\gamma_k \omega_k}$	Jacobians transformations with respect to process noise
H_k	observation matrix
∇H_i	Jacobian of h with respect to X_{k+1} evaluated at $\hat{X}^-_{k+1}$
∇H_k	Jacobians transformations of parameter H_k in nonlinear H_∞ filter
I_n	identity matrix with n dimension
J	cost function
k	specific time
K_k	Kalman gain
ℓ_m	position of landmark
L_a	landmark vector (absolute map)
L_r	landmark vector (relative map)

m	number of landmarks in the environment
n	dimension of a vector or matrix
P	covariance matrix
P_0	initial state error covariance matrix
P_{0rr}	robot initial state error covariance
P_{0mm}	landmarks initial state error covariance
P_k	state error covariance matrix
P_k^-	predicted or priori state covariance matrix
P_k^+	estimated or posteriori state covariance matrix
P_{mm}	cross-covariance matrix of the robot and landmark position or cross-correlation between them
P_{rm}	covariance matrix of the landmark position
P_{rr}	covariance matrix of the robot position
Q_k	covariance matrix of process noise
r_i	relative distance or range
R_k	covariance matrix of measurement noise
S_k	covariance matrix for the innovation
T	sampling rate or time interval of one movement step
u_k	control input
v_k	measurement noise
w_k	process noise of the system
x_i	x coordinate of landmark position
x_k^r	x coordinate of robot position
X	x coordinate of global reference frame

X_k	state vector
$\hat{X}_k$	estimated state
$\tilde{X}_k$	error of the estimation
$\hat{X}_k^+$	posteriori estimate of state or estimated state
$\hat{X}_k^-$	priori estimate of state or predicted state
X_m	state vector of landmark position
X_r	state vector of robot position
$X_{r(0)}$	initial position of mobile robot
X_R	x coordinate of robot reference frame
y_k	linear combination of the state
$\hat{y}_k$	estimation of y_k
$\tilde{y}_k$	estimation error of y_k
y_k^r	y coordinate of robot position
y_i	y coordinate of landmark position
Y	y coordinate of global reference frame
Y_R	y coordinate of robot reference frame
z_k	observation model or measurement model
$\hat{z}_k$	predicted observation
Z_k	observations vector
γ	scalar value or level of noise attenuation
γ_k	mobile robot turning rate
γ_{k+1}	Bernoulli random variable
$\delta\gamma$	process noise of mobile robot turning rate

$\delta\omega$	process noise of mobile robot velocity
θ_k	robot heading angle
λ	eigenvalue
μ	mean
μ_k	innovation matrix
σ^2	variance
υ_r	measurement noise of range observation
υ_ϕ	measurement noise of bearing observation
ϕ_i	relative angle or bearing
ψ_n	diagonal element of a matrix
ω_k	velocity of mobile robot
Ω_k	Fisher information matrix
0_n	null matrix with n dimension
$\cos(\cdot)$	cosines of
$\det(\cdot)$	determinant of
$\mathrm{diag}(\cdot)$	diagonal matrix of
$\exp(\cdot)$	exponential function of
$\mathrm{eig}(\cdot)$	eigenvalue of
$\sin(\cdot)$	sine of
$\tan^{-1}(\cdot)$	inverse tangent of
$f(\cdot)$	function of
$h(\cdot)$	function of
$p(\cdot)$	density distribution function of
$\mathrm{tr}(\cdot)$	trace of a matrix

$\forall$	for all
$\in$	element of
$\mathbb{E}[\cdot]$	expectation of
$\mathbb{R}$	real number
$\mathbb{R}^{n\times m}$	dimension of matrix, all real number with *n* rows and *m* columns
$\mathcal{O}(\cdot)$	order of magnitude of
$\mathcal{N}\left(\mu,\sigma^2\right)$	normal (Gaussian) distribution with mean μ and variance σ^2
$\mathcal{N}\left(X;\mu,\sigma^2\right)$	pdf of a normal (Gaussian) variable X with mean μ and variance σ^2
$\square$	end of the proof
$[\cdot]^T$	transpose of matrix
$\lvert\cdot\rvert$	determinant of a matrix or magnitude of a scalar
$\lVert\cdot\rVert$	norm of a vector or matrix
$\rightarrow$	much more towards
$\Rightarrow$	therefore implies that
$\bigcap$	logical “AND” operation (set intersection)
$\approx$	approximately equal
$\sim$	distributed as

LIST OF ABBREVIATIONS

1D	one dimensional
2D	two dimensional
3D	three dimensional
CML	concurrent mapping and localization
e.g.	example
EKF	extended Kalman filter
Eq.	equation
et al.	and others
FIM	Fisher information matrix
H_∞	*H* infinity
HF	*H* infinity filter
i.e.	that is
KF	Kalman filter
MAV	micro air vehicle
pdf	probability density function
PF	particle filter
PSD	positive semidefinite
RMSE	root-mean-square error
RWI	Real World Interfaces, Inc.
SLAM	simultaneous localization and mapping
(w/o)	without

CHAPTER 1

INTRODUCTION

1.1 Background of Study

This chapter presents a general overview of navigation activities in mobile robot. The simultaneous localization and mapping of mobile robot is discussed; the problem that motivates this study is explained. The objectives are listed and followed by the scopes, of which this thesis is limited to. This chapter is concluded by underlining the organization of the thesis.

1.1.1 Navigation of Mobile Robot

Navigation is a non-trivial issue in mobile robots. In order to autonomously navigate and perform useful tasks such as surveillance, exploration in hazardous area, or as a transporter in the industry, a mobile robot is required to know its exact position and orientation in either known or unknown environment. Navigation of mobile robot is normally based on three activities: path planning, localization and map building. In these activities, the mobile robot either has been provided by a prior map, in which its current position is located, or the mobile robot is well notified about its current position, therefore requires it to build the map of that environment. In path planning, the mobile robot may also have been provided with an initial map. Its task is to find the path to move from its initial position to the target position without collision.

However, if the mobile robot needs to be operated in an unknown environment or in an area where no information of the map is available, the robot needs to perform two simultaneous activities for the navigation purposes. It needs to map the environment and concurrently localize its position. This technique is called simultaneous localization and mapping (SLAM), or concurrent mapping and localization (CML).

1.1.2 Simultaneous Localization and Mapping

Simultaneous localization and mapping provides a condition where a mobile robot is assigned to observe an unknown environment and incrementally constructs a map of the environment that it has recognized. It attempts to localize itself on the constructed map recursively until its task is achieved. These activities are accomplished with the aid of proprioceptive (e.g., from an odometer) and exteroceptive (e.g., from a laser scanner) sensors on board. Using the data fed by the sensors and the algorithm to transfer the information obtained in the robot reference frame, the mobile robot is able to estimate its current position and the position of the detected feature or landmarks to the users or operators. However, this process is susceptible to errors that may be generated from various sources such as the sensors, modeling, system, and algorithm. Figure 1.1 depicts a simple notion of SLAM.

Studies were conducted in an attempt to minimize the errors by either focusing on developing a better algorithm or using appropriate sensors based on the environment conditions and situations. In developing a better algorithm, applying estimation technique or estimators is one of the preferred approaches. Estimators such as Gaussian or nonparametric filters will provide an estimation of the robot and landmarks position from the noisy data recorded by the sensors. Several Gaussian filters have been implemented in mobile robot SLAM such as the Kalman filter and its derivatives (extended Kalman filter, unscented Kalman filter, and compressed Kalman filter), information filter, and H_∞ filter. These filters are commonly based on the probabilistic theory. On the other hand, particle filter and histogram filter are the examples of nonparametric filters.

Nevertheless, there are several concerns with regards to using estimators in mobile robot SLAM, such as uncertainties, high computational cost, data association, dynamic environment and model approximation. These issues have brought up a huge area for the researchers to focus on.

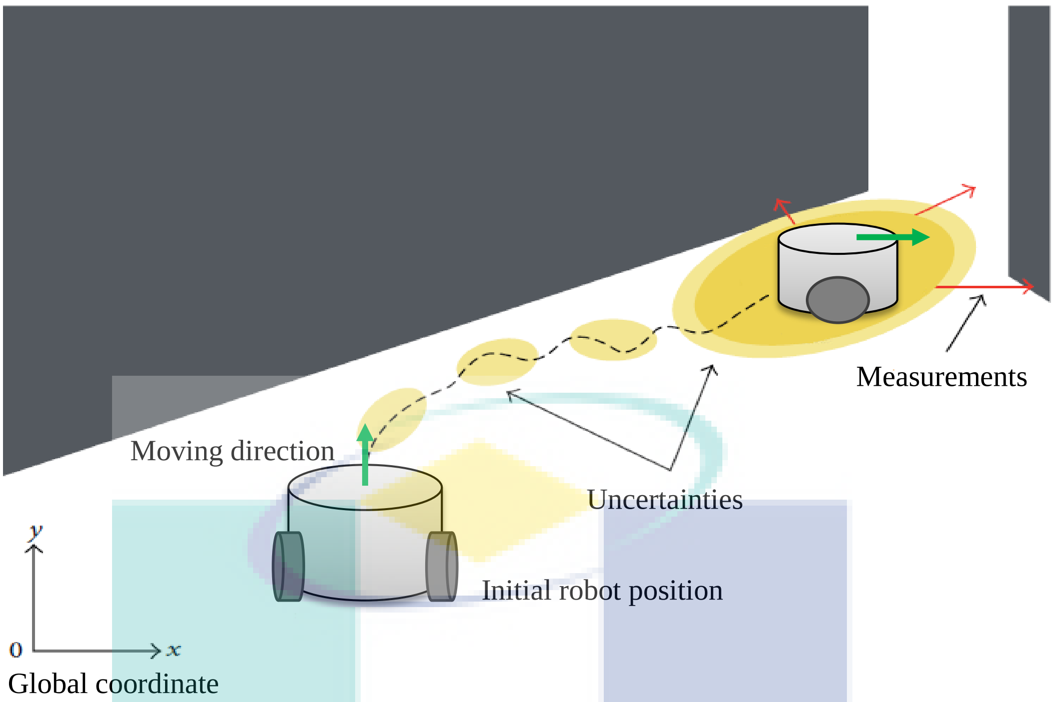

Figure 1.1. Simultaneous localization and mapping: mobile robot makes relative measurements with some uncertainties.

1.2 Problem Statement

The main goal in the development of SLAM algorithm using an estimator is to obtain an optimum solution of the estimation. This includes an accurate and consistent estimation, low computational cost, ability to handle uncertainties and robustness. One of the most popular estimators in mobile robot SLAM is Kalman filter due to its simplicity and computational efficiency (Thrun et al., 2005). In Kalman filter-based SLAM, the estimation is based on how effective the filter reduces the uncertainties generated during the observation. These uncertainties, calculated iteratively to continuously identify the efficiency of estimation, are presented in the form of covariance matrix, which is an essential parameter in this thesis. Larger uncertainties would normally lead to more erroneous estimation; hence, smaller uncertainties are an apparent target to ensure a better estimation.

One of the drawbacks of Kalman filter-based SLAM is the process to generate the covariance matrix. The Kalman filter generally requires an update time of $\mathcal{O}(m^2)$, in which most of the computation time is used to update and calculate the covariance matrix. Parameter m stands for the landmarks, and this number increases as more landmarks are detected. Therefore one of the aims of this thesis is to find the possible technique to simplify the structure of covariance matrix to reduce the computational cost of Kalman filter-based SLAM.

However, the structure of the covariance matrix cannot be easily simplified since each term in the covariance matrix represents the variances and correlations of different parameters. The major parts of the covariance matrix are the cross-correlation elements, which represent the correlation between the position of the robot and the landmarks. Some of the approaches in simplifying the covariance structure are by eliminating the cross-correlation elements. Nonetheless, the cross-correlation cannot be simply removed and it is important to ensure the accuracy of the estimation. In this thesis, the importance of cross-correlation terms is proven by means of the mathematical approach.

In general, SLAM of mobile robot is assumed to be conducted under the optimal conditions: no dynamic obstacles, the environment is planar and the measurement data are always available. In reality, these are not always the case, especially in terms of the continuous availability of the measurement data. In implementing the Kalman filter in SLAM, the measurement data are required at each time step for the correction of the state estimation. Sudden loss of these data might have an impact on the state estimation and the covariance value. Hence, this thesis also aims to investigate the impact of this phenomenon; addressed as intermittent measurement condition on the Kalman filter-based SLAM to investigate the ability of Kalman filter in handling uncertainties.

The main characteristic in implementing Kalman filter as an estimator is the Gaussian behavior of the noises. The noises in the system and from the measurement process must hold white Gaussian distribution criterion, in which zero mean with associated variance based on probability distribution function. This requirement has degraded the performance of the Kalman filter for the application of SLAM under the condition of unknown noise characteristics. Therefore, H_∞ filter could be an alternative in this particular condition. However, there are certain conditions and parameters that need to be well defined to ensure the H_∞ filter-based SLAM performs better than

Kalman filter-based SLAM. Improper tuning of the parameters will lead to the inconsistency of the estimation and to the finite escape time problem phenomenon.

1.3 Research Objectives

This research is conducted using an inductive method of general research strategy. The inductive methods analyze the observed phenomenon and identify the general principles, structures, or processes underlying the phenomenon observed (Nik Mohamed, 2014). The purpose of this kind of research is to develop explanations on the investigated subject or phenomenon. Therefore this thesis aims to perform a theoretical analysis to provide the knowledge on the abovementioned problems of mobile robot SLAM in the direction of system analysis, perspective of control, and estimation theory. By employing the inductive method and based on the research problems, this thesis was guided by the following research objectives:

(i) To prove that the cross-correlation is important in Kalman filter-based SLAM of mobile robot by means of a mathematical approach.

(ii) To diagonalize the covariance matrix of Kalman filter-based SLAM with the aim to reduce the computational cost.

(iii) To scrutinize the behavior of covariance matrix and state estimation of Kalman filter-based SLAM under intermittent condition.

(iv) To propose a sufficient gamma value in H_∞ filter-based SLAM under specific conditions.

1.4 Scope of Study

The topic for each research problems of this thesis will be discussed in separate chapters. The covariance matrix is the main dependent variable of the study in all research problems. The issues are investigated and theoretically analyzed based on the pre-defined specific environmental conditions and assumptions. The results of the analysis are presented by several propositions, lemmas and theorems. The proposed theorems are validated against simulation analysis. The scope of this thesis is, however, limited to the theoretical framework and simulation. The experimental work is not a part of the present study, but it is going to be addressed in the future work. Generally, the area of study involved in this thesis is shown by Figure 1.2.

1.5 Thesis Overview

The remainder of this thesis is organized as follows.

Chapter 2 presents the literature review of the development in simultaneous localization and mapping of the mobile robot by focusing on the issues of estimators and uncertainties. However, the related works concerning the research problems of this thesis are discussed in detail in each dedicated chapter.

Chapter 3 explains the research methodology used in this thesis and provides the fundamental of the theoretical formulations of the models, the Kalman filter, and H_∞ filter.

Chapter 4 covers the first research problem, the importance of cross-correlation elements. This chapter concludes the first objective of the thesis.

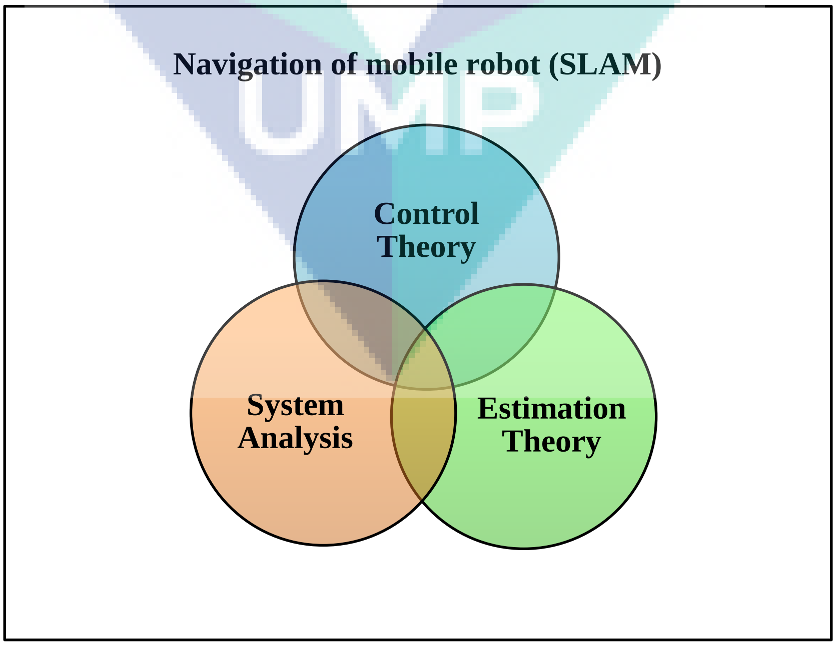

Figure 1.2. Area of study.

Chapter 5 delineates the whole discussion of the second research problems, the simplification of the covariance matrix by means of the diagonalization method. This completes the second objective of this thesis.

Chapter 6 describes the impact of the intermittent measurement on the Kalman filter-based SLAM; this is to prove the ability of Kalman filter in handling uncertainties, and therefore fulfills the third objective of the thesis.

Chapter 7 discusses the performance analysis of H_∞ filter in mobile robot SLAM, and provides the conditions and a boundary of parameter selection in which H_∞ filter outperforms the Kalman filter. This chapter accomplishes the final objective of the thesis.

Chapter 8 concludes the thesis with a summary of the contributions and suggestions of the future research directions with regards each issue.

The publications generated from this thesis are listed in the appendix.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In this chapter, a brief overview of the state of art in simultaneous localization and mapping of mobile robot is presented. The first section describes the history and types of localization and mapping in mobile robot in brief. Section 2.3 emphasizes on the development of SLAM in mobile robot in general. The subsequent section focuses on the improvement of Kalman filter-based SLAM and related issues that have been discussed in the Chapter 1. However, the related literatures on each research problem are not presented here since they are discussed in the dedicated chapters.

2.2 Localization and Mapping in Mobile Robot

Prior to the merging of the localization and mapping in mobile robot, these two activities were performed individually. Localization is a process of defining the position of mobile robot in a map. Generally, there are two types of localization strategies: relative or local localization, and absolute or global localization. In the local localization (e.g. wheel-based odometry and dead-reckoning), the mobile robot tracks its position based on its initial position using the information provided by the on-board sensors (Barshan and Durrant-Whyte, 1995, and Borenstein and Liqiang, 1996). In global localization, the mobile robot determines its position with respect to a global reference frame with an assist from the beacons, landmarks or satellite-based signals (e.g. GPS) (Leonard and Durrant-Whyte, 1991a; Betke and Gurvits, 1997; Eom et al., 2010, and Park and Park, 2014). Local localization suffers from the sensor noises and limitations, uncertain measurements, and wheel slippage, while costly installation of beacons, dependencies towards landmarks, and poor availability of satellite signal are some of the disadvantages of global localization (Betke and Gurvits, 1997; Goel et al., 1999 and González et al., 2009). Therefore, an alternative technique was proposed, which employed a probabilistic localization that utilizes the probability function to estimate the position of the mobile robot using the measurement data with prior knowledge of the environment and measuring devices. Chatila, Laumond and Crowley were the first to employ Kalman filter in mobile robot localization with the aid of a priori map (Chatila and Laumond, 1985 and Crowley, 1989).

With regards to the mapping, the earliest map was represented by fine-grained grids developed by Elfes and Moravec (Elfes, 1987 and Moravec, 1988); it is termed as the occupancy grid maps. The grids model the occupied and the free space of the environment by means of a collection of discretized pixels. This type of representation was then being used by (Borenstein and Koren, 1991 and Thrun, 2003). Other types of mapping techniques are the topological and metric map. The topological map represents an environment using a collection of nodes connected via arcs and their interconnections (Kuipers and Byun, 1991, and Choset and Keiji, 2001). The metric representations directly describe the robot environment in an absolute coordinate system. This type of map is also known as the feature-based map, since the data are in fact the collection of landmarks locations and the correlated uncertainty between them (Leonard and Durrant-Whyte, 1991b and Dissanayake et al., 1999). An example of a metric approach is the simultaneous localization and mapping algorithms.

2.3 Development of Simultaneous Localization and Mapping

The earliest development of the simultaneous localization and mapping in mobile robot was initiated by Cheeseman and Smith (Smith and Cheeseman, 1986 and Smith et al., 1990). They introduced a representation of the mobile robot position and the position of all landmarks in a joined state vector combination with a full covariance matrix, which is known as a stochastic map. Smith et al. discussed the concept in the context of feature-based mapping with point landmarks. These works pioneered the probabilistic approach by means of an estimator such as the Kalman filter to concurrently solve the localization and mapping problem in mobile robot.

The research in navigation, and simultaneous localization and mapping of mobile robot was started by the Robotics Research Group, University of Oxford in 1990s. Hugh Durrant-Whyte, J. J. Leonard, J. K. Uhlman, M. Csorba, S. J. Julier, and P. M. Newman were among the researchers from the group that have strengthened the fundamentals of mobile robot SLAM using Kalman filter, and implemented SLAM on the ground and underwater mobile robot (Leonard, 1990; Leonard and Durrant-Whyte, 1991a, b; Uhlmann, 1995; Csorba, 1997; Julier, 1997 and Newman, 1999). Since then, SLAM has become a highly active field of research.

SLAM has been applied in a wide range of applications such as in mining (Nuechter et al., 2004, and Zlot and Bosse, 2014), underwater robot or vehicle (Newman, 1999; West and Syrmos, 2006; Ribas, 2008; Wang et al., 2013 and Paull et al., 2014) and unmanned air vehicles (Kim and Sukkarieh, 2003 and Grzonka et al., 2012) using various techniques such as the 3D visualization (Henry et al., 2014, and Zlot and Bosse, 2014), multiple robot navigation (Saeedi et al., 2011; Forster et al., 2013 and Lazaro et al., 2013), vision-based strategies (Davison et al., 2007, and Celik and Somani, 2013), and learning strategies by means of artificial intelligence (Chatterjee and Matsuno, 2007; Saeedi et al., 2011 and Fu et al., 2014). Figure 2.1 shows some of the applications of SLAM in robotics. Recently, SLAM has been implemented not only in the robot or vehicle but also in medical instruments such as the wireless capsule endoscopy (Body-SLAM) (Bao, 2014) and home appliances (Lee et al., 2012).

Besides the type of applications mentioned above, there are huge research areas and issues that can be explored in SLAM as depicted in Figure 2.2. Since SLAM can be applied in a multitude of devices and applications, the issues and problems in SLAM differ from one application to another. For instance, in selecting the sensors (sonar, laser scanners (Rogers et al., 2014), wheel encoder, gyroscope or cameras), the problems that might arise are such as noises, accuracy, cost of implementation, in addition to the quality and quantity of information gained depending on the application of interest. Camera is the most preferred device nowadays due to its fast reaction and feedback, high accuracy, and low in cost. Nonetheless, it suffers from the initialization problem of landmark position (Mohammadloo et al., 2013; Guerra et al., 2014 and Valiente et al., 2015).

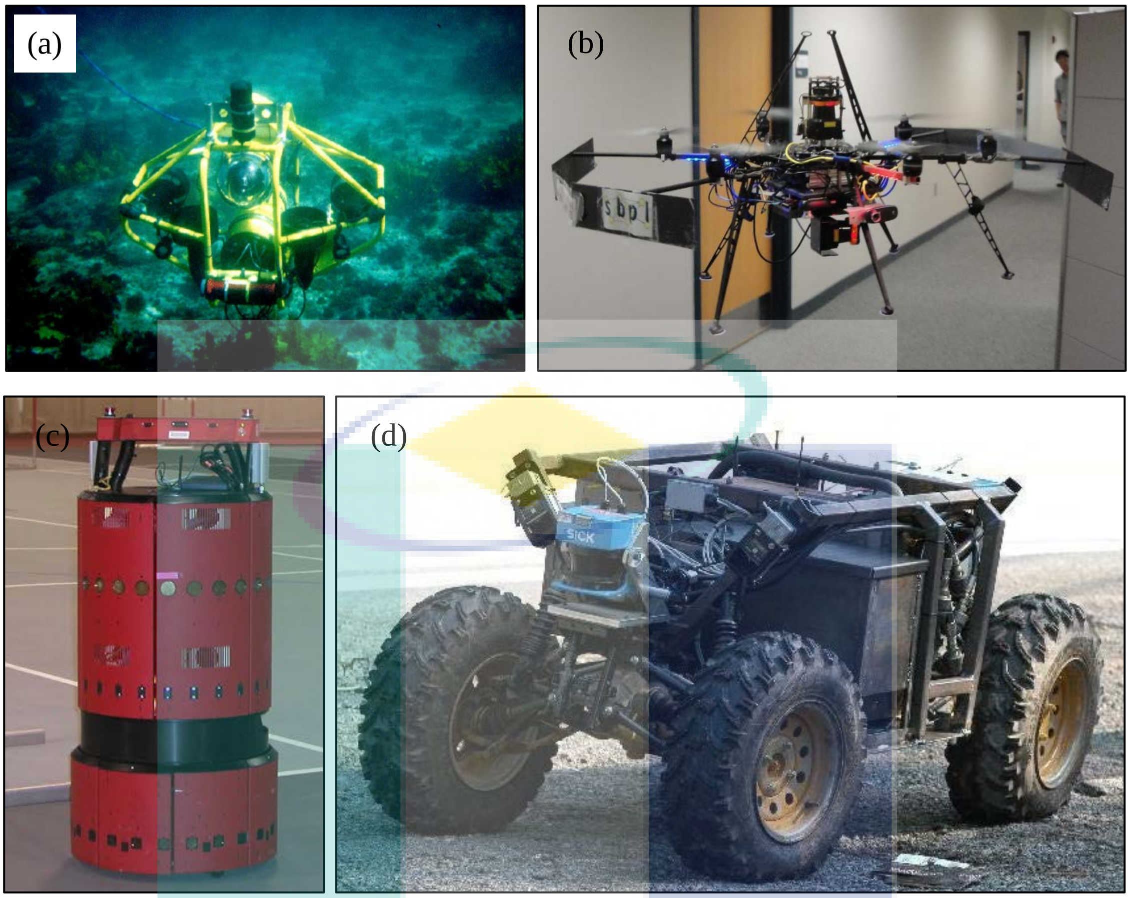

Figure 2.1. Application of SLAM in various terrains and environments.

(a) Autonomous underwater vehicle (Oberon) developed by Australian Centre for Field Robotics at the University of Sydney, Australia.
Source: Newman (1999)

(b) Hexarotor MAV with on-board sensing and processing developed by Robotics Institute of Carnegie Mellon University, USA.
Source: Likhachev (2013)

(c) RWI B21 indoor mobile robot at the Computer Science and Artificial Intelligence Laboratory, Massachusetts Institute of Technology, USA.
Source: Thrun et al. (2005)

(d) The Groundhog robot built to map abandoned mines, developed by Stanford Artificial Intelligence Laboratory, Stanford University, USA.
Source: Thrun et al. (2005)

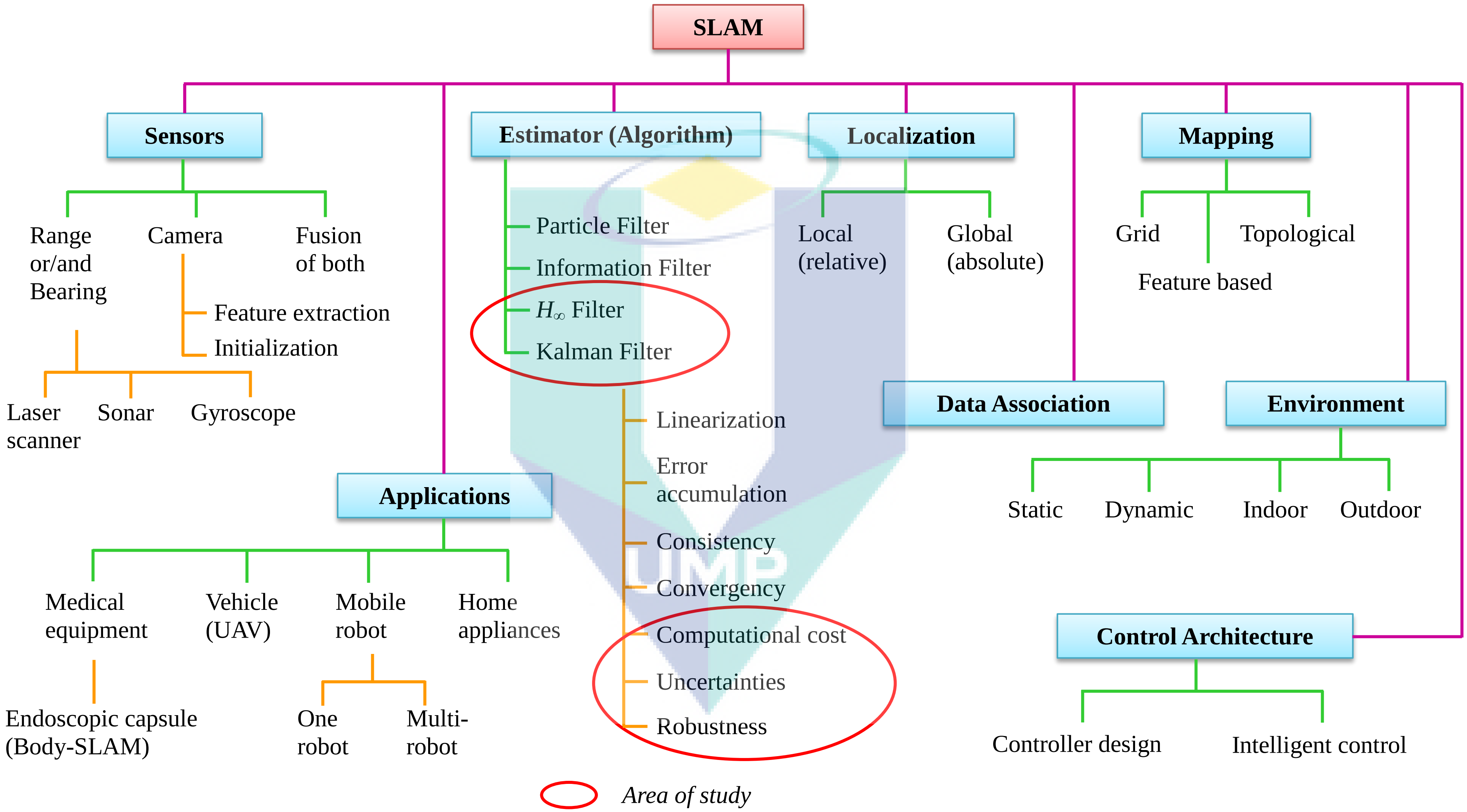

Figure 2.2. Issues and studies on the problems in SLAM.

Other than the sensors, an adequate knowledge of the environment is also vital to ensure correct implementation of SLAM. SLAM is developed based on the conditions and limitations of the environment: either in indoor or outdoor application (Oh et al., 2014; Brand et al., 2015; Dong-Il et al., 2015 and López et al., 2015), or in static or dynamic environment (Todoran and Bader, 2015). The localization and mapping technique, in addition to the control architecture are equally important in establishing an optimal and robust SLAM. Moreover, data association is one of the challenging problems in SLAM. The problem arises when the detected landmark from the observation cannot be properly identified from the landmarks stored in the existing map. It is important to identify the correct correspondences between sensed and mapped landmarks. Various solutions to the problem have been proposed (Neira and Tardos, 2001; Gil et al., 2006; Bosse and Zlot, 2008; Baum et al., 2015; Brekke and Chitre, 2015; and Zhang et al., 2016). This thesis, on the other hand, focuses on the estimator used in SLAM, particularly the H_∞ and Kalman filter in investigating the possible technique to reduce the computational cost, handling the uncertainties and robustness as explained in Section 1.2. Literatures regarding these issues are presented in Section 4.2, 5.2, 6.2 and 7.2. Moreover, linearization error, consistency and convergency are some of other concerns in applying the estimator in SLAM (Castellanos et al., 2004; Bailey and Durrant-Whyte, 2006; Bailey et al., 2006; Castellanos et al., 2007; Huang and Dissanayake, 2007; Paz et al., 2008; Li et al., 2009, and Saha and Chakravorty, 2016). However these problems are beyond the scope of the thesis.

2.4 Solutions to SLAM

An approach that is able to tolerate uncertainties in SLAM is the probabilistic technique. This technique, which is based on Bayesian approach is preferred to the more complex, computationally expensive behavior-based SLAM and mathematical-based SLAM (Thrun et al., 2005). The probability-based approach takes into account these limitations reasonably well as it does not require extensive mathematical computation or the demand of high reliability sensors to estimate the position. The extended Kalman filter (EKF) is an example of the probabilistic techniques. EKF's application in SLAM has become popular in the early 2000s and still receives high attention among researchers (Dissanayake et al., 2001; Nieto et al., 2006; Castellanos et al., 2007; Huang and Dissanayake, 2007; Paz et al., 2008 and D'Alfonso et al., 2013). An early development of EKF-based SLAM was proposed by Gamini Dissanayake et al. (Dissanayake et al., 2001, and Huang and Dissanayake, 2007) notably due to its simple application and low computational cost as compared to other probabilistic approaches. In extended Kalman filter, the estimation is based on how effective the filter reduces the uncertainties that are generated during the observation. These uncertainties are calculated iteratively to continuously identify the efficiency of estimation; this is presented in the form of covariance matrix. Larger uncertainties lead to more erroneous estimation; hence, smaller uncertainties are an obvious target. The results of Huang et al.'s work (Huang and Dissanayake, 2007) suggest that the covariance matrix or uncertainties will converge if the mobile robot is able to continuously observe the landmarks.

Unfortunately, SLAM demands further considerations for the environmental conditions. An assumption of Gaussian noise has restricted extended Kalman filter performance as the main player thus allowing space for a more robust approach such as the particle filter (PF) (Montemerlo and Thrun, 2003 and Pei et al., 2014). Nonetheless, the particle filter is more complex and computationally expensive in addition to being difficult to be applied online. Therefore, the H_∞ filter approach for SLAM is proposed to mitigate the aforementioned issues as this particular technique is more robust as compared to the extended Kalman filter as far as the non-Gaussian noise is concerned (Simon, 2006, and West and Syrmos, 2006) while having a much lower computational cost as compared to the particle filter (Simon, 2006 and Lewis et al., 2008).

2.5 Summary

The localization and mapping processes used to be performed separately until they were merged and executed simultaneously. Today, SLAM is used in a large number of applications such as in the mining sector, military technology, home appliances and medical devices. There are several issues that need to be addressed when employing SLAM in any application. The Kalman filter is preferred due to its simplicity, ability to be applied online and computationally inexpensive. The related works concerning the specific research problems are presented in detail in each dedicated chapter: Section 4.2, Section 5.2, Section 6.2 and Section 7.2.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

This chapter details the research methodology used in this thesis. The chapter begins with the theoretical formulations of the models employed to represent a mobile robot, the environment and the estimation method used in order to solve the simultaneous localization and mapping of a mobile robot. The discussions emphasize on the process and the observation models, in Section 3.2, and the map characteristics in Section 3.3. This is followed by a brief overview of the estimation techniques used in the study; Kalman filter and H_∞ filter. Section 3.5 elaborates the overall process of the research in the form of a flow chart and a short description for each step. However, the detail explanation on the methodology used in each research problems and the algorithms of the simulations are presented in each dedicated chapter.

3.2 System Models

This thesis deals with two dimensional (2D) SLAM and the map is also represented as a 2D map. SLAM is represented through discrete time dynamical system equation using process and observation model.

3.2.1 Process Model

The process model in mobile robot SLAM describes the kinematics and movements of a mobile robot. The mobile robot moves in an environment and measures its relative distance to existing landmarks using sensors. This process is performed in order to locate its position and simultaneously detect and verify the position of the landmarks. On the other hand, the measurement process is represented using the observation model.

The process model of the mobile robot localization and mapping at time $k+1$, described as a function of state vector X_k, control input u_k and process noise w_k evaluated at time k, is defined as:

$$X_{k+1} = f\,(X_k, u_{k+1}, w_{k+1}, k) \tag{3.1}$$

The state vector of a 2D SLAM $X_k \in \mathbb{R}^{3+2m}$ is a joint state-vector of robot position X_r and position of landmark X_m, which has the following structure:

$$X_k = \begin{bmatrix} X_r & X_m \end{bmatrix}^T \tag{3.2}$$

where the position of the mobile robot $X_r = [\theta_k \quad x_k^r \quad y_k^r]^T$ is represented by the robot heading angle θ_k and the coordinates of the center of mobile robot with respect to the global coordinate frame (x_k^r, y_k^r), as depicted in Figure 3.1. The state of the landmarks $X_m = \begin{bmatrix} \ell_1 & \ell_2 & \cdots & \ell_m \end{bmatrix}^T$ are modeled as a set of point landmarks and described by the Cartesian coordinate (x_i, y_i), $i = 1, 2, \ldots, m$, where m refers to the number of landmarks in the environment. Therefore, the full state of the SLAM can be described as follows:

$$X_k = \begin{bmatrix} \theta_k & x_k^r & y_k^r & x_1 & y_1 & \cdots & x_i & y_i \end{bmatrix}^T \tag{3.3}$$

The control input of the robot movement is designated by $u_k = [\gamma_k \quad \omega_k]^T$, where γ_k is a mobile robot turning rate and ω_k is its velocity with associated process noises, $\delta\gamma$ and $\delta\omega$. The process noise w_k is a zero-mean Gaussian noise of $\delta\gamma$ and $\delta\omega$ with covariance Q_k, i.e. $w_k \sim \mathcal{N}(0, Q_k)$. Therefore, the whole process model for the complete system of mobile robot SLAM may be written as

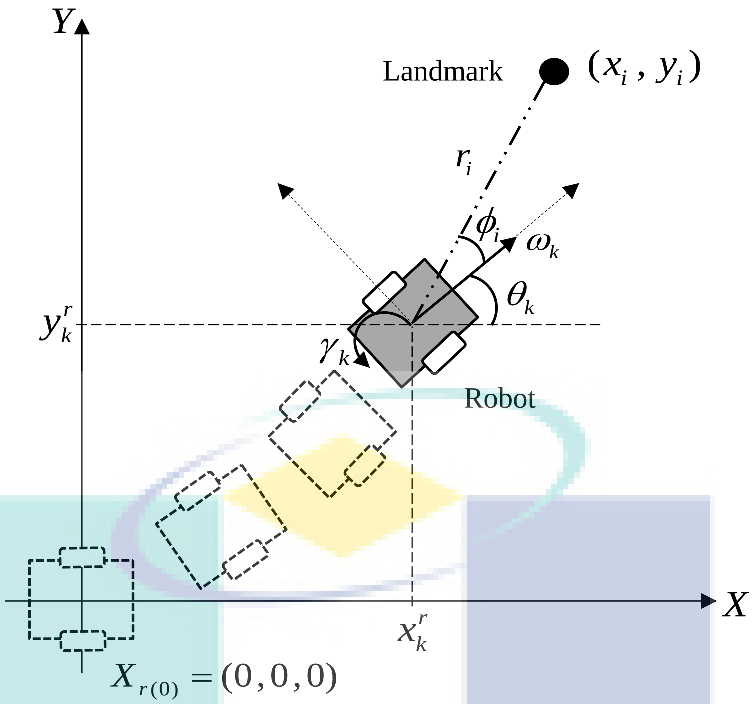

Figure 3.1. Process and observation model of mobile robot SLAM.

$$X_{k+1} = F_k X_k + u_{k+1} + w_{k+1}$$

$$\begin{bmatrix} X_{r(k+1)} \\ \ell_1 \\ \vdots \\ \ell_m \end{bmatrix} = \begin{bmatrix} F_{r(k)} & 0 & \cdots & 0 \\ 0 & I_{\ell_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & I_{\ell_m} \end{bmatrix} \begin{bmatrix} X_{r(k)} \\ \ell_1 \\ \vdots \\ \ell_m \end{bmatrix} + \begin{bmatrix} u_{r(k+1)} \\ 0_{\ell_1} \\ \vdots \\ 0_{\ell_m} \end{bmatrix} + \begin{bmatrix} w_{r(k+1)} \\ 0_{\ell_1} \\ \vdots \\ 0_{\ell_m} \end{bmatrix} \tag{3.4}$$

where I_{ℓ_i} is an identity matrix with dimension of $\mathbb{R}^{(\ell_i \times \ell_i)}$ and 0_{ℓ_i} is the null matrix with a similar dimension as I_{ℓ_i}. Both matrices are dedicated for the landmarks state due to the stationary behavior of the landmark.

3.2.1.1 Mobile Robot Model

The mobile robot considered in this thesis is a two-wheel (uni-cycle) differential drive with the center of mass located below the axle of the robot, and are equipped with the range and bearing sensors onboard, as illustrated in Figure 3.2. The mobile robot is

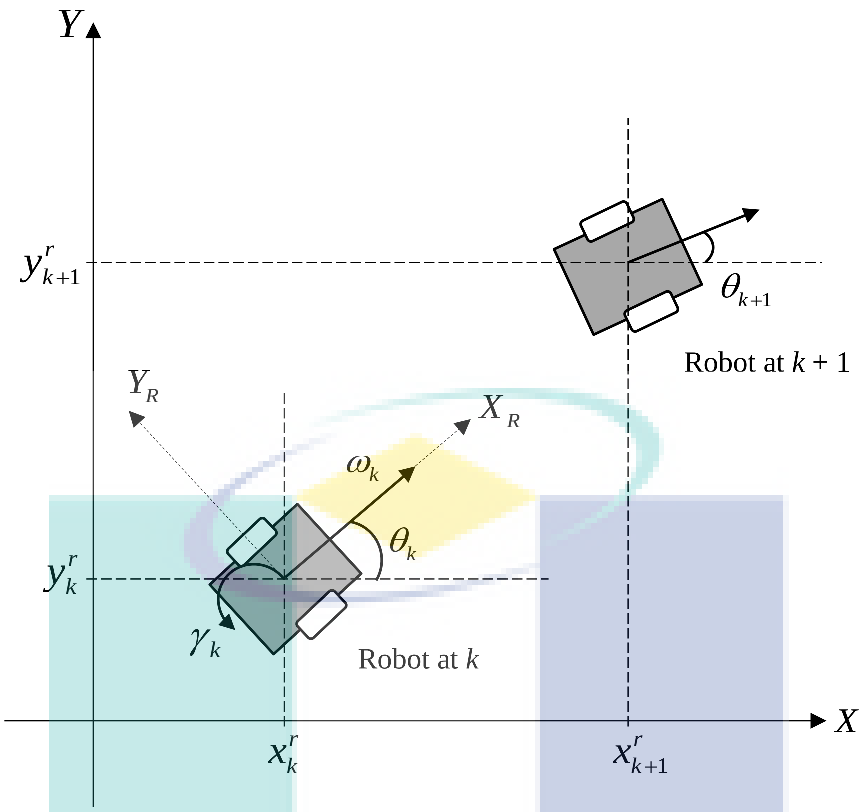

Figure 3.2. Process model of mobile robot.

assumed to be operated on a planar plane with the wheel always remain vertical and that there is, in all cases, one single point of contact between the wheel and the ground plane. Furthermore, there is no slippage occurred at this single point of contact for all time. Thus, the process model of this type of mobile robot after the translation from robot reference frame to global reference frame is defined as follows (Martinelli et al., 2005, and Huang and Dissanayake, 2007):

$$X_{r(k+1)} = \begin{bmatrix} \theta_{k+1} \\ x^r_{k+1} \\ y^r_{k+1} \end{bmatrix} = \begin{bmatrix} \theta_k + (\gamma_k + \delta\gamma)T \\ x^r_k + (\omega_k + \delta\omega)T\cos(\theta_k) \\ y^r_k + (\omega_k + \delta\omega)T\sin(\theta_k) \end{bmatrix} \tag{3.5}$$

where T is the sampling rate or time interval of one movement step.

3.2.1.2 Landmark Model

There are many different types of landmarks and landmark sensors that have been proposed in the literature. Among the common examples are (Frese, 2004):

- *Artificial landmarks:* ultrasonic beacons, radio transmitters, infrared transmitters, laser reflectors, visual markers of specific color or pattern, inductive loops in the ground.
- *Natural landmarks:* corners, walls, vertical lines, visual corners, doors, ceiling grates, trees.
- *Landmark sensors:* ultrasonic transducers, laser range scanners, cameras.

These landmarks are normally used in the hardware implementations. However, since this thesis involves no experimental work , the landmarks used for the discussion will be restricted to point landmarks in the planar plane, and are assumed to be stationary at all time. The landmark is described by two Cartesian coordinates (x_i, y_i) and it is assumed that the sensors measure the location of the landmark relative to the robot's current position (x_k^r, y_k^r), as depicted in Figure 3.1. Hence the process model for the landmarks at time $k+1$ is

$$X_{m(k+1)} = X_{m(k)}$$
$$\begin{bmatrix} x_{1(k+1)} & y_{1(k+1)} & \cdots & x_{i(k+1)} & y_{i(k+1)} \end{bmatrix}^T = \begin{bmatrix} x_{1(k)} & y_{1(k)} & \cdots & x_{i(k)} & y_{i(k)} \end{bmatrix}^T \tag{3.6}$$

3.2.2 Observation Model

The observation or measurement process performed by the sensors of the mobile robot in localization and mapping is represented by an observation model. In mobile robot SLAM, the observation of the *i*-th landmark possesses range and bearing readings, which indicate relative distance r_i and relative angle ϕ_i of the mobile robot to any observed landmarks in the environment. It is assumed that the mobile robot is equipped with the range and bearing sensors onboard and encoders at the wheels to measure the speed. However the type of encoders is not of interest of this study since the thesis only focuses on the simulation work. The speed of the mobile robot is determined by the user. It is defined in the control input, u_k.

The observation model of the mobile robot SLAM is defined as in Eq. (3.7), where $v_k = [\upsilon_r \quad \upsilon_\phi]^T$ is the zero-mean Gaussian observation noise applied to the range and bearing observations with covariance R_k, i.e. $v_k \sim \mathcal{N}(0\,,\,R_k)$. In this thesis, the observation model is also addressed as the measurement model. Process and observation models of the mobile robot localization and mapping are illustrated in Figure 3.1 on page 17.

$$
\begin{aligned}
z_k &= h(X_k, v_k, k) \\
&= \begin{bmatrix} r_i + \upsilon_r \\ \phi_i + \upsilon_\phi \end{bmatrix} \\
&= \begin{bmatrix} \sqrt{(x_i - x_k^r)^2 + (y_i - y_k^r)^2} + \upsilon_r \\ \tan^{-1}\left(\dfrac{y_i - y_k^r}{x_i - x_k^r}\right) - \theta_k + \upsilon_\phi \end{bmatrix}
\end{aligned} \tag{3.7}
$$

3.3 Characteristics of Maps

As the mobile robot moves in the environment and scans the available landmarks, it maps the environment based on the measurement data gathered. The map consists of a set of landmarks with defined location. The landmarks may or may not be known in the mapping process. However in SLAM, the latter is always the case.

The map can be defined in two forms; absolute and relative map (Newman, 1999). An absolute map is the map in which all landmarks are registered in a Cartesian coordinate system by referring to the global coordinate frame. This map has a simple form, and is written in form of vector as follows:

$$L_a = \begin{bmatrix} \ell_1 \\ \ell_2 \\ \vdots \\ \ell_m \end{bmatrix} \tag{3.8}$$

$$\ell_i = (x_i, y_i)$$

Figure 3.3 illustrates the absolute map with three landmarks.

In the relative map form, the landmarks are represented by the relationship between individual landmarks, as shown in Figure 3.4. These relationships are stored in the relative map states, such as that between the landmarks ℓ_p and ℓ_q is written as $\ell_{(p,q)}$. In the point landmarks, the state are simply the vector subtraction of two absolute landmark locations, $\ell_{(p,q)} = \ell_q - \ell_p$. The relative map is written in the vector form as follows:

$$L_r = \begin{bmatrix} \vdots \\ \ell_{(p,q)} \\ \ell_{(q,s)} \\ \vdots \end{bmatrix} \tag{3.9}$$

In this thesis, an absolute map form is used to present the state of the landmarks. The selection is due to its simplicity, and to maintain the whole coordinates of both robot and landmarks, which are referred to the global coordinate frame. Moreover, the observations are made from the mobile robot relative to the landmarks position. Therefore, the absolute map form is superior than its counterpart to represent the whole SLAM system in this thesis.

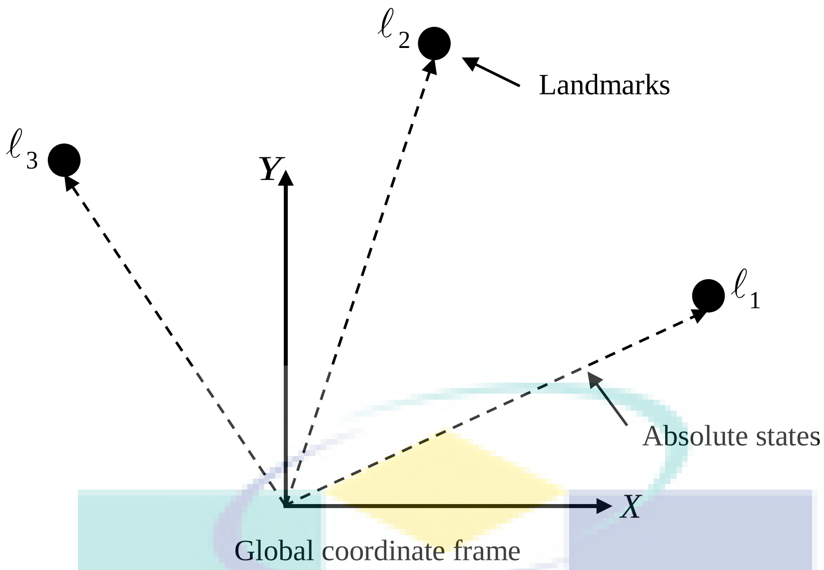

Figure 3.3. An absolute map with three landmarks.

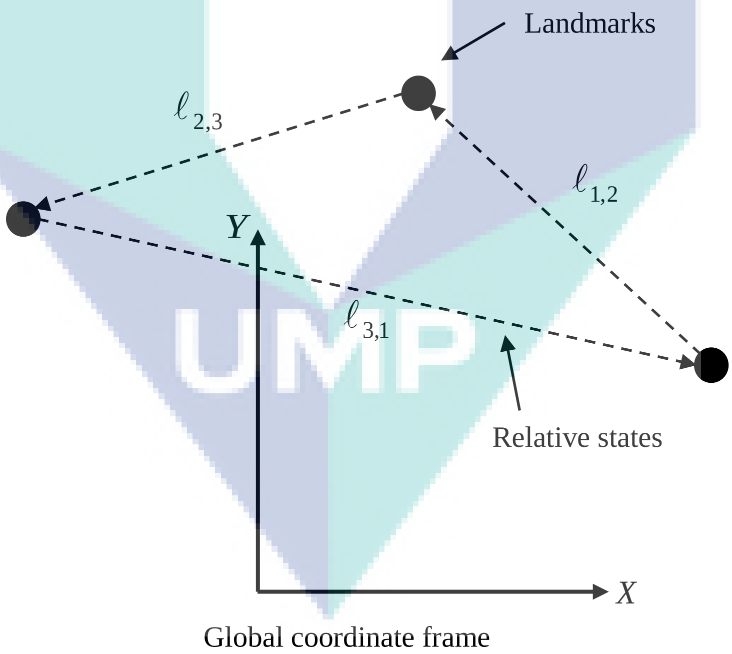

Figure 3.4. Three landmarks representation through a relative map.

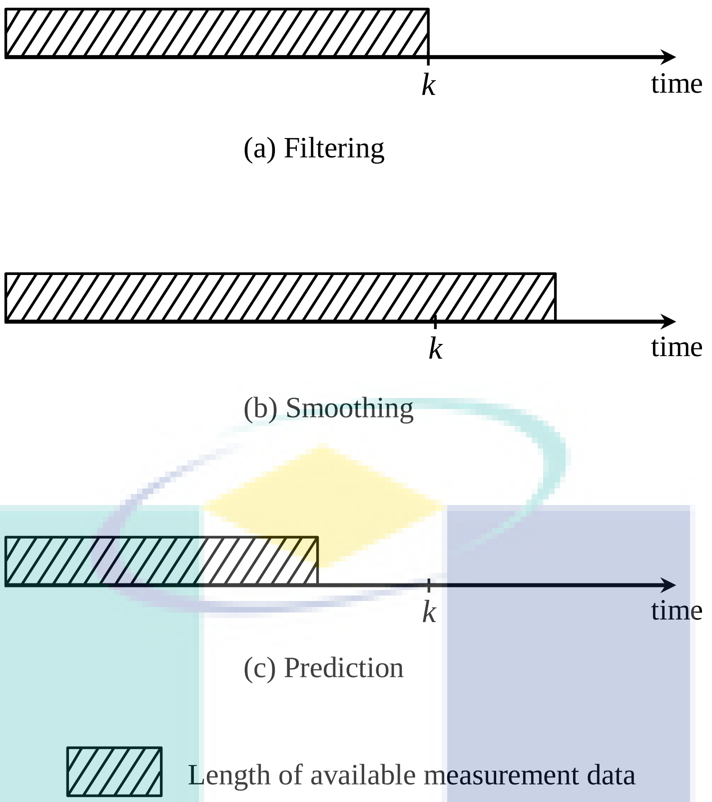

Figure 3.5. Three types of estimation problem.

Source: Gelb et al. (2001)

3.4 Estimation of the State

Estimation is a process of extracting desired information (i.e. parameters) from the indirect and uncertain observations (measurements) by utilizing the measurement errors, effects of disturbances on the system, control actions of the system, and prior knowledge of the information. There are three types of estimation problems namely filtering, smoothing, and prediction. If the time at which an estimation is desired coincides with the last measurement, the problem is called as filtering. When the time of estimation falls within the span of the measurement data, the problem is referred to as smoothing. The prediction, on the other hand, describes the problem in which the time of interest occurs after the last available measurement. The timeline that describes these three problems is illustrated in Figure 3.5 (Gelb et al., 2001).

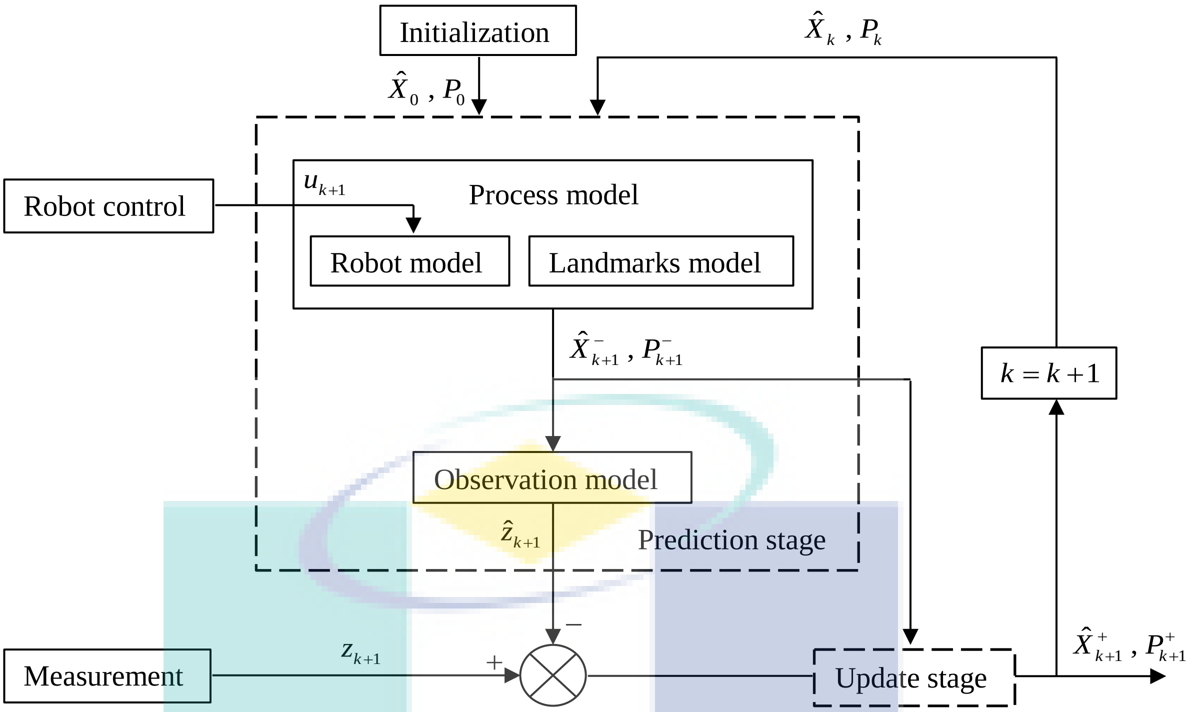

Figure 3.6. Estimation in mobile robot SLAM using Kalman filter.

The estimation approach in mobile robot SLAM is to develop a filtering process for the system, as the estimation is performed at the instant time of the measurement data. In this thesis, two types of filter (also known as estimators) will be examined; Kalman filter and H_∞ filter. Both filters are recursive least squares estimators. They produce at time k a minimum mean squared error of an estimate $\hat{X}^+_k$ of a state vector X_k. This estimate is obtained by fusing a state estimate prediction $\hat{X}^-_k$, that evolves over time as a function of robot controls u_k, with an observation z_k of the state vector X_k. The estimate $\hat{X}^+_k$ is the conditional mean of X_k given all observations $Z_k = [z_1, \cdots, z_k]$ until the time k is available. The state prediction and the measurements process describe the dynamical stochastic system of the mobile robot and its environment as illustrated in Figure 3.1 (on page 17). The state at time $k+1$ is stochastically dependent on the state at time k and the control u_{k+1}. Further, the measurement z_{k+1} depends stochastically on the predicted state at time $k+1$. Figure 3.6 illustrates the example of the whole estimation process using Kalman filter.

3.4.1 Kalman Filter

The Kalman filter is one of the Gaussian filters that model the quantities such as sensor measurements, controls, and the states of the robot and its environment as random variables. These variables are determined through probability functions, and represented as a belief. A belief is a concept to represent the state transition probability that reflects the robot's internal knowledge about the state of the environment. Gaussian techniques possess the basic idea that the beliefs are represented by multivariate normal distributions and unimodal; they possess a single maximum, which is generally denoted as $\mathcal{N}\left(X;\mu,\sigma^2\right)$, a random variable that is represented through its mean, and its variance. Often, X will be a multi-dimensional vector and are characterized by the density distribution functions of the following form:

$$p(X)=\det(2\pi P)^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}(X-\mu)^T P^{-1}(X-\mu)\right\} \tag{3.10}$$

where μ is the mean vector, and P is a positive semidefinite and symmetric matrix known as the covariance matrix. In general, if X is a scalar value, $P=\sigma^2$. More explanations on the probabilistic robotic, Gaussian filters and all the concepts of probability distribution function of mobile robot focusing on SLAM are presented in (Thrun et al., 2005). In this thesis, only the important parameters will be explained for the sake of brevity.

In the mobile robot SLAM, the position of the robot and landmarks are estimated by means of the Kalman filter (KF) based on the probabilistic concept explained earlier. The state vector is predicted based on the system's previous information and is then estimated based on the measurement data obtained from the sensors as illustrated in Figure 3.6. The Kalman filter provides the mean that indicates the updated state vector $\hat{X}_k$, and the covariance of the estimation P_k, which designates the estimation error. The steps for localizing and mapping the mobile robot and the landmarks under linear and nonlinear assumptions using Kalman filter are described in the following subsections.

3.4.1.1 Linear Assumption

In a linear system, the process model of SLAM from time k to time $k+1$ is described as

$$X_{k+1} = F_{k+1}X_k + G_{k+1}u_{k+1} + w_{k+1} \tag{3.11}$$

where X_k is the state of the mobile robot and landmarks, F_{k+1} is the state transition matrix, G_{k+1} is the control matrix that mapped the control inputs u_{k+1} into the state space, and w_{k+1} is the zero-mean Gaussian process noise with covariance Q_{k+1}. On the other hand, the observation model is defined as

$$z_{k+1} = \begin{bmatrix} r_i + \upsilon_r \\ \phi_i + \upsilon_\phi \end{bmatrix} = H_{k+1}X_{k+1}^- + v_{k+1} \tag{3.12}$$

where H_{k+1} is the observation matrix that describes the parameter captured from the observation and Eq. (3.12) is equivalent to the Eq. (3.7).

The Kalman filter is used to estimate the mobile robot pose and landmark location. It recursively computes the estimation of a state X_k according to the process and observation model in Eq. (3.11) and Eq. (3.12). An estimate at the time $k+1$ can be obtained through the expected values of the process model on the first k observations,

$$\hat{X}_{k+1}^- = F_{k+1}\hat{X}_k + G_{k+1}u_{k+1} \tag{3.13}$$

with the corresponding state error covariance matrix

$$P_{k+1}^- = F_{k+1}P_kF_{k+1}{}^T + Q_{k+1} \tag{3.14}$$

These two equations are normally being described as a prediction or time update stage of Kalman filter on the system behavior before the measurement data are cooperated for the correction, as shown in Figure 3.6. Throughout the thesis, parameter $\hat{X}_{k+1}^{-}$ will be addressed as a predicted or priori state while P_{k+1}^{-} will be denoted as predicted or priori state error covariance. These parameters are required in the analysis discussed in Chapter 5.

Using the information from predicted state, the observation at time $k+1$ is predicted. The predicted observation $\hat{z}_{k+1}$ has the following characteristic:

$$\begin{aligned} \hat{z}_{k+1} &= \mathbb{E}\left[z_{k+1} \middle| Z_k\right] \\ &= \mathbb{E}\left[H_{k+1} X_{k+1} \mid Z_k\right] \\ &= H_{k+1}\hat{X}_{k+1}^{-} \end{aligned} \tag{3.15}$$

The difference between the actual observation at time $k+1$ and the predicted observation is known as the innovation matrix μ_{k+1}, whereas S_{k+1} is associated covariance matrix for the innovation. Both parameters are defined as follows:

$$\begin{aligned} \mu_{k+1} &= z_{k+1} - \hat{z}_{k+1} \\ &= z_{k+1} - H_{k+1}\hat{X}_{k+1}^{-} \\ &= H_{k+1}X_{k+1}^{-} + v_{k+1} - H_{k+1}\hat{X}_{k+1}^{-} \\ &= H_{k+1}\tilde{X}_{k+1}^{-} + v_{k+1} \end{aligned} \tag{3.16}$$

$$\begin{aligned} S_{(\mu\mu)k+1} &= \mathbb{E}\left[\mu_{k+1}\mu_{k+1}^{T}\right] \\ &= H_{k+1}P_{k+1}^{-}H_{k+1}^{T} + R_{k+1} \end{aligned} \tag{3.17}$$

$$S_{(X\mu)k+1} = \mathbb{E}\left[\tilde{X}^-_{k+1}\mu^T_{k+1}\right]$$

$$= \mathbb{E}\left[\tilde{X}^-_{k+1}\left(H_{k+1}\tilde{X}^-_{k+1} + v_{k+1}\right)^T\right] \tag{3.18}$$

$$= P^-_{k+1}H^T_{k+1}$$

where $\tilde{X}^-_{k+1} = X^-_{k+1} - \hat{X}^-_{k+1}$ is the error of the state estimation in the prediction stage. The Kalman filter will attempt to minimize the expected mean squared error of the distribution. Therefore, a weighting matrix is chosen for this purpose, which defines the correction factor that needs to be implied based on the value of covariance of the prediction error and the innovation $S_{(X\mu)k+1}$, and the covariance of the innovation $S_{(\mu\mu)k+1}$. This weighting matrix is known as the Kalman gain K_{k+1} and is defined as

$$K_{k+1} = S_{(X\mu)k+1}\left(S_{(\mu\mu)k+1}\right)^{-1}$$
$$= P^-_{k+1}H^T_{k+1}\left(H_{k+1}P^-_{k+1}H^T_{k+1} + R_{k+1}\right)^{-1} \tag{3.19}$$

Therefore the Kalman update step or measurement update stage for the state estimate and the covariance of the estimate is given by

$$\hat{X}^+_{k+1} = \hat{X}^-_{k+1} + K_{k+1}\ \mu_{k+1} \tag{3.20}$$

$$P^+_{k+1} = \left(I_n - K_{k+1}H_{k+1}\right)P^-_{k+1} \tag{3.21}$$

Both parameters are denoted as the updated state or posteriori state $\hat{X}^+_{k+1}$ and updated state error covariance or posteriori state error covariance P^+_{k+1} throughout the thesis. These parameters are vital in accomplishing the first, second and third objectives. Besides, the updated state error covariance can also be expressed in terms of innovation matrix as follows:

$$P^+_{k+1} = P^-_{k+1} - K_{k+1}S_{(\mu\mu)k+1}K^T_{k+1} \tag{3.22}$$

In Kalman filter, the estimation error, process and observation noise are all uncorrelated. The results of the Kalman filter in the linear case are summarized in Table 3.1 on page 42.

3.4.1.2 Nonlinear Extended Kalman Filter

In the nonlinear form, the process and observation model of nonlinear SLAM are described as

$$X_{k+1} = f\left(X_k, u_{k+1}, w_{k+1}, k\right) \tag{3.23}$$

$$z_k = \begin{bmatrix} r_i + \upsilon_r \\ \phi_i + \upsilon_\phi \end{bmatrix} = h\left(X_{k+1}^-, v_{k+1}, k\right) \tag{3.24}$$

The detail explanations of the parameters are presented in Section 3.2. The estimation of a nonlinear system using Kalman filter is performed through a modified form of the Kalman filter, known as the extended Kalman filter (EKF). In EKF, the process and observation models are assumed to be locally linear and the respective noises w_k and v_k are small. The process model of the Eq. (3.23) is linearized as a Taylor series expansion about $\hat{X}_k$, under the conditions as defined in Assumption 3.1 on page 33.

$$X_{k+1} = f\left(\hat{X}_k, u_{k+1}, 0, k\right) + \nabla F_X \tilde{X}_k + \nabla F_w w_{k+1} + \text{higher order terms} \tag{3.25}$$

∇F_X is the Jacobian of f with respect to X_k evaluated at $\hat{X}_k$ and ∇F_w is the Jacobian with respect to w_k. These Jacobians are calculated from the mobile robot model, Eq. (3.5) and possess following equations,

$$\nabla F_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\omega T \sin\hat{\theta}_k & 1 & 0 & 0 \\ \omega T \cos\hat{\theta}_k & 0 & 1 & 0 \\ 0 & 0 & 0 & I_m \end{bmatrix} \tag{3.26}$$

$$\nabla F_w = \begin{bmatrix} \nabla f_{\gamma\omega} \\ 0_m \end{bmatrix} \tag{3.27}$$

However, assuming that: (i) the higher order terms in the Taylor series expansion are negligible since their values are too small and very close to zero (Ogata, 2010), and (ii) $\tilde{X}_k$ and w_k are small and they are zero mean random variables, this leads to the following prediction equation of the EKF

$$\begin{aligned} \hat{X}_{k+1}^- &= \mathbb{E}\left[X_{k+1} \mid Z_k \right] \\ &\approx \mathbb{E}\left[f\left(\hat{X}_k, u_{k+1}, 0, k \right) + \nabla F_X \tilde{X}_k + \nabla F_w w_{k+1} \right] \\ &= f\left(\hat{X}_k, u_{k+1}, 0, k \right) \end{aligned} \tag{3.28}$$

$$\begin{aligned} \tilde{X}_{k+1}^- &= X_{k+1} - \hat{X}_{k+1}^- \\ &\approx \nabla F_X \tilde{X}_k + \nabla F_w w_{k+1} \end{aligned} \tag{3.29}$$

$$\begin{aligned} P_{k+1}^- &= \mathbb{E}\left[\tilde{X}_{k+1}^- \left(\tilde{X}_{k+1}^- \right)^T \right] \\ &= \nabla F_X P_k \nabla F_X^T + \nabla F_w Q_{k+1} \nabla F_w^T \end{aligned} \tag{3.30}$$

where $\hat{X}_{k+1}^-$ is the predicted state of EKF while P_{k+1}^- is the respective predicted error covariance of the estimation.

In the update step, the update equations are formed by linearizing the observation model of Eq. (3.24) through Taylor series expansion about $\hat{X}_{k+1}^-$,

$$z_{k+1} = h\left(\hat{X}_{k+1}^-, 0, k \right) + \nabla H_i \tilde{X}_{k+1}^- + v_{k+1} + \text{higher order terms} \tag{3.31}$$

where ∇H_i is the Jacobian of h with respect to X_{k+1} evaluated at $\hat{X}_{k+1}^{-}$. Based on the assumption of $\hat{X}_{k+1}^{-} \approx \mathbb{E}\left[X_{k+1} \mid Z_k\right]$ and assuming that $\tilde{X}_{k+1}^{-}$ and v_{k+1} are small and zero mean random variables, in addition to the higher order terms in the Taylor series that are negligible, the updated equations of EKF lead to the following expressions,

$$\begin{aligned} \hat{z}_{k+1} &\approx \mathbb{E}\left[h\left(\hat{X}_{k+1}^{-},0,k\right)+\nabla H_i \tilde{X}_{k+1}^{-}+v_{k+1}\right] \\ &= h\left(\hat{X}_{k+1}^{-},0,k\right) \end{aligned} \tag{3.32}$$

where the innovation matrix and its respective covariances have the following terms,

$$\begin{aligned} \mu_{k+1} &= z_{k+1}-\hat{z}_{k+1} \\ &\approx \nabla H_i \tilde{X}_{k+1}^{-}+v_{k+1} \end{aligned} \tag{3.33}$$

$$\begin{aligned} S_{(\mu\mu)k+1} &= \mathbb{E}\left[\mu_{k+1}\mu_{k+1}^{T}\right] \\ &\approx \nabla H_i P_{k+1}^{-} \nabla H_i^{T}+R_{k+1} \end{aligned} \tag{3.34}$$

$$\begin{aligned} S_{(X\mu)k+1} &= \mathbb{E}\left[\tilde{X}_{k+1}^{-}\mu_{k+1}^{T}\right] \\ &\approx \mathbb{E}\left[\tilde{X}_{k+1}^{-}\left(\nabla H_i \tilde{X}_{k+1}^{-}+v_{k+1}\right)^{T}\right] \\ &= P_{k+1}^{-}\nabla H_i^{T} \end{aligned} \tag{3.35}$$

Therefore, by assuming $\tilde{X}_{k+1}^{-}$ and v_{k+1} are not correlated, the updated state $\hat{X}_{k+1}^{+}$ and updated state error covariance P_{k+1}^{+} are given by the following characteristics,

$$\hat{X}_{k+1}^{+} = \hat{X}_{k+1}^{-}+K_{k+1}\mu_{k+1} \tag{3.36}$$

with the error of the estimation defined by

$$\tilde{X}_{k+1}^{+} = X_{k+1} - \hat{X}_{k+1}^{+} \tag{3.37}$$

and therefore induce the respective state error covariance matrix

$$\begin{aligned} P_{k+1}^{+} &= \mathbb{E}\left[\tilde{X}_{k+1}^{+} \left(\tilde{X}_{k+1}^{+} \right)^{T} \right] \\ &= \left(I_n - K_{k+1} \nabla H_i \right) P_{k+1}^{-} \end{aligned} \tag{3.38}$$

with K_{k+1} is the Kalman gain and has the following definition:

$$\begin{aligned} K_{k+1} &= S_{(X\mu)k+1} \left(S_{(\mu\mu)k+1} \right)^{-1} \\ &= P_{k+1}^{-} \nabla H_i^{T} \left(\nabla H_i P_{k+1}^{-} \nabla H_i^{T} + R_{k+1} \right)^{-1} \end{aligned} \tag{3.39}$$

Linearization of the measurement model Eq. (3.24) yields a Jacobian matrix ∇H_i with respect to X_{k+1}. This Jacobian matrix is important in EKF-based SLAM and will be one of the essential variables in the thesis. ∇H_i is defined as

$$\begin{aligned} \nabla H_i &= \frac{\partial h\left(\hat{X}_{k+1}^{-} \right)}{\partial X_k} \\ &= \begin{bmatrix} 0 & -\dfrac{dx}{r} & -\dfrac{dy}{r} & \dfrac{dx}{r} & \dfrac{dy}{r} \\ -1 & \dfrac{dy}{r^2} & -\dfrac{dx}{r^2} & -\dfrac{dy}{r^2} & \dfrac{dx}{r^2} \end{bmatrix} \\ dx &= \hat{x}_i^{-} - \hat{x}_{k+1}^{r-} \\ dy &= \hat{y}_i^{-} - \hat{y}_{k+1}^{r-} \\ r &= \sqrt{dx^2 + dy^2} \end{aligned} \tag{3.40}$$

where ∇H_i indicates the changes of the range and bearing measurement as the robot position $\left(\theta_k \quad x_k^r \quad y_k^r\right)$ changes relative to the position of the detected landmark $\left(x_i \quad y_i\right)$ at the particular time. By using the Jacobians, the linear Kalman filter form could be used in the nonlinear condition. Besides, Calleja et al. (Vidal-Calleja et al., 2004a) showed that by using this linearized model, the resulting Jacobians are still capable of producing a partially observable solution of localization and mapping. Furthermore, the higher order terms in the Taylor series expansion for the linearization are negligible.

Assumption 3.1: *The predicted and updated state vectors, error covariance matrixes and Jacobians in Eq.* (3.13) – *Eq.* (3.40) *are evaluated with the assumption that the process and observation models are locally linear, in which* w_k *and* v_k *are small and uncorrelated. Both noise hold the following characteristics with* $Q_k \geq 0$ *and* $R_k > 0$.

$$\mathbb{E}\begin{bmatrix} w_k & 0 \\ 0 & v_k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{3.41}$$

$$\mathbb{E}\left[\begin{bmatrix} w_k & 0 \\ 0 & v_k \end{bmatrix}\begin{bmatrix} w_k & 0 \\ 0 & v_k \end{bmatrix}^T\right] = \begin{bmatrix} Q_k & 0 \\ 0 & R_k \end{bmatrix} \tag{3.42}$$

The equations of extended Kalman filter are summarized in the Table 3.1 on page 42.

3.4.1.3 Covariance Matrix

The covariance matrix of a state estimation in SLAM is a combination of the matrix of mobile robot and landmark position covariance matrixes and correlation between mobile robot and landmarks. Correlations between mobile robot position and landmarks estimation arise when the measurements are incorporated; thus, the covariance matrix becomes dense. The covariance matrix P_k is defined generally as

$$P_k = \begin{bmatrix} P_{rr} & P_{rm} \\ P_{rm}^T & P_{mm} \end{bmatrix} \tag{3.43}$$

P_{rr} : Covariance matrix of the robot position.

P_{mm} : Covariance matrix of the landmark position.

P_{rm} : Cross-covariance matrix of the robot and landmark position or cross-correlation between them.

The dimension of state error covariance in SLAM is $(3+2m)\times(3+2m)$, a quadratic, symmetric and positive semidefinite matrix, with m is the total detected landmarks. The size of the covariance matrix will grow as the robot continuously observed new landmarks in the environment. State error covariance in SLAM is fully represented in Eq. (3.45). From Eq. (3.14), the complete predicted state covariance may be written in the following form:

$$\begin{bmatrix} P_{rr(k+1)}^{-} & P_{rm(k+1)}^{-} \\ \left(P_{rm(k+1)}^{-}\right)^T & P_{mm(k+1)}^{-} \end{bmatrix} = \begin{bmatrix} F_{r(k)} P_{rr(k)} F_{r(k)}^T + Q_{k+1} & F_{r(k)} P_{rm(k)} \\ P_{rm(k)}^T F_{r(k)}^T & P_{mm(k)} \end{bmatrix} \tag{3.44}$$

The covariance indicates the error associated with the robot and landmark state estimations. From the covariance, the increment or decrement of uncertainties and

$$\begin{bmatrix} P_{\theta\theta} & P_{\theta x} & P_{\theta y} & P_{\theta m_{1,x}} & P_{\theta m_{1,y}} & .. & .. & P_{\theta m_{n,x}} & P_{\theta m_{n,y}} \\ P_{x\theta} & P_{xx} & P_{xy} & P_{x m_{1,x}} & P_{x m_{1,y}} & .. & .. & P_{x m_{n,x}} & P_{x m_{n,y}} \\ P_{y\theta} & P_{yx} & P_{yy} & P_{y m_{1,x}} & P_{y m_{1,y}} & .. & .. & P_{y m_{n,x}} & P_{y m_{n,y}} \\ P_{m_{1,x}\theta} & P_{m_{1,x}x} & P_{m_{1,x}y} & P_{m_{1,x}m_{1,x}} & P_{m_{1,x}m_{1,y}} & .. & .. & P_{m_{1,x}m_{n,x}} & P_{m_{1,x}m_{n,y}} \\ P_{m_{1,y}\theta} & P_{m_{1,y}x} & P_{m_{1,y}y} & P_{m_{1,y}m_{1,x}} & P_{m_{1,y}m_{1,y}} & .. & .. & P_{m_{1,y}m_{n,x}} & P_{m_{1,y}m_{n,y}} \\ : & : & : & : & : & .. & .. & : & : \\ : & : & : & : & : & .. & .. & : & : \\ P_{m_{n,x}\theta} & P_{m_{n,x}x} & P_{m_{n,x}y} & P_{m_{n,x}m_{1,x}} & P_{m_{n,x}m_{1,y}} & .. & .. & P_{m_{n,x}m_{n,x}} & P_{m_{n,x}m_{n,y}} \\ P_{m_{n,y}\theta} & P_{m_{n,y}x} & P_{m_{n,y}y} & P_{m_{n,y}m_{1,x}} & P_{m_{n,y}m_{1,y}} & .. & .. & P_{m_{n,y}m_{n,x}} & P_{m_{n,y}m_{n,y}} \end{bmatrix} \tag{3.45}$$

errors of the estimation could be observed, which represents the accuracy and consistency of the estimation. Therefore, the study on the behavior of covariance matrix is one of the important issues in designing the technique of mobile robot SLAM.

3.4.2 H_∞ Filter

The initial development of the Kalman filter was meant for the aerospace application back in the 1960s. As the Kalman filter has been employed in various applications and systems such as in common industrial application, the limitation of Kalman filter was discovered. The understanding of the statistical properties of the noise behavior in the system is the major limitation of the Kalman filter. Besides, Kalman filter may not be robust against modeling errors and noise uncertainty of certain estimation problems. Some of the limitations are (Simon, 2006):

- The mean and correlation of the process w_k and measurement noise v_k need to be known at each time instant.
- The covariances Q_k and R_k of the noises must be determined beforehand, since the Kalman filter uses these covariances as design parameters.
- The Kalman filter is the minimum variance estimator if the noise is Gaussian. However, if a different cost function (such as the worst-case estimation error) should be applied to the system, then the Kalman filter may not be suitable to accomplish the objectives.

Therefore, to compensate those limitations of Kalman filter, the H_∞ filter could be one of the alternatives, in which H_∞ filter does not make any assumptions about the noise statistical properties, and it minimizes the worst-case estimation error.

Generally, the H_∞ filter is operated as follows. A standard linear discrete-time system model is defined as

$$\begin{aligned} X_{k+1} &= F_k X_k + u_k + w_k \\ z_k &= H_k X_k + v_k \end{aligned} \tag{3.46}$$

where w_k and v_k are the noises that may be random with possibly unknown statistics or may be deterministic with non-zero mean. The goal is to estimate the linear combination of the state y_k based on the measurements up to the time $N-1$ which is given by

$$y_k = D_k X_k \tag{3.47}$$

where D_k is a user-defined matrix that is assumed to be full rank. The estimation of y_k is denoted by $\hat{y}_k$. If $D_k = I$, then the estimation is similar to the Kalman filter. In H_∞ filter, the noises are assumed to be bounded by certain noise energy and has been explained in (Lewis et al., 2008). The H_∞ filter is operated by minimizing the estimation error $\tilde{y}_k = y_k - \hat{y}_k$ for any input noises w_k and v_k, which is the energy of the estimation error normalized by the energy of the input noises. The performance is measured by the following cost function:

$$J = \frac{\sum_{k=0}^{N-1} \left\| y_k - \hat{y}_k \right\|_{Y_k}^2}{\left\| X_0 - \hat{X}_0 \right\|_{P_0^{-1}}^2 + \sum_{k=0}^{N-1} \left(\left\| w_k \right\|_{Q_k^{-1}}^2 + \left\| v_k \right\|_{R_k^{-1}}^2 \right)} \tag{3.48}$$

where $\left((X_0 - \hat{X}_0), w_k, v_k \right) \neq 0$, $\hat{X}_0$ is a priori estimate of the initial state, y_k, P_0, Q_k and R_k are symmetric positive definite matrices chosen by the user based on the specific problem, and generally $\left\| c_k \right\|_{B_k}^2 = c_k^T B_k c_k$. It should be noted that P_0, Q_k and R_k in Eq. (3.48) are not the covariances that was defined in Kalman filter. However there are analogous as in Kalman filter if those quantities are known with zero-mean and covariances. In that case, those parameters should be used in the estimation problem by using H_∞ filter.

The direct minimization of J is not tractable, therefore the above fraction of Eq. (3.48) should be bounded by $\gamma^2, \gamma \in \mathbb{R}$ for the worst-case scenario, that is, the least favorable noises w_k and v_k. This means that, H_∞ filter will try to find an estimate $\hat{y}_k$ for some minimum γ^2 that results in

$$\sup \frac{\sum_{k=0}^{N-1} \left\| y_k - \hat{y}_k \right\|_{Y_k}^2}{\left\| X_0 - \hat{X}_0 \right\|_{P_0^{-1}}^2 + \sum_{k=0}^{N-1} \left(\left\| w_k \right\|_{Q_k^{-1}}^2 + \left\| v_k \right\|_{R_k^{-1}}^2 \right)} < \gamma^2 \tag{3.49}$$

which represents the ratio of the energy by the estimation error (numerator) and the energy of the input noises (denominator) is smaller than a certain value γ^2 defined by the user. Therefore, the minimum value for γ^2 needs to be defined, however if γ^2 is too small, the solution for H_∞ filter will not exist, in which the divergence might occur and this could lead to the finite escape time problem of H_∞ filter. On the other hand, if γ^2 is too large, the behavior of the H_∞ filter will be similar to the Kalman filter. Thus, the parameter γ may be thought as a tuning parameter to control the tradeoff between H_∞ performance and minimum variance performance.

The equations for the calculation of estimated state and the covariance for the H_∞ filter are listed below. H_∞ filter equations are initially derived from the one step equation of Kalman filter, in which either the prediction or update step is ignored. For example, the predicted state of $k+1$ can be computed by integrating the predicted state of time k, without estimating the estimated state at time k, as follows:

$$\begin{aligned}
\hat{X}_k^- &= F_{k-1} X_{k-1}^+ + u_{k-1} \\
\hat{X}_k^+ &= \hat{X}_k^- + K_k \left(z_k - \hat{z}_k \right) \\
\hat{X}_{k+1}^- &= F_k \hat{X}_k^+ + u_k \\
&= F_k \left(\hat{X}_k^- + K_k \left(z_k - \hat{z}_k \right) \right) + u_k \\
&= F_k \hat{X}_k^- + F_k K_k \left(z_k - \hat{z}_k \right) + u_k
\end{aligned} \tag{3.50}$$

Then by using the same approach and applying the matrix inversion lemma for the inverse term of the Kalman gain $\left(H_{k+1} P_{k+1}^- H_{k+1}^T + R_{k+1} \right)^{-1}$ (see Eq. (3.19)), a modified covariance matrix and Kalman gain can be written as follows:

$$K_k = P_k^- \left(I + H_k^T R_k^{-1} H_k P_k^- \right)^{-1} H_k^T R_k^{-1} \tag{3.51}$$

$$P_{k+1}^- = F_k P_k^- \left(I + H_k^T R_k^{-1} H_k P_k^- \right)^{-1} F_k^T + Q_k \tag{3.52}$$

Detailed derivation of above equations and more explanation on the matrix inversion lemma can be obtained from (Simon, 2006).

Therefore by integrating the new cost function and the new parameter γ^2 to ensure the best performance of H_∞ filter as discussed earlier, the estimation equations for H_∞ filter are summarized as follows:

$$\begin{aligned} \hat{X}_{k+1} &= F_k \hat{X}_k + F_k K_k \left(z_k - H_k \hat{X}_k \right) + u_k \\ K_k &= P_k \left(I - \gamma^{-2} P_k + H_k^T R_k^{-1} H_k P_k \right)^{-1} H_k^T R_k^{-1} \\ P_{k+1} &= F_k P_k \left(I - \gamma^{-2} P_k + H_k^T R_k^{-1} H_k P_k \right)^{-1} F_k^T + Q_k \end{aligned} \tag{3.53}$$

Moreover, the following condition must hold at each time step k in order for the H_∞ filter to consistently hold a positive semidefinite (PSD) solution and provide a better estimation than Kalman filter (Ahmad and Namerikawa, 2010b, 2011c):

$$P_k^{-1} - \gamma^{-2} + H_k^T R_k^{-1} H_k > 0 \tag{3.54}$$

In comparison to the Kalman filter, H_∞ filter is a worst-case estimator in the sense that it assumes that noises w_k and v_k, and X_0 will be chosen by the nature to maximize the cost function. On the other hand, these parameters need to be defined by the user beforehand to be implemented in Kalman filter. Therefore, H_∞ filter is considered more robust with regards to the design and it can normally be addressed as a robust version of the Kalman filter. The main concept and estimation equations explained in this section are extracted from the following literatures (Shen and Deng, 1997; Einicke and White, 1999; Simon, 2006; Lewis et al., 2008, and Ahmad and

Namerikawa, 2011c). A summary of the non-linear H_∞ filter equations is tabulated in Table 3.1 on page 42.

3.4.2.1 Finite Escape Time Phenomenon

One of the major issues in implementing H_∞ filter is to solve the estimation problem that lies in the selection of γ value defined by the user. This issue has been addressed as a proper tuning in the H_∞ filter. The main goal is to choose the minimum value of γ to minimize the cost function of Eq. (3.49), however the smaller the γ value, the more sensitive the H_∞ filter to the effect of the measurement noise; this might cause an instability (Ahmad and Othman, 2014). Nonetheless, if the chosen γ value is too large, the H_∞ filter behaves more likely as the Kalman filter. Therefore there exists a certain boundary for the selection of γ based on the specific system conditions to achieve the optimal performance of H_∞ filter.

Furthermore, one of the effects on the improper tuning of γ is a finite escape time problem. This effect can be seen from the covariance value, in which the filter become suddenly diverged for a short time, and continue to converge after that particular time. Finite escape time problem may increase the estimation error. This phenomenon is illustrated in Figure 3.7.

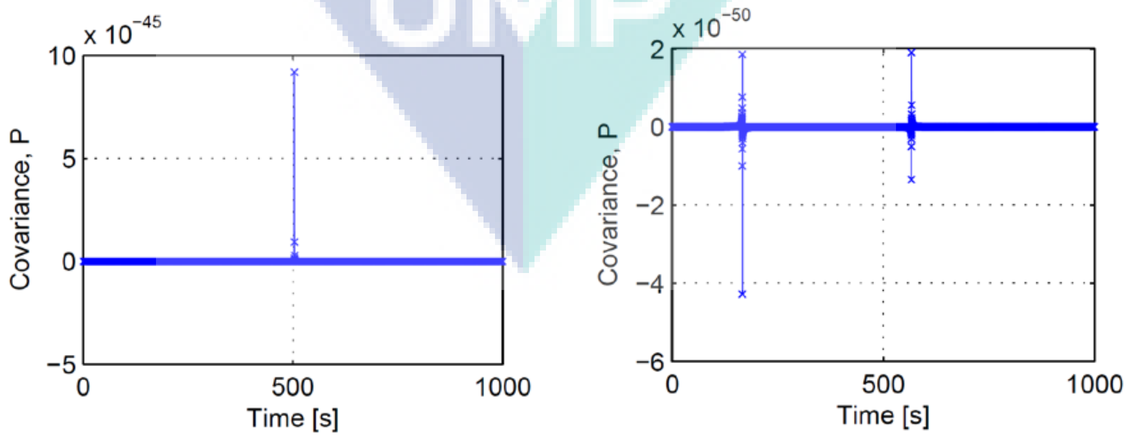

Figure 3.7. Example of finite escape time phenomenon in H_∞ filter.

Source: Ahmad and Namerikawa (2009a)

3.5 Research Process

The whole process of the research is explained through a flow chart as depicted in Figure 3.8 on page 41. The flow chart describes the general research flow of all objectives. The in-depth methodology of each research objective is elaborated in section "Scope of Analysis" in each dedicated chapter: Chapter 4.3, 5.3, 6.3 and 7.3.

The initial phase of the work involves a background study on the control theory, estimation theory, and simultaneous localization of mapping of mobile robot in addition to the review of related works. The analysis begins with the definition of the environmental conditions for the mobile robot and the whole SLAM process, and assumptions based on each research problem and the objectives. The defined conditions and assumptions represent the actual conditions of the mobile robot. Then, several case studies were defined to organize the analyses according to the independent variables involved or the situations of mobile robot. This was done to facilitate the observation of the significant effect of each independent variables and specific conditions on the dependent variables in each case study.

The analyses were conducted based on the control theory by means of the mathematical approach to provide the solution for the problems. This approach is chosen since the field areas of the thesis are system analysis, control theory, and estimation theory, which are based on mathematical formulation. The results of the analyses were presented in terms of propositions, lemmas or theorems. Finally, the findings were validated through simulation analysis using MATLAB and Simulink.

3.6 Summary

In this chapter, the model of the mobile robot, landmarks and estimators used in the thesis are presented. The equations governing the parameters are derived and explained. The estimators of interest, the Kalman and H_∞ filters are discussed. The Kalman filter initially was developed to be used in a linear system. Through a linearization process using Taylor series expansion, the Kalman filter is able to be applied in a nonlinear system. The H_∞ filter, on the other hand can be applied in both conditions, linear and nonlinear system.

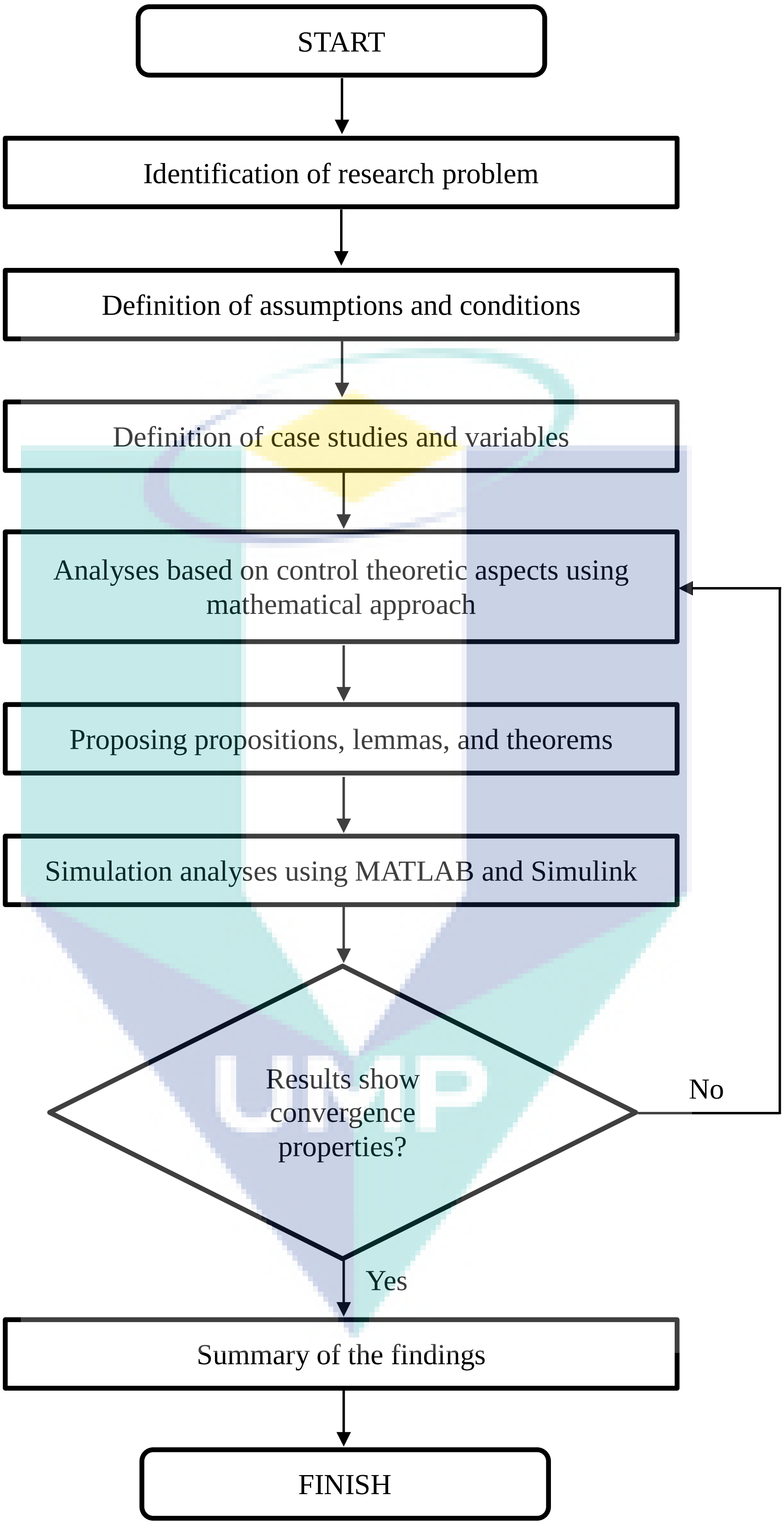

Figure 3.8. Flow chart of the research process.

Table 3.1
The equations of Kalman filter, extended Kalman filter, and H_∞ filter.

	The Kalman Filter	**The Extended Kalman Filter**	**The H_∞ Filter**
The System	$X_{k+1} = F_k X_k + G_k u_k + w_k$	$X_{k+1} = f\left(X_k, u_k, w_k, k\right)$	$X_{k+1} = f\left(X_k, u_k, w_k, k\right)$
	$z_k = H_k X_k + v_k$	$z_k = h\left(X_k, v_k, k\right)$	$z_k = h\left(X_k, v_k, k\right)$
			$y_k = D_k X_k$
The Prediction Step	$\hat{X}^-_{k+1} = F_k \hat{X}_k + G_k u_k$	$\hat{X}^-_{k+1} = f\left(\hat{X}_k, u_k, 0, k\right)$	
	$P^-_{k+1} = F_k P_k F_k^T + Q_k$	$P^-_{k+1} = \nabla F_X P_k \nabla F_X^T + \nabla F_w Q_k \nabla F_w^T$	$\hat{X}_{k+1} = \nabla F_k \hat{X}_k + \nabla F_k K_k \left(z_k - \nabla H_k \hat{X}_k\right) + u_k$
The Update Step	$\hat{X}^+_{k+1} = \hat{X}^-_{k+1} + K_{k+1}\ \mu_{k+1}$	$\hat{X}^+_{k+1} = \hat{X}^-_{k+1} + K_{k+1}\ \mu_{k+1}$	$P_{k+1} = \nabla F_k P_k \left(I - \gamma^{-2} P_k + \nabla H_k^T R_k^{-1} \nabla H_k P_k\right)^{-1} \nabla F_k^T + Q_k$
	$P^+_{k+1} = \left(I_n - K_{k+1} H_{k+1}\right) P^-_{k+1}$	$P^+_{k+1} = \left(I_n - K_{k+1} \nabla H_i\right) P^-_{k+1}$	
where	$\mu_{k+1} = z_{k+1} - H_{k+1} \hat{X}^-_{k+1}$	$\mu_{k+1} = z_{k+1} - h\left(\hat{X}^-_{k+1}, 0, k\right)$	$K_k = P_k \left(I - \gamma^{-2} P_k + \nabla H_k^T R_k^{-1} \nabla H_k P_k\right)^{-1} \nabla H_k^T R_k^{-1}$
	$S_{(\mu\mu)k+1} = H_{k+1} P^-_{k+1} H^T_{k+1} + R_{k+1}$	$S_{(\mu\mu)k+1} = \nabla H_i P^-_{k+1} \nabla H_i^T + R_{k+1}$	
	$K_{k+1} = P^-_{k+1} H^T_{k+1} \left(S_{(\mu\mu)k+1}\right)^{-1}$	$K_{k+1} = P^-_{k+1} \nabla H_i^T \left(S_{(\mu\mu)k+1}\right)^{-1}$	

CHAPTER 4

CROSS-CORRELATION OF STATE COVARIANCE

This chapter details the theoretical investigation of the importance of cross-correlation in mobile robot simultaneous localization and mapping. Two different case studies were conducted to examine the effects of cross-correlation terms on the estimation. The first case is a situation where a mobile robot moves and calculates its position relative to a landmark, whereas in the second case, the mobile robot is independent of the landmark position. The preliminary results obtained indicate that the updated state covariance of the latter case could decrease or increase compared to that of the former case. Similar conditions were then simulated to validate the results. A good agreement was reached between the theoretical and simulation results. Parts of this chapter were published in (Ahmad et al., 2013) and (Ahmad and Othman, 2015). This chapter concludes the first objective of this thesis.

4.1 Introduction

One of the main issues of the extended Kalman filter in mobile robot SLAM is the high computational cost due to the updating process of the state covariance matrix. To reduce the cost, researchers attempt to find the suitable technique to simplify the structure of the state covariance matrix by means of diagonalization process, i.e. to simplify the multiplication of the covariance matrix with other parameters. However, the cross-correlation elements in the covariance structure cannot be simply eliminated because they represent the correlations between each parameter in the state vector. Therefore, this study attempts to theoretically prove the importance of cross-correlation elements, thus they should not be neglected in the calculations. The structure of the state covariance matrix was generally explained earlier in Subsection 3.4.1.3 and will be elaborated in the following chapter, Section 5.4.1.

4.2 Related Work

One of the proposed technique to diagonalize the state covariance was by means of the decorrelation approach; this is known as the covariance inflation method (Guivant and Nebot, 2003). The algorithm will decorrelate a subset of the states that is weakly correlated and eliminate the weak cross-correlation terms in the state covariance. However, the approach might lead to filter instability (Julier, 2003 and Andrade-Cetto et al., 2005). The cross-correlation in the state covariance defines the relationship between the mobile robot and the observed objects or landmarks. The cross-correlation is essential in assuring better performance (Smith et al., 1990; Hébert et al., 1996; Castellanos et al., 1997 and Thrun et al., 2005) as the mobile robot relies and depends on these landmarks to determine its location. Thus, it should not be neglected in the calculation.

The importance of the correlation has been proven through simulation and experimental results (Smith et al., 1990; Hébert et al., 1996; Castellanos et al., 1997 and Thrun et al., 2005), but none of these studies have theoretically proven the findings. The researchers have claimed that if the mobile robot was configured to be independent from landmarks, the uncertainties become smaller, so does the state covariance and the estimation is considered to be optimistic. The findings have been presented through representation of the covariance ellipse. To date, no theoretical proof has been proposed. Therefore, this study is conducted to prove that the state covariance is smaller if the correlation between a mobile robot and the landmarks is ignored by means of theoretical analysis of state covariance behavior.

Moreover, Hébert et al. conducted the study with the assumption that the system behaves linearly to eliminate the non-linearity effect (Hébert et al., 1996). Since the mobile robot SLAM is a nonlinear problem, this study includes the nonlinear behavior into consideration to investigate the real effect of the importance of cross-correlation on the estimation. Further, it has been found that the direction of the robot's movement may influence the behavior of state covariance through the simulation analysis.

In addition, previous studies were conducted based on the assumption that both the robot and the landmarks possess positive coordinates. In reality, this might not always be the case. Hence, the condition in which the mobile robot and the positions of the landmarks are located in the negative side of the global coordinate frame is investigated.

4.3 Scope of Analysis

The discussion was divided into two case studies: a mobile robot that moves while simultaneously measuring its relative *x*–*y* positions to a landmark, and a mobile robot that moves and refers only to its initial position. The latter was proposed to define the situation that eliminates the correlation between the mobile robot and the landmark. Moreover, the study was extended by simulating the movement of the mobile robot with a change in direction in both conditions to investigate the effect of moving direction on the results.

The remainder of this chapter is organized as follows. Section 4.4 presents the analysis of cross-correlation in the state covariance matrix based on the proposed case studies. The discussion of the analysis is elaborated in Section 4.5, whereas Section 4.6 presents the validation against simulation analysis. Finally, Section 4.7 concludes the chapter. The models, equations and general conditions used in this chapter are referred to the theoretical formulations presented in Chapter 3.

4.4 Impact of Cross-correlation

This section is divided into two subsections to examine the statistical behavior of the state error covariance matrix, especially on its cross-correlation elements. The first subsection identifies the behavior of the covariance matrix when the mobile robot locates itself by referring to a landmark. The first contribution of this study is highlighted in this section, in which the analysis is conducted with the assumption that mobile robot and landmark are located on the negative side of the global coordinate. The second subsection determines the condition where the mobile robot depends only on its initial position for estimation purposes, which indicates no dependencies on the landmark for localization and mapping process; thus the effect of the cross-correlation between its position and landmarks can be neglected. The results of these two conditions are discussed and explained in the next section. This serves as the second contribution of the study.

Proposition 4.1. *Let the initial state covariance matrix be* $P_0 = I_n$*, where* I_n *is an identity matrix with n-dimension. Under this condition, the estimation of the state is significantly dependent on the measurement matrix* ∇H_i *and the measurement noise covariance* R_{k+1}.

Proof. By assuming the initial state covariance matrix $P_0 = I_n$ when the mobile robot starts to observe its environment at $k=1$ sampling time, from Eq. (3.38) and Eq. (3.39), the Kalman gain K_{k+1} and the state covariance matrix P_{k+1}^{+} can be defined as follows:

$$\begin{aligned} K_{k+1} &= P_{k+1}^{-}\nabla H_i^T \left(\nabla H_i P_{k+1}^{-} \nabla H_i^T + R_{k+1}\right)^{-1} \\ &= P_0 \nabla H_i^T \left(\nabla H_i P_0 \nabla H_i^T + R_{k+1}\right)^{-1} \\ &= \nabla H_i^T \left(\nabla H_i \nabla H_i^T + R_{k+1}\right)^{-1} \end{aligned} \tag{4.1}$$

$$\begin{aligned} P_{k+1}^{+} &= P_{k+1}^{-} - K_{k+1}\nabla H_i P_{k+1}^{-} \\ &= P_0\left(I_n - K_{k+1}\nabla H_i\right) \\ &= I_n - K_{k+1}\nabla H_i \\ &= I_n - \nabla H_i^T \left(\nabla H_i \nabla H_i^T + R_{k+1}\right)^{-1} \nabla H_i \end{aligned} \tag{4.2}$$

Equation (4.1) and Eq. (4.2) show that the estimation of the state is now strongly dependent on the measurement matrix ∇H_i and the measurement noise covariance R_{k+1}. However, R_{k+1} is normally known in extended Kalman filter, especially when the designer has prior knowledge of the sensors efficiency and environment conditions. Besides, R_{k+1} does not correlate to other noises or states as mentioned in Assumption 3.1 on page 33. The only remaining element to be analyzed is the measurement matrix ∇H_i. Thus, it is expected that the measurement matrix plays an important role in describing the cross-correlation behavior in the state covariance matrix, and therefore will be treated as an independent variable throughout the study. □

The following definition in describing the behavior of measurement matrix ∇H_i is continuously used in this chapter. This characteristic is a general assumption in the extended Kalman filter state estimation of mobile robot navigation (Huang and Dissanayake, 2007). It should be noted that simpler notation for mobile robot position is used.

Definition 4.2. *The Jacobian for the measurement matrix when the robot observes only one new landmark in its surroundings at point A and makes n observations is written as*

$$\nabla H_A = \begin{bmatrix} 0 & -\frac{dx_A}{r_A} & -\frac{dy_A}{r_A} & \frac{dx_A}{r_A} & \frac{dy_A}{r_A} \\ -1 & \frac{dy_A}{r_A{}^2} & -\frac{dx_A}{r_A{}^2} & -\frac{dy_A}{r_A{}^2} & \frac{dx_A}{r_A{}^2} \end{bmatrix} = \begin{bmatrix} -e & -A & A \end{bmatrix} \tag{4.3}$$

where

$$e = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad A = \begin{bmatrix} \frac{dx_A}{r_A} & \frac{dy_A}{r_A} \\ -\frac{dy_A}{r_A{}^2} & \frac{dx_A}{r_A{}^2} \end{bmatrix} \tag{4.4}$$

evaluated at the true positions of the landmark (x_i, y_i) *and the true position of the robot* (x_A, y_A), *and its elements are defined by*

$$\begin{aligned} dx_A &= x_i - x_A \\ dy_A &= y_i - y_A \\ r_A &= \sqrt{dx_A{}^2 + dy_A{}^2} \end{aligned} \tag{4.5}$$

This definition will be used to investigate the effect of cross-correlation elements on the estimation accuracy based on the conditions specified in Proposition 4.1 on page 46.

4.4.1 Dependency of Landmark Existence

The following definition and assumption are proposed to describe the conditions used in this study. The assumption that the mobile robot does not know its prior position is the main motivation of the study. Therefore, to investigate the impact of the mobile robot position on the Jacobian and covariance matrix, Definition 4.3 is proposed:

Definition 4.3. *The initial position of the mobile robot is configured at* $(x_o, y_o) = (0, 0)$ *with respect to the global coordinate system, the region of the robot's movement in the environment can be generally divided into four quadrants or areas:*

(i) x and y coordinates are both positive,

(ii) x and y coordinates are both negative,

(iii) x coordinate is positive, y coordinate is negative,

(iv) x coordinate is negative, y coordinate is positive.

Based on the stated areas, this study is conducted with the following assumption.

Assumption 4.4. *Consider the mobile robot moves consistently and linearly in only one direction along the x–axis, starting from its initial position towards the location of the landmark in the second area defined in Definition 4.3. The position of the landmark is* $(-x_z, -y_z)$ *and that of the robot is* $(-x_A, -y_A)$*. Furthermore, the landmark's x–position is located behind the mobile robot, such that* $x_z > x_A$.

If Assumption 4.4 is applied and as long as Eq. (3.7) and Definition 4.2 are referred, the Jacobian for the measurement matrix under these conditions is described as

$$\nabla H_z = \begin{bmatrix} 0 & \frac{dx_z}{r_z} & \frac{dy_z}{r_z} & -\frac{dx_z}{r_z} & -\frac{dy_z}{r_z} \\ -1 & -\frac{dy_z}{r_z^2} & \frac{dx_z}{r_z^2} & \frac{dy_z}{r_z^2} & -\frac{dx_z}{r_z^2} \end{bmatrix} \tag{4.6}$$

$$= \begin{bmatrix} -e & A_z & -A_z \end{bmatrix},$$

where

$$e = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad A_z = \begin{bmatrix} \frac{dx_z}{r_z} & \frac{dy_z}{r_z} \\ -\frac{dy_z}{r_z^2} & \frac{dx_z}{r_z^2} \end{bmatrix} \tag{4.7}$$

$$\begin{aligned} dx_z &= -x_z - (-x_A) = -x_z + x_A = -(x_z - x_A) \\ dy_z &= -y_z - (-y_A) = -y_z + y_A = -(y_z - y_A) \\ r_z &= \sqrt{dx_z^{\,2} + dy_z^{\,2}} \end{aligned} \tag{4.8}$$

This equation is in fact correlated with Definition 4.2, and Eq. (4.3) is obtained if the conditions are the opposite of the abovementioned case. The state covariance of this case has been investigated before (Huang and Dissanayake, 2007) but the study was based on the assumption that both the positions of the robot and the landmark are positive; most of the previous studies were based on the same assumption. However, since the mobile robot is unable to locate itself beforehand, other possibilities of robot and landmark positions as defined in Definition 4.3 should be considered. This study was conducted to investigate the influence of this condition on the state estimation, particularly on the measurement matrix, which probably will produce a different behavior of the estimation. Thus, the Jacobian of the measurement matrix under the conditions defined in Assumption 4.4 is proposed, which produces the following theorem.

Theorem 4.5. *If the movement of the mobile robot and the conditions of the environment are similar as defined in Assumption 4.4 and by referring to Proposition 4.1, the state covariance matrix of the estimation is given by*

$$P_{zk+1}^{+} = \begin{bmatrix} I - ec_z e & eA_z c_z & -eA_z c_z \\ eA_z c_z & I - A_z c_z A_z & A_z c_z A_z \\ -eA_z c_z & A_z c_z A_z & I - A_z c_z A_z \end{bmatrix} \tag{4.9}$$

where

$$c_z = \left[e^2 + 2A_z{}^2 + R_{k+1} \right]^{-1} \tag{4.10}$$

Proof. The Kalman gain and state covariance matrix for the aforementioned conditions are calculated through Eq. (4.1) and Eq. (4.2) with the Jacobian defined in Eq. (4.6). As the initial state covariance is defined as an identity matrix, the Kalman gain for the state covariance update is

$$\begin{aligned} K_{zk+1} &= \nabla H_z^T \left(\nabla H_z \nabla H_z^T + R_{k+1} \right)^{-1} \\ &= \begin{bmatrix} -e \\ A_z \\ -A_z \end{bmatrix} \left(\begin{bmatrix} -e & A_z & -A_z \end{bmatrix} \begin{bmatrix} -e \\ A_z \\ -A_z \end{bmatrix} + R_{k+1} \right)^{-1} \\ &= \begin{bmatrix} -e \\ A_z \\ -A_z \end{bmatrix} \left[e^2 + 2A_z{}^2 + R_{k+1} \right]^{-1} \\ &= \begin{bmatrix} -e \left[e^2 + 2A_z{}^2 + R_{k+1} \right]^{-1} \\ A_z \left[e^2 + 2A_z{}^2 + R_{k+1} \right]^{-1} \\ -A_z \left[e^2 + 2A_z{}^2 + R_{k+1} \right]^{-1} \end{bmatrix} \end{aligned} \tag{4.11}$$

Since the initial covariance matrix is defined as an identity matrix and the Kalman gain is defined as in Eq. (4.11), the state covariance matrix can be written as

$$
\begin{aligned}
P_{zk+1}^{+} &= I_n - K_{k+1}\nabla H_z \\
&= I_n - \begin{bmatrix} -e\,\left[e^2 + 2A_z^{\;2} + R_{k+1} \right]^{-1} \\ A_z\left[e^2 + 2A_z^{\;2} + R_{k+1} \right]^{-1} \\ -A_z\left[e^2 + 2A_z^{\;2} + R_{k+1} \right]^{-1} \end{bmatrix} \begin{bmatrix} -e \\ A_z \\ -A_z \end{bmatrix}^T \\
&= I_n - \begin{bmatrix} ec_z e & -ec_z A_z & ec_z A_z \\ -A_z c_z e & A_z c_z A_z & -A_z c_z A_z \\ A_z c_z e & -A_z c_z A_z & A_z c_z A_z \end{bmatrix} \\
&= \begin{bmatrix} I - ec_z e & ec_z A_z & -ec_z A_z \\ A_z c_z e & I - A_z c_z A_z & A_z c_z A_z \\ -A_z c_z e & A_z c_z A_z & I - A_z c_z A_z \end{bmatrix}
\end{aligned}
\tag{4.12}
$$

□

4.4.2 Estimation Based on Initial Position of Mobile Robot

To consistently evaluate the cross-correlation characteristics throughout the estimation, the previous assumptions are referred. Consider a mobile robot that operates in an environment that has non-identifiable landmarks to be observed. Under this condition, the mobile robot only has the information about its initial position and the following assumption is proposed.

Assumption 4.6. *Assume that the mobile robot moves from its initial position to the second quadrant defined in Definition 4.3. The position of the robot becomes* $\left(-x_A, -y_A\right)$. *With this assumption, and with respect to Definition 4.2, the Jacobian for the measurement matrix possesses the following parameters:*

$$
\nabla H_v = \begin{bmatrix} 0 & -\dfrac{dx_v}{r_v} & -\dfrac{dy_v}{r_v} & \dfrac{dx_v}{r_v} & \dfrac{dy_v}{r_v} \\ -1 & \dfrac{dy_v}{r_v^{\;2}} & -\dfrac{dx_v}{r_v^{\;2}} & -\dfrac{dy_v}{r_v^{\;2}} & \dfrac{dx_v}{r_v^{\;2}} \end{bmatrix} \tag{4.13}
$$

$$
= \left[-e \quad -A_v \quad A_v \right]
$$

where

$$e = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad A_v = \begin{bmatrix} \frac{dx_v}{r_v} & \frac{dy_v}{r_v} \\ -\frac{dy_v}{r_v^{\;2}} & \frac{dx_v}{r_v^{\;2}} \end{bmatrix} \tag{4.14}$$

$$\begin{aligned} dx_v &= 0-(-x_A) = x_A \\ dy_v &= 0-(-y_A) = y_A \\ r_v &= \sqrt{dx_v^{\;2} + dy_v^{\;2}} = \sqrt{x_A^{\;2} + y_A^{\;2}} \end{aligned} \tag{4.15}$$

Hence, Assumption 4.6 leads to the following theorem.

Theorem 4.7. *If there is no observable landmark in the environment and the mobile robot refers only to its initial position for localization, the state covariance matrix becomes*

$$P_{vk+1}^{+} = \begin{bmatrix} I - ec_v e & -eA_v c_v & eA_v c_v \\ -eA_v c_v & I - A_v c_v A_v & A_v c_v A_v \\ eA_v c_v & A_v c_v A_v & I - A_v c_v A_v \end{bmatrix} \tag{4.16}$$

with

$$c_v = \left[e^2 + 2A_v^{\;2} + R_{k+1} \right]^{-1} \tag{4.17}$$

Proof. Similar to the previous subsection, the Kalman gain is computed using Eq. (4.1) with the Jacobian defined in Eq. (4.13). Therefore the Kalman gain for this condition is

$$K_{vk+1} = \nabla H_v^T \left(\nabla H_v \nabla H_v^T + R_{k+1} \right)^{-1}$$

$$= \begin{bmatrix} -e \\ -A_v \\ A_v \end{bmatrix} \left(\begin{bmatrix} -e & -A_v & A_v \end{bmatrix} \begin{bmatrix} -e \\ -A_v \\ A_v \end{bmatrix} + R_{k+1} \right)^{-1}$$

$$= \begin{bmatrix} -e \\ -A_v \\ A_v \end{bmatrix} \left[e^2 + 2A_v^{\ 2} + R_{k+1} \right]^{-1} \tag{4.18}$$

$$= \begin{bmatrix} -e \left[e^2 + 2A_v^{\ 2} + R_{k+1} \right]^{-1} \\ -A_v \left[e^2 + 2A_v^{\ 2} + R_{k+1} \right]^{-1} \\ A_v \left[e^2 + 2A_v^{\ 2} + R_{k+1} \right]^{-1} \end{bmatrix}$$

Hence, by using Eq. (4.18), the state covariance matrix is updated as follows:

$$P_{vk+1}^{+} = I_n - K_{k+1} \nabla H_v$$

$$= I_n - \begin{bmatrix} -e \left[e^2 + 2A_v^{\ 2} + R_{k+1} \right]^{-1} \\ -A_v \left[e^2 + 2A_v^{\ 2} + R_{k+1} \right]^{-1} \\ A_v \left[e^2 + 2A_v^{\ 2} + R_{k+1} \right]^{-1} \end{bmatrix} \begin{bmatrix} -e \\ -A_v \\ A_v \end{bmatrix}^T$$

$$= I_n - \begin{bmatrix} ec_v e & ec_v A_v & -ec_v A_v \\ A_v c_v e & A_v c_v A_v & -A_v c_v A_v \\ -A_v c_v e & -A_v c_v A_v & A_v c_v A_v \end{bmatrix} \tag{4.19}$$

$$= \begin{bmatrix} I - ec_v e & -ec_v A_v & ec_v A_v \\ -A_v c_v e & I - A_v c_v A_v & A_v c_v A_v \\ A_v c_v e & A_v c_v A_v & I - A_v c_v A_v \end{bmatrix}$$

□

Both results obtained from this and previous subsections are evaluated in the following section.

4.5 Discussions

The state covariance matrices for the mobile robot localization and mapping process based on the existence and absence of the landmarks are derived in the previous section (Eq. (4.9) and Eq. (4.16), respectively). In this section, both conditions are compared and analyzed to examine the differences between the two cases, and to understand the impact of both conditions on the state estimation.

In the structure of state covariance matrix, diagonal elements are important since it initiates the covariance of the pose of the robot and the position of the landmark as shown in Eq. (3.45). The accuracy analysis of the state estimation is normally focused on the diagonal elements. The smaller the values of the elements, the uncertainties of the estimation are lower, which indicate a better state estimation. The diagonal elements of Eq. (4.9) and Eq. (4.16) show similar form but the difference can be observed when the particular elements of matrix A_z and A_v are analyzed.

Lemma 4.8. *If the mobile robot continues to move in the negative direction of x–axis with respect to Assumption 4.6, the Jacobian for the measurement matrix* ∇H_v *will increase since the relative distance of the mobile robot increases from its initial position, and the characteristics of each element in* A_v *also increases.*

Proof. If the mobile robot moves in the negative direction of *x*–axis, i.e. it moves further from its initial position in the second quadrant as defined in Definition 4.3, $x_{A_k} < x_{A_{k+1}} < x_{A_{k+2}} < \cdots < x_{A_{k+n}}$, based on Eq. (4.15), the elements in matrix A_v increases as dx_v and dy_v increased. □

Moreover, even though the diagonal elements in Eq. (4.9) and Eq. (4.16) are almost similar, matrix A_v has a larger value than that of matrix A_z if Lemma 4.8 is referred. This is proven through the following theorem with the conditions as described in Assumption 4.4 and Assumption 4.6.

Theorem 4.9. *The state covariance matrix of the mobile robot, when the position of the robot is calculated from its initial position* P^{+}_{vk+1}*, is smaller than that of the position measured with respect to the landmark* P^{+}_{zk+1}*. This proves that the estimation is too optimistic.*

Proof. For a Jacobian matrix of a mobile robot observing one landmark in the environment, matrices A_z and A_v have a dimension of 2×2. Thus, with reference to Eq. (4.7), Eq. (4.8), Eq. (4.14), and Eq. (4.15), the magnitude of A_z and A_v are calculated as follows:

$$
\begin{aligned}
\det(A_z) &= \left(\left(\frac{dx_z}{r_z}\right)\left(\frac{dx_z}{r_z^{\,2}}\right)\right)-\left(\left(-\frac{dy_z}{r_z^{\,2}}\right)\left(\frac{dy_z}{r_z}\right)\right)\\
&= \left(\left(\frac{-x_z+x_A}{r_z}\right)\left(\frac{-x_z+x_A}{r_z^{\,2}}\right)\right)-\left(\left(\frac{y_z-y_A}{r_z^{\,2}}\right)\left(\frac{-y_z+y_A}{r_z}\right)\right)\\
&= \frac{x_A^{\,2}+y_A^{\,2}+x_z^{\,2}+y_z^{\,2}-2x_zx_A-2y_zy_A}{r_z^{\,3}} \qquad (4.20)\\
&= \frac{x_A^{\,2}+y_A^{\,2}+x_z^{\,2}+y_z^{\,2}-2x_zx_A-2y_zy_A}{\left(x_A^{\,2}+y_A^{\,2}+x_z^{\,2}+y_z^{\,2}-2x_zx_A-2y_zy_A\right)^{3/2}}\\
&= \frac{1}{\left(x_A^{\,2}+y_A^{\,2}+x_z^{\,2}+y_z^{\,2}-2x_zx_A-2y_zy_A\right)^{1/2}}
\end{aligned}
$$

with

$$
\begin{aligned}
r_z &= \left(dx_z^{\,2}+dy_z^{\,2}\right)^{1/2}\\
&= \left(\left(-x_z+x_A\right)^2+\left(-y_z+y_A\right)^2\right)^{1/2} \qquad (4.21)\\
&= \left(x_A^{\,2}+y_A^{\,2}+x_z^{\,2}+y_z^{\,2}-2x_zx_A-2y_zy_A\right)^{1/2}
\end{aligned}
$$

For the sake of brevity, let $B_z = x_A{}^2 + y_A{}^2 + x_z{}^2 + y_z{}^2 - 2x_z x_A - 2y_z y_A$, therefore

$$\det(A_z) = \frac{1}{\sqrt{B_z}} \tag{4.22}$$

whereas

$$\begin{aligned} \det(A_v) &= \left(\left(\frac{dx_v}{r_v}\right)\left(\frac{dx_v}{r_v{}^2}\right)\right) - \left(\left(-\frac{dy_v}{r_v{}^2}\right)\left(\frac{dy_v}{r_v}\right)\right) \\ &= \left(\left(\frac{x_A}{r_v}\right)\left(\frac{x_A}{r_v{}^2}\right)\right) - \left(\left(-\frac{y_A}{r_v{}^2}\right)\left(\frac{y_A}{r_v}\right)\right) \\ &= \frac{x_A{}^2 + y_A{}^2}{r_v{}^3} \\ &= \frac{x_A{}^2 + y_A{}^2}{\left(x_A{}^2 + y_A{}^2\right)^{3/2}} \\ &= \frac{1}{\left(x_A{}^2 + y_A{}^2\right)^{1/2}} \end{aligned} \tag{4.23}$$

with

$$\begin{aligned} r_v &= \left(dx_v{}^2 + dy_v{}^2\right)^{1/2} \\ &= \left(x_A{}^2 + y_A{}^2\right)^{1/2} \end{aligned} \tag{4.24}$$

Similar as Eq. (4.22), let $B_v = x_A{}^2 + y_A{}^2$, therefore

$$\det(A_v) = \frac{1}{\sqrt{B_v}} \tag{4.25}$$

It is clearly shown that $B_v < B_z$ for any x_A , $x_i \in \mathbb{R}$ and y_A , $y_i \in \mathbb{R}$; therefore from Eq. (4.20) and Eq. (4.23), $|A_v| > |A_z|$. Hence, referring to the diagonal elements of Eq. (4.9) and Eq. (4.16), the results induce the state covariance matrices $P^+_{vk+1} < P^+_{zk+1}$. This is proven using trace matrix of both covariances that indicate $tr\left(P^+_{vk+1}\right) < tr\left(P^+_{zk+1}\right)$ since $\left(I - A_v c_v A_v\right) < \left(I - A_z c_z A_z\right)$. □

Theorem 4.9 indicates that the state covariance matrix, when the position of the mobile robot is measured from its initial position, is smaller than that of the position relative to the landmark, which contradicts to the preliminary results. The existence of the landmarks and cross-correlation are important in ensuring a better estimation, therefore the state covariance for this case should be smaller. However, this scenario has indirectly indicated that the state estimation becomes too optimistic and could lead to erroneous results in estimating the position. Optimistic estimation, defined as the estimation with smaller estimated uncertainty than the true uncertainty, may cause inconsistency in the estimation. The results are true and have been proven through simulation in the next section. These findings are in agreement with that of other studies (Smith et al., 1990; Hébert et al., 1996; Castellanos et al., 1997 and Thrun et al., 2005), which have shown the importance of correlation, optimistic behavior of its ignorance, and the biased estimation. Moreover, this fact has been mathematically proven through the proposed theorems in this thesis.

Based on the calculation derived from the observation model in Definition 4.2 and how the mobile robot depends on the landmarks, the updated state covariance may have two distinctive results. The state covariance could become larger or smaller depending on the region where the mobile robot is moving, as explained in Definition 4.3. Some results of this scenario will be reviewed through simulation in the next section, which proves that the direction of the mobile robot does influence the behavior of state covariance matrix.

Table 4.1
Control parameters for the simulation.

Parameter	Symbol	Value
Sampling time (s)	T	0.1
Process noise covariance	Q_k	1×10^{-6}
Measurement noise covariance	R_k	0.01
Robot initial state covariance	P_{0rr}	$10\times I_3$
Landmark initial state covariance	P_{0mm}	$1000\times I_2$
Robot initial position	(x_A, y_A)	$(0,0)$
Landmark position	(x_z, y_z)	$(-2,6)$

4.6 Simulation Results

The results previously obtained were analyzed through simulation using control parameters as listed in Table 4.1. The parameters were adopted from the published experimental works (Ahmad and Namerikawa, 2011a, 2013). In the simulation, the mobile robot moved at a constant speed. Two cases were examined in order to investigate the consistency of the proposed analysis as discussed in the previous sections. In the first case, the mobile robot refers to only one landmark for localization and mapping purposes; in the second case, the mobile robot refers only to its initial position in estimating its movement within the environment. In both cases, the environment is assumed to be planar and there were no moving objects during mobile robot observations. Moreover, data association is assumed available at all time.

Figure 4.1 depicts the results obtained for the mobile robot SLAM in both cases, in which the green curve indicates the first case, while the red curve represents the second case. When the mobile robot is not referring to any landmark, the estimation becomes inaccurate and consequently is totally deviated from the true path as shown by the blue curve. However, as shown in Figure 4.2, the updated state error has smaller uncertainties when the robot is only referring to its initial position compared to that of the robot is referring to a landmark. The results obtained are in agreement with the

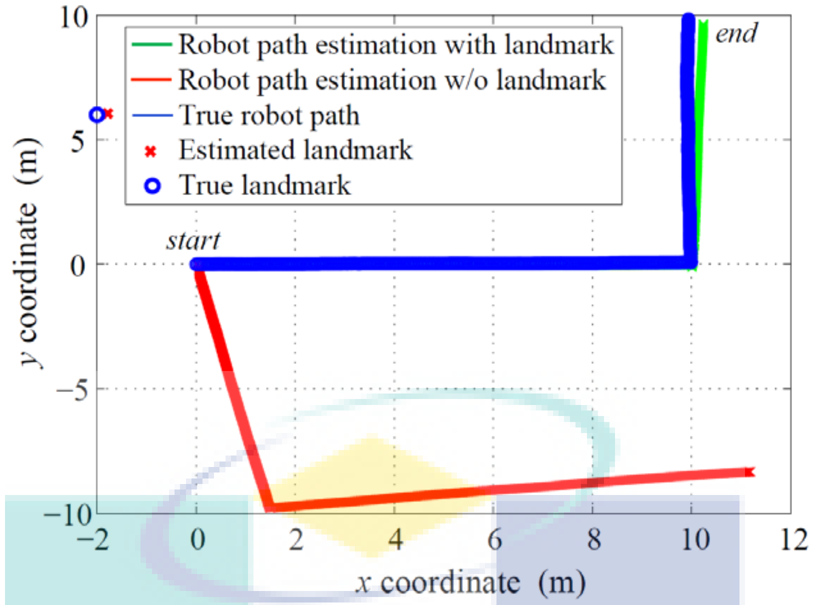

Figure 4.1. Comparison of the estimated robot path between the estimation with landmarks and the estimation without (w/o) landmarks.

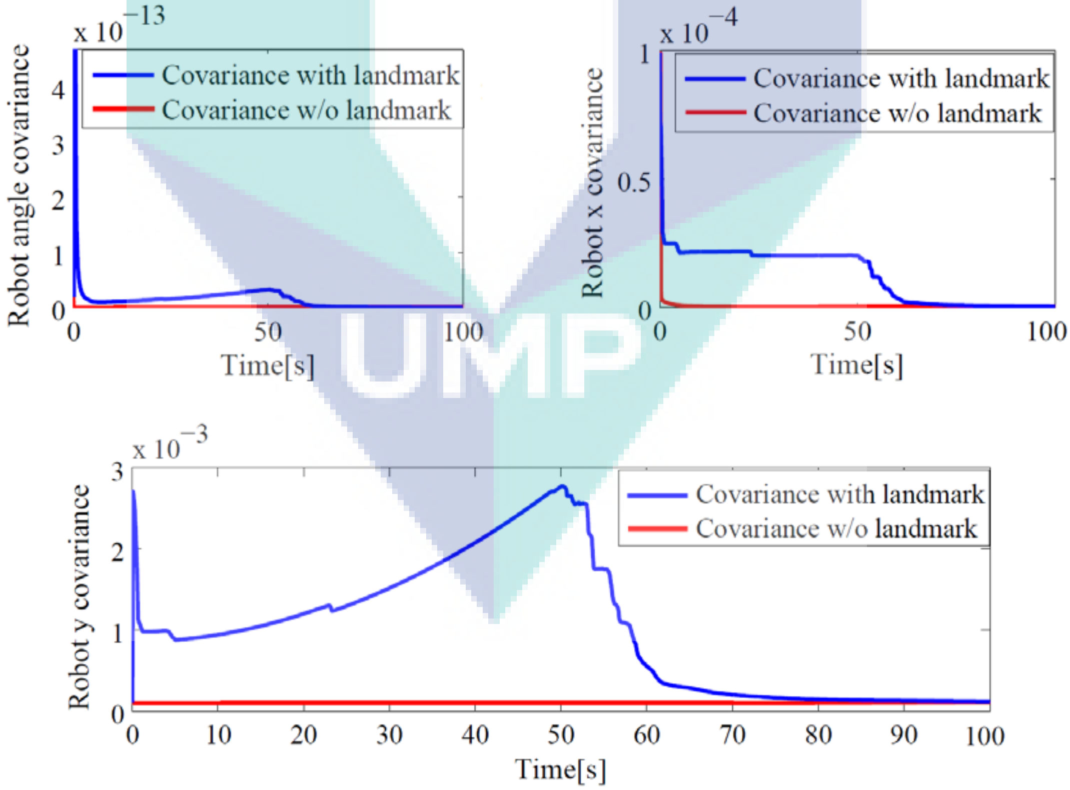

Figure 4.2. Comparison of the estimated covariance matrix between the estimation with landmarks and the estimation without (w/o) landmarks.

previous findings, which stated that the estimation becomes too optimistic even though the results do meet the expectations. Moreover, it is observed that for the normal case of mobile robot SLAM, the state covariance is larger than that of the case with no landmark. Despite of the large uncertainties generated during the estimation, the mobile robot estimation has smaller error as shown in Figure 4.1. By comparing these outcomes, the mobile robot that refers to a landmark produces a more reliable performance and has less error. This clarifies the importance of the cross-correlation, which defines the relationship between landmarks and the mobile robot in obtaining higher accuracy during the mobile robot SLAM. If the mobile robot is configured to be independent of landmarks, then the estimation may become unstable or, even worse, causes the mobile robot becoming lost.

To ensure that the results are consistent, the mobile robot was assigned to move in different paths in the environment. The second simulation is performed to investigate the condition where the mobile robot moves and changes its direction halfway within the area of interest as depicted in Figure 4.3. Different scale of y coordinate is used compared to that of Figure 4.1 to clearly highlight the changes of robot path. Note that along the area of interest, the updated state error covariance is larger in the second case compared to that of the first case, as shown in Figure 4.4. Other than that area, the results describe similar characteristics as in Figure 4.1 and Figure 4.2. Therefore, depending on the direction of mobile robot's movement, it is seen that the updated state error covariance could increase or decrease when compared to that of the first case.

It is apparent that the position of the mobile robot during its observation with the reference to its initial position has a significant effect on the updated state covariance. Particularly, this occurs due to the calculation during the mobile robot measurement model, which may influence the Jacobian matrix. If the mobile robot does not depend on or being correlated to any landmarks, then the updated state covariance would become smaller, although the position estimation is incorrect. This also explains the results reported by Castellanos et al. (Castellanos et al., 1997). However, if the mobile robot attempts to change its direction, then the updated state error covariance would increase as compared to the case of a mobile robot that locates its position based on a landmark. Further investigation is required to corroborate these findings.

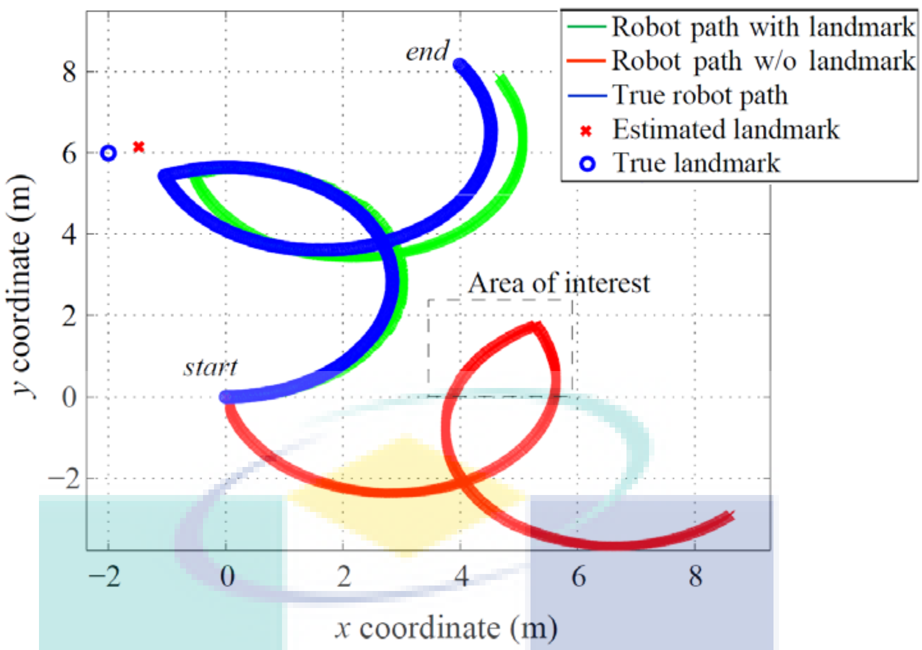

Figure 4.3. Comparison of the estimated robot path between the estimation with and without (w/o) landmarks while the robot changed its path.

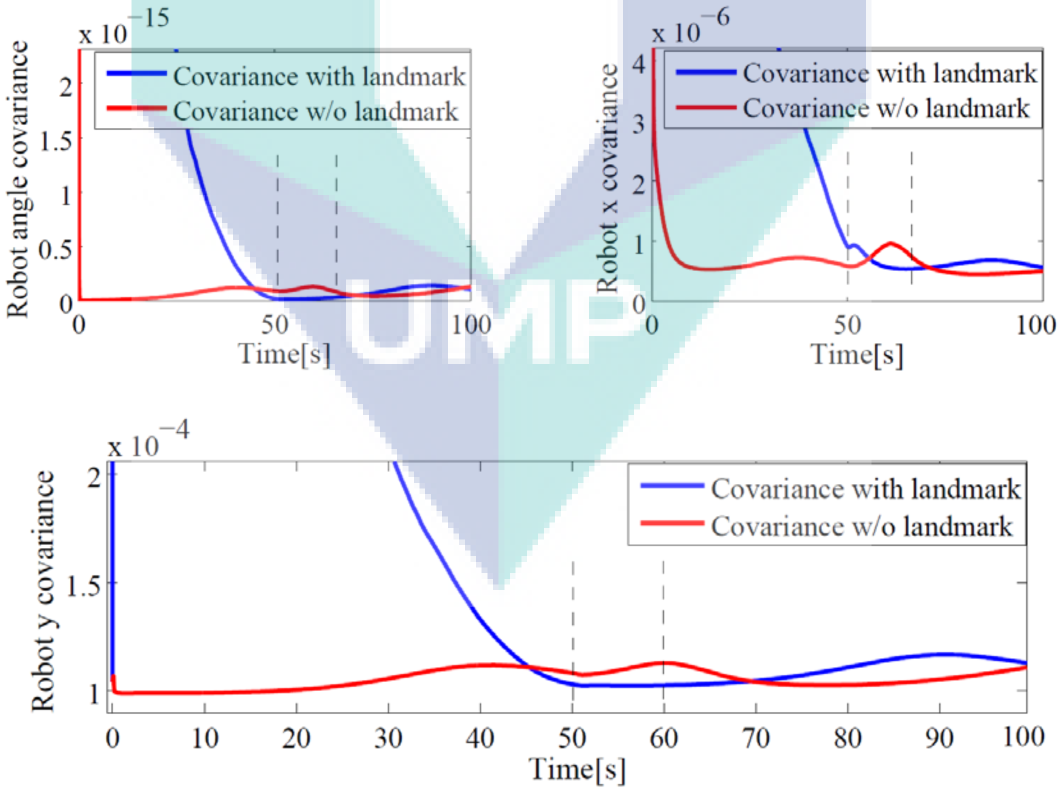

Figure 4.4. Comparison of the estimated covariance matrix between robot estimation with and without (w/o) landmarks while the robot changed its path.

4.7 Summary

This chapter discusses the effect of cross-correlation in the state covariance matrix of mobile robot SLAM. The analysis has been conducted in two cases: a mobile robot SLAM based on the existence of landmark and without the existence of landmark where the robot relies on its initial position. The analysis was conducted with the assumption that both the mobile robot and the landmark have negative coordinates. It has been theoretically proven that the state covariance matrix for the estimation in the absence of landmark is smaller than that of the case when the robot refers to a landmark. This indicates that the estimation has lower uncertainties and the estimation is optimistic, since the simulation conducted in this study has shown that the estimation under this condition is inaccurate. Thus, cross-correlation is important in the mobile robot SLAM as the robot requires a landmark to identify and determine its position in the environment. If the mobile robot cannot identify its location, the 'kidnapped robot' problem may occur. Besides, the computational cost may increase if the mobile robot needs to reconfirm its location by re-observing the environment. Furthermore, the direction of mobile robot's movement may also influence the results as it could affect the Jacobian measurement matrix in the state covariance matrix calculation. The results obtained from the simulation are in a good agreement with the analytical results.

CHAPTER 5

DIAGONALIZATION OF COVARIANCE MATRIX

One of the biggest factors that contribute to the computational cost of extended Kalman filter-based SLAM is the calculation of the covariance update. This is due to the multiplications of the covariance matrix with other parameters along with the increment of its dimension, which is twice the number of landmarks. Therefore a study is conducted to decrease the computational complexity of the covariance matrix without compromising the accuracy of the state estimation using eigenvalue approach. This chapter presents a preliminary study on the matrix-diagonalization technique, which is applied to the covariance matrix in EKF-based SLAM to simplify the multiplication process. The behaviors of estimation and covariance are observed based on four case studies to analyze the performance of the proposed technique. Part of this chapter has been published in (Othman and Ahmad, 2014a) and has fulfilled the second objective of the thesis.

5.1 Introduction

Extended Kalman filter (EKF) has been widely used to solve the estimation problem in SLAM due to the simplicity of the algorithm, its robustness and ability to apply the algorithm online compared to other approaches such as particle filter. However, the whole covariance matrix in EKF-based SLAM needs to be updated every time a new landmark is detected. This process involves a lot of mathematical operation, thus will increase the computational cost. Moreover, the dimension of covariance matrix will increase to twice the number of landmark, as more landmarks are detected. The classical EKF-based SLAM algorithm is known to have a cost of $\mathcal{O}(m^2)$, in which m

denotes the number of landmarks within the map. This limits the use of EKF in a large environment (only a few hundred landmarks). Besides, the full-covariance structure is also very sensitive to the effects of linearization errors, which will accumulate through time and cause the divergence to the filter (Julier and Uhlmann, 2007). Therefore, researchers have been trying to find the solution to mitigate the shortcomings either by (i) dividing the map into sub-local, local, and global map, (ii) using non-full SLAM, or (iii) focusing on the simplification of the covariance structure. This thesis focuses on the third approach; the simplification of the covariance structure.

5.2 Related Work

Guivant and Nebot (2003) introduced a decorrelation algorithm to simplify the covariance matrix. The algorithm will decorrelate a subset of the states that is weakly correlated and cancel the weakly cross-correlation terms in the covariance matrix. A positive semi definite matrix is added to the covariance matrix to reduce the computational and storage costs in SLAM. However, this technique has some drawbacks that lead to filter instability. For that reason the cross-correlation of the structure needs to be preserved (Thrun et al., 2005). Besides, the technique used by Guivant and Nebot (2003) was applied on the map with the relative state representation.

Moreover, a study was conducted to improve the technique through diagonalization of the only part of the state error covariance (Vidal-Calleja et al., 2004a). The technique is known as covariance inflation method, in which a pseudo-noise covariance is added to the covariance matrix to maintain the suboptimality of the filter, given that SLAM is considered as a partially observable system (Andrade-Cetto and Sanfeliu, 2004, and Ahmad et al., 2014). However, the method only diagonalized the priori covariance matrix of the landmarks and Vidal-Calleja et al. (2004a) conducted the simulation analysis under the assumption of linear SLAM.

Besides covariance inflation, Julier and Uhlmann (2007) introduced a covariance intersection method for SLAM, a fusion technique that combines two covariances when the correlations between them are unknown, and this method has been implemented not only in SLAM, but also in other applications (Jiang and Xiao, 2014). In this technique, the updating process is carried out in two independent steps;

updating the robot, then updating the landmark. However a new parameter ω exists in the algorithm that needs to be chosen through an optimization process. This might increase the computational complexity and computation time. Therefore, other approaches should be proposed.

5.3 Scope of Analysis

This preliminary study was conducted to find an alternative technique in diagonalizing the covariance matrix of EKF-based SLAM. As an initial approach, the matrix will be diagonalized by finding its eigenvalues and rebuilding a new diagonal-covariance from these values. The objective is to simplify the multiplication steps in the covariance calculation to minimize the computational complexity as well as computational cost. Multiplication of a matrix with another diagonal matrix is much easier and faster since only diagonal elements are incorporated. The analysis was conducted base on four case studies:

(i) Predicted covariance when both (robot and landmarks) is diagonalized.

(ii) Estimated covariance when both (robot and landmarks) is diagonalized.

(iii) Only estimated covariance of landmarks is diagonalized.

(iv) Only estimated covariance of certain landmarks is diagonalized.

These four cases were analyzed individually to determine the consistency and reliability of the proposed method. These cases are also known as partial observability as certain states of either mobile robot or landmarks, or both covariances are decorrelated to reduce the computational cost.

The preliminary results of the effect on the estimation and covariance behavior were presented, which have been obtained through simulations. In this study, the new diagonalized covariance matrix is the independent variable, whereas the state estimation and the final covariance are the dependent variables.

The remainder of this chapter is structured as follows. Section 5.4 contains a brief explanation on the structure of covariance matrix and the technique of matrix diagonalization. Section 5.5 explains the diagonalization process based on four case studies. The simulated results are presented and discussed in Section 5.6. Finally, the conclusion is drawn in Section 5.7.

5.4 Mathematical Formulation

In this section, the whole structure of covariance matrix is represented, since it is the parameter of study in this chapter. Besides, a technique of matrix diagonalization by finding the eigenvalues is also introduced.

5.4.1 Structure of Covariance Matrix

The covariance matrix of a state estimation in SLAM is a combination matrix of mobile robot and landmark position covariance matrixes and correlation between mobile robot and landmarks. Correlation between mobile robot position and landmarks estimation arise when the measurements are incorporated and thus, the state error covariance becomes dense. The state error covariance P_k is generally defined as

$$P_k = \begin{bmatrix} P_{rr} & P_{rm} \\ P_{rm}^T & P_{mm} \end{bmatrix} \tag{5.1}$$

P_{rr} : Covariance matrix of the robot position

P_{mm} : Covariance matrix of the landmarks position

P_{rm} : Cross-covariance matrix of the robot and landmarks position or cross-correlation between them

Proposition 5.1. *The determinant of the state error covariance matrix is a measure of the volume of the uncertainty ellipsoid associated with the state estimate, which indicates the total uncertainty of that particular state estimation* (Dissanayake et al., 2001).

$$
\left[\begin{array}{ccccccccc}
P_{\theta\theta} & P_{\theta x} & P_{\theta y} & P_{\theta m_{1,x}} & P_{\theta m_{1,y}} & .. & .. & P_{\theta m_{n,x}} & P_{\theta m_{n,y}} \\
P_{x\theta} & P_{xx} & P_{xy} & P_{x m_{1,x}} & P_{x m_{1,y}} & .. & .. & P_{x m_{n,x}} & P_{x m_{n,y}} \\
P_{y\theta} & P_{yx} & P_{yy} & P_{y m_{1,x}} & P_{y m_{1,y}} & .. & .. & P_{y m_{n,x}} & P_{y m_{n,y}} \\
P_{m_{1,x}\theta} & P_{m_{1,x}x} & P_{m_{1,x}y} & P_{m_{1,x}m_{1,x}} & P_{m_{1,x}m_{1,y}} & .. & .. & P_{m_{1,x}m_{n,x}} & P_{m_{1,x}m_{n,y}} \\
P_{m_{1,y}\theta} & P_{m_{1,y}x} & P_{m_{1,y}y} & P_{m_{1,y}m_{1,x}} & P_{m_{1,y}m_{1,y}} & .. & .. & P_{m_{1,y}m_{n,x}} & P_{m_{1,y}m_{n,y}} \\
: & : & : & : & : & .. & .. & : & : \\
: & : & : & : & : & .. & .. & : & : \\
P_{m_{n,x}\theta} & P_{m_{n,x}x} & P_{m_{n,x}y} & P_{m_{n,x}m_{1,x}} & P_{m_{n,x}m_{1,y}} & .. & .. & P_{m_{n,x}m_{n,x}} & P_{m_{n,x}m_{n,y}} \\
P_{m_{n,y}\theta} & P_{m_{n,y}x} & P_{m_{n,y}y} & P_{m_{n,y}m_{1,x}} & P_{m_{n,y}m_{1,y}} & .. & .. & P_{m_{n,y}m_{n,x}} & P_{m_{n,y}m_{n,y}}
\end{array}\right] \Big\} * \tag{5.2}
$$

* Diagonal elements of the state error covariance matrix

The dimension of state error covariance in SLAM is $(3+2m)\times(3+2m)$, where m is the number of detected landmarks. The size of the covariance matrix grows as the robot continuously observes new landmarks in the environment. The structure of the state error covariance for SLAM is fully represented in Eq. (5.2). The state error covariance indicates the error associated with the robot and landmarks state estimations, as defined in Proposition 5.1. From the state error covariance, the increment or decrement of the uncertainties and errors of the estimation could be observed, which represent the precision and consistency of the estimation. The better the estimation, the smaller the covariance value. However, if the predicted covariance value is too small compared to the actual value, that is, if the estimation contains an error, but the covariance indicates a lower value, then the estimation in this situation is known as an optimistic estimation. This is one of the open issues in SLAM and must be considered in EKF-based SLAM (Huang and Dissanayake, 2007).

5.4.2 Diagonalization of a Matrix

A diagonal matrix is a matrix in which the upper and lower sides of its elements are zero. The diagonal elements on the other hand may or may not be zero. The diagonal matrix of a $n\times n$ square matrix can be defined as follows:

$$\begin{aligned}&\text{let the elements of } D=(d_{i,j})\\&d_{i,j}=0 \ \ if \ \ i\neq j \ \ \forall i,j\in\{1,2,\cdots,n\}\end{aligned} \tag{5.3}$$

The operation of matrix multiplication is simpler in a diagonal matrix. Only the diagonal elements are involved and this speeds up the operation and requires less computation cost if it is going to be applied in SLAM since the calculation of covariance matrix involves a lot of matrix multiplication, Eq. (3.30) and Eq. (3.38).

Let A be a $n\times n$ square matrix. Square matrix A is similarly equivalent to a diagonal matrix D, if and only if there exist an invertible matrix C such that $A=C^{-1}DC$. Suppose there exists a number λ and a correspondent column matrix B with dimension of $n\times 1$ such that

$$C=\begin{bmatrix}B_1 & B_2 & \cdots & B_n\end{bmatrix} \tag{5.4}$$

in which λ is said to be an eigenvalue of A with the corresponding eigenvector B. Then A is diagonalizable to a matrix D. In general, for each $n\times n$ matrix, there will be normally n number of eigenvalues; the eigenvalues might be real, complex or combination of both numbers.

Definition 5.2. *Let A be a $n\times n$ square matrix and D is a diagonal matrix in which its diagonal elements are the eigenvalues of A, such as follows:*

$$D=\begin{bmatrix}\lambda_1 & 0 & 0 & 0\\ 0 & \lambda_2 & 0 & 0\\ 0 & 0 & \ddots & 0\\ 0 & 0 & 0 & \lambda_n\end{bmatrix} \tag{5.5}$$

Therefore, there exists the following relationship between matrix A and matrix D:

$$\begin{aligned}\det(A)&=\det(D)\\ \operatorname{norm}(A)&=\operatorname{norm}(D)\end{aligned} \tag{5.6}$$

Referring to Proposition 5.1 and the behavior of diagonal matrix presented in Eq. (5.6), there is a possibility that the diagonalization by means of the eigenvalues can be one of the alternative techniques to minimize the computational cost in EKF-based SLAM. The technique is also inspired by the previous works of Guivant and Nebot (2003), and Vidal-Calleja et al. (2004a), but with some differences. This method will be implemented on the map with absolute state representation, and tested on both predicted and updated covariances.

5.5 Diagonalization of Covariance Matrix in EKF-Based SLAM

The study attempts to investigate the effect of diagonalization of covariance matrix on the estimation performance and the covariance behavior through simulation analysis. The analysis is conducted base on four case studies, as defined in Section 5.3. The first and second case studies are proposed by diagonalizing the whole structure of covariance matrix and the process is discussed in Subsection 5.5.1. On the other hand, only the part of landmarks covariance is diagonalized in the third and fourth case studies to evaluate the possibilities of the proposed approach. These cases are also known as partial observability as certain states of either mobile robot or landmarks or both covariances are decorrelated to reduce the computational cost. The cases are analyzed individually to determine the consistency and reliability of the proposed method.

5.5.1 Full Diagonalization of Covariance Matrix

Equation (5.2) is fully diagonalized by finding the eigenvalues for the whole covariance values. Then the eigenvalues are collected and a new diagonal covariance matrix is built by referring to these values. Algorithm 5.1 and Algorithm 5.2 describe the diagonalization steps of the first and second case studies. Therefore the new diagonal covariance matrix has the following structure, Eq. (5.7):

Algorithm 5.1: Diagonalization of predicted covariance matrix (case 1)

```
Diagonalize_predicted_all( X̂_k, P_k, u_k, z_k )
```

compute predicted state $\hat{X}_{k+1}^{-}$

compute predicted covariance P_{k+1}^{-}

find eigenvalues $\lambda_n = eig(P_{k+1}^{-})$

build diagonal matrix $P_{(D),k+1}^{-} = diag(\lambda_n)$

compute predicted measurement $h(\hat{X}_{k+1}^{-}, k)$

compute estimated state $\hat{X}_{k+1}^{+}$

compute Kalman gain K_{k+1}

compute estimated covariance P_{k+1}^{+}

return $\hat{X}_{k+1}^{+}$, P_{k+1}^{+}

Algorithm 5.2: Diagonalization of estimated covariance matrix (case 2)

```
Diagonalize_estimated_all( X̂_k, P_k, u_k, z_k )
```

compute predicted state $\hat{X}_{k+1}^{-}$

compute predicted covariance P_{k+1}^{-}

compute predicted measurement $h(\hat{X}_{k+1}^{-}, k)$

compute estimated state $\hat{X}_{k+1}^{+}$

compute Kalman gain K_{k+1}

compute estimated covariance P_{k+1}^{+}

find eigenvalues $\lambda_n = eig(P_{k+1}^{+})$

build diagonal matrix $P_{(D),k+1}^{+} = diag(\lambda_n)$

return $\hat{X}_{k+1}^{+}$, $P_{(D),k+1}^{+}$

$$P_{(D),k+1} = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \lambda_n \end{bmatrix} \tag{5.7}$$

By using this new covariance matrix, the multiplication process in Eq. (3.30) and Eq. (3.38) is simplified and therefore the computational cost reduced. This is shown later by the duration of the simulation listed in Table 5.2 on page 78.

5.5.2 Partial Diagonalization of Covariance Matrix

In SLAM, it is important for the mobile robot to have the knowledge of its current position. Initially the mobile robot needs to locate itself before sensing the landmark in the environment. Thus, it is important to retain the covariance of mobile robot P_{rr} as accurate as possible. In third case study, only the covariance of landmark P_{mm} is diagonalized using Algorithm 5.3. Hence the estimated covariance matrix for the third case study has the structure as indicates in following equation:

$$P_{(D),k+1} = \begin{bmatrix} \begin{matrix} P_{\theta\theta} & P_{\theta x} & P_{\theta y} \\ P_{x\theta} & P_{xx} & P_{xy} \\ P_{y\theta} & P_{yx} & P_{yy} \end{matrix} & P_{rm} \\ P_{rm}^{T} & \begin{matrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \lambda_n \end{matrix} \end{bmatrix} \tag{5.8}$$

This method was implemented by Vidal-Calleja et al. (2004a) on the priori state error covariance of the landmarks with the assumption of the system is partially observable, and in the linear case SLAM. Therefore, this study attempts to extend the aforementioned work by diagonalizing the posteriori state error covariance of the landmarks of a nonlinear case SLAM.

Algorithm 5.3: Diagonalization of estimated covariance matrix of the landmark position (case 3)

`Diagonalize_estimated_landmark` $(\hat{X}_k, P_k, u_k, z_k)$

---

compute predicted state $\hat{X}^-_{k+1}$

compute predicted covariance P^-_{k+1}

compute predicted measurement $h(\hat{X}^-_{k+1}, k)$

compute estimated state $\hat{X}^+_{k+1}$

compute Kalman gain K_{k+1}

compute estimated covariance P^+_{k+1}

find eigenvalues of $P^+_{(mm),k+1}$

$$\lambda_n = eig\left(P^+_{k+1}(4:\text{end}, 4:\text{end})\right)$$

build diagonal matrix $P^+_{(mm,D),k+1} = diag(\lambda_n)$

build new diagonal matrix

$$P^+_{(D),k+1} = \begin{bmatrix} P^+_{k+1}(1:3,1:3) & P^+_{k+1}(1:3,4:\text{end}) \\ P^+_{k+1}(4:\text{end},1:3) & P^+_{(mm,D),k+1} \end{bmatrix}$$

Return $\hat{X}^+_{k+1}$, $P^+_{(D),k+1}$

Generally in SLAM, a mobile robot normally travels in a cycle, whereby the loop of the algorithm is closed as the mobile robot detects the first landmark for the second time. By successfully closing the loop, the error and the uncertainties of the estimation is reduced. Therefore, in such case, the first landmark position and correlation are important to be identified and preserved. Hence, in the fourth case study, the effect of diagonalization are only conducted on the second and forth landmarks covariance. The covariance structure of the new diagonalized covariance matrix for the fourth case is depicted in Eq. (5.9) and the diagonalization process is conducted base on Algorithm 5.4.

$$
P_{(D),k+1} = \left[\begin{array}{ccc|ccccccc}
P_{\theta\theta} & P_{\theta x} & P_{\theta y} & & & & & & & \\
P_{x\theta} & P_{xx} & P_{xy} & & & & P_{rm} & & & \\
P_{y\theta} & P_{yx} & P_{yy} & & & & & & & \\
\hline
 & & & P_{m_{1,x}m_{1,x}} & P_{m_{1,x}m_{1,y}} & P_{m_{1,x}m_{2,x}} & P_{m_{1,x}m_{2,y}} & \cdots & P_{m_{1,x}m_{n,x}} & P_{m_{1,x}m_{n,y}} \\
 & & & P_{m_{1,y}m_{1,x}} & P_{m_{1,y}m_{1,y}} & P_{m_{1,y}m_{2,x}} & P_{m_{1,y}m_{2,y}} & \cdots & P_{m_{1,y}m_{n,x}} & P_{m_{1,y}m_{n,y}} \\
 & & & P_{m_{2,x}m_{1,x}} & P_{m_{2,x}m_{1,y}} & \lambda_1 & 0 & 0 & 0 & 0 \\
 & P_{rm}^T & & P_{m_{2,y}m_{1,y}} & P_{m_{2,y}m_{1,y}} & 0 & \lambda_2 & 0 & 0 & 0 \\
 & & & \vdots & \vdots & 0 & 0 & \lambda_3 & 0 & 0 \\
 & & & P_{m_{n,x}m_{1,x}} & P_{m_{n,x}m_{1,y}} & \vdots & \vdots & \vdots & \ddots & \vdots \\
 & & & P_{m_{n,y}m_{1,y}} & P_{m_{n,y}m_{1,y}} & 0 & 0 & 0 & 0 & \lambda_n
\end{array} \right] \quad (5.9)
$$

Algorithm 5.4: Diagonalization of estimated covariance matrix of certain landmark position (case 4)

`Diagonalize_estimated_landmark` ($\hat{X}_k, P_k, u_k, z_k$)

`compute predicted state` $\hat{X}_{k+1}^-$

`compute predicted covariance` P_{k+1}^-

`compute predicted measurement` $h(\hat{X}_{k+1}^-, k)$

`compute estimated state` $\hat{X}_{k+1}^+$

`compute Kalman gain` K_{k+1}

`compute estimated covariance` P_{k+1}^+

`find eigenvalues of` $P_{(mm,n\geq 2),k+1}^+$

$$\lambda_n = eig\left(P_{k+1}^+(6:\text{end},6:\text{end})\right)$$

`build diagonal matrix` $P_{(mm,D),k+1}^+ = diag(\lambda_n)$

`build new diagonal matrix`

$$P_{(D),k+1}^+ = \begin{bmatrix} P_{k+1}^+(1:5,1:5) & P_{k+1}^+(1:5,6:\text{end}) \\ P_{k+1}^+(6:\text{end},1:5) & P_{(mm,D),k+1}^+ \end{bmatrix}$$

`Return` $\hat{X}_{k+1}^+$, $P_{(D),k+1}^+$

Table 5.1
Parameters for the simulation analysis.

Parameter	Symbol	Value
Sampling time (s)	T	0.1
Process noise covariance	Q_k	1×10^{-6}
Measurement noise covariance	R_k	1×10^{-3}
Initial covariance matrix	P_0	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$
Robot initial position	(x_0^r, y_0^r)	$(0,0)$
Landmarks position	(x_1, y_1)	$(-25,-20)$
	(x_2, y_2)	$(-25,20)$
	(x_3, y_3)	$(25,20)$
	(x_4, y_4)	$(25,-20)$

5.6 Simulation Results and Discussions

The simulation analyses of the proposed case studies were conducted based on a SLAM model developed by (HSO, 2013), incorporating the proposed diogonalization technique. The behavior of the estimation and covariance matrix were analyzed. The parameters used in the simulation are listed in Table 5.1 which were selected from the published experimental work (Ahmad and Namerikawa, 2011a). The mobile robot moved circularly in an anticlockwise-direction with a constant velocity and turning rate.

Figure 5.1 shows the estimation of the mobile robot's position and landmarks' position in the normal condition, (i.e. using normal covariance matrix). The mobile robot moved for k = 1000 s and continuously observed the landmarks in every cycle of movement. The figure illustrates the final position and orientation of the mobile robot at the end of the simulation, which designates the final state of the mobile robot. Besides, the covariance-ellipses indicate the uncertainties of the estimation, which represent the state error covariance value. The smaller the ellipse, the better the estimation.

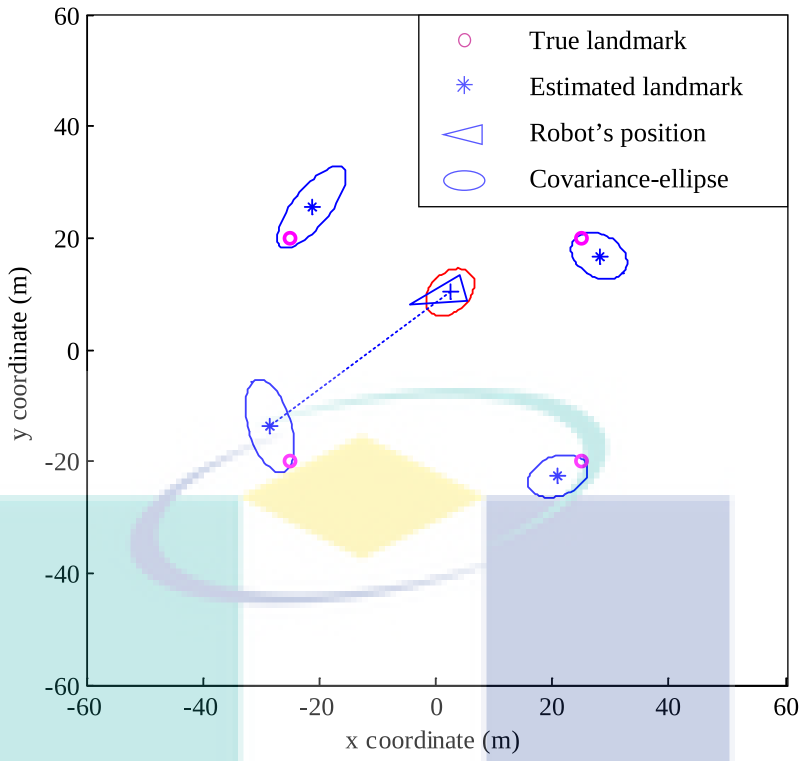

Figure 5.1. Position estimation and covariance under normal condition.

Using the same parameters, the simulations of the four case studies as defined in Section 5.3 were conducted. Figure 5.2 depicts the estimation and covariance behavior of the first case study, while the result from the second case study is illustrated in Figure 5.3. It is apparent from both figures that the estimation of mobile robot and landmark position is possible when the whole covariance is diagonalized through the technique defined in previous section. However the estimation has some degree of errors, but it is considered acceptable. The final position of the mobile robot is similar to that of a normal EKF-based SLAM (Figure 5.1), which indicates that the mobile robot is able to localize its position. Besides, the estimation and covariance behavior were found to be insensitive to the number of landmarks and their position as depicted in Figure 5.4. Nonetheless, the covariance behaves abnormally in these cases, in which the covariance decreases drastically and it is too small compared to the true covariance as depicted in Figure 5.1. This scenario describes an optimistic estimation (Huang and Dissanayake, 2007) as explained in Subsection 5.4.1 of this chapter.

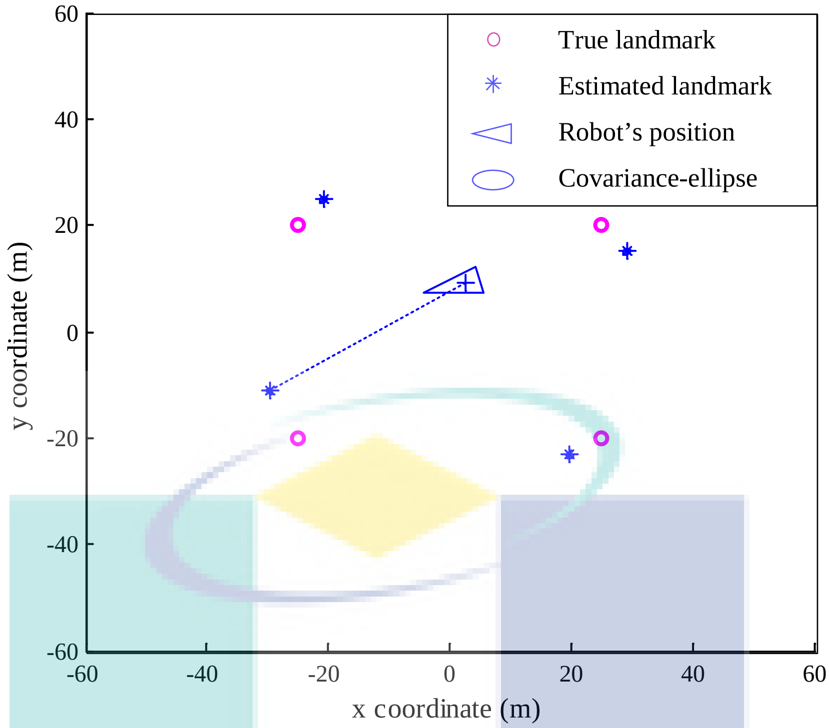

Figure 5.2. State estimation and covariance for case one.

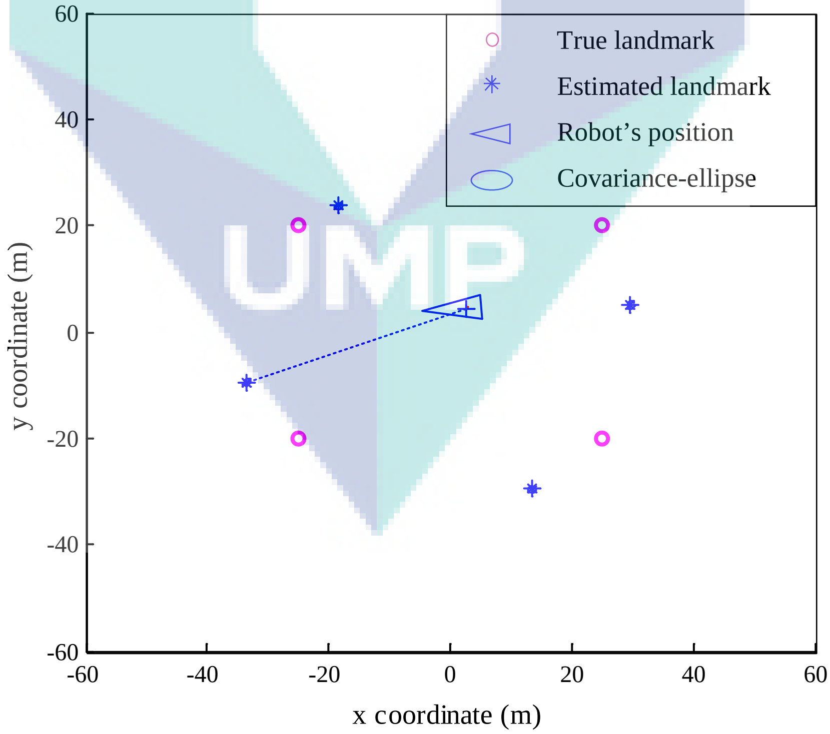

Figure 5.3. Estimation of the state and covariance behavior of case two.

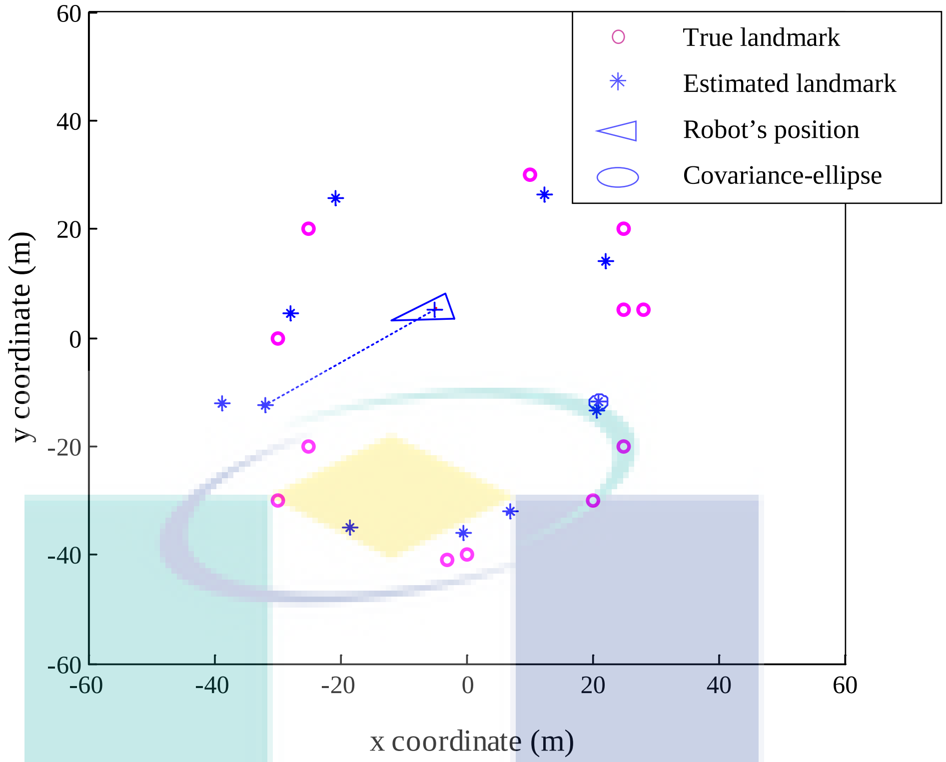

Figure 5.4. State estimation and covariance behavior of case one with different landmarks positions in addition to more landmarks.

Moreover, Table 5.2 shows the comparison of the computation time of all conditions recorded using the *tic and toc* function in MATLAB. The computation time of the four cases are compared to that of the normal EKF-based SLAM. This comparison is important to investigate the possibility of eigenvalue approach in diagonalizing the covariance matrix. It is clearly shown that the times taken to complete the SLAM process for the first and second case study are comparable to that of the normal condition. Even though there are additional steps taken in diagonalizing the covariance matrix, the time taken to complete the whole process is faster. This is demonstrated by the duration of the second case study, which is shorter than that of the normal condition. The results suggest that the technique may be applied in the EKF-based SLAM, but it requires some modification regarding to its approach, for example, the robot covariance and the covariance of the landmark should be managed separately.

Table 5.2
Computation time for all cases.

Covariance type	Simulation time (s)	Computation time (s)
Normal	1000	103.570
Case study 1	1000	104.382
Case study 2	1000	102.844
Case study 3	300	34.779
Case study 4	300	34.084

On the other hand, Figure 5.5 shows the result of the third case study, in which only the covariance of landmark is being diagonalized. It is evident that the mobile robot is unable to detect the landmarks and also unable to localize itself. As a result, the mobile robot has lost its way within the environment. Note that the scale of the environment has been enlarged and the landmarks are similar to other case studies. Besides, the simulation is conducted for only 300 s since the estimation is unsuccessful. Identical results were also obtained in the fourth case as depicted in Figure 5.6.

Based on a thorough analysis on the covariance values, it is observed that the diagonal elements of the covariance in both conditions (third and fourth) are not able to fulfill the requirement of positive semidefinite (PSD) behavior. This misleading result is believed to occur due to the structure of the new covariance, Eq. (5.8) and Eq. (5.9) in which only the covariance of the landmarks is being diagonalized, whereas the cross-correlation terms are neglected. The cross-correlation terms should be included in the diagonalization process since these terms are important in ensuring the optimal estimation performance, as demonstrated in Chapter 4 as well as in literature (Castellanos et al., 1997; Ahmad et al., 2013, and Ahmad and Othman, 2015). Thus, the new suggested structure of the diagonalized matrices for both cases should be as shown in Eq. (5.10) and Eq. (5.11), in which ψ_n is a new diagonal element that integrates the cross-correlation values.

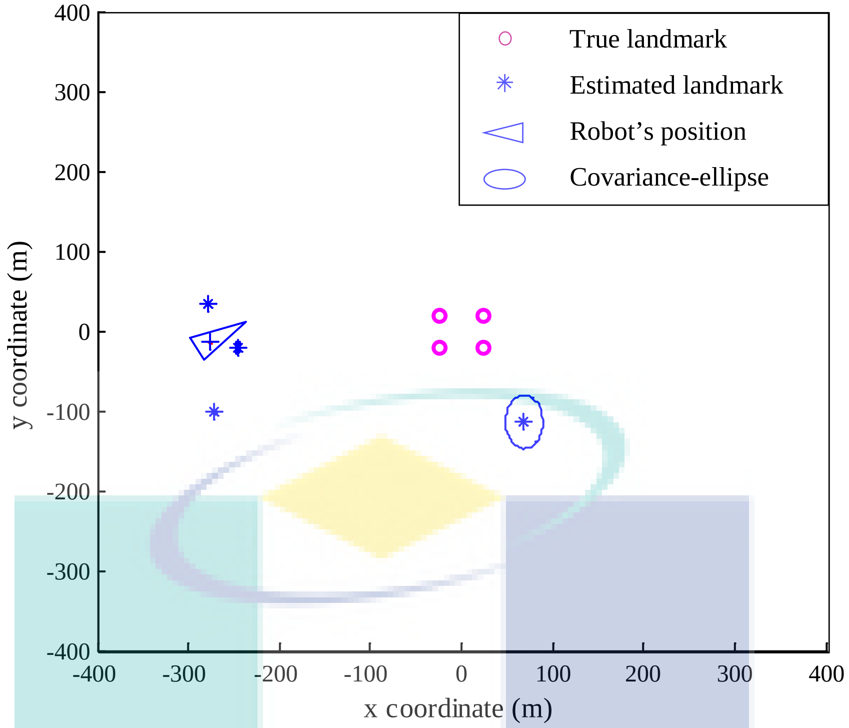

Figure 5.5. Erroneous state and covariance estimation of case three.

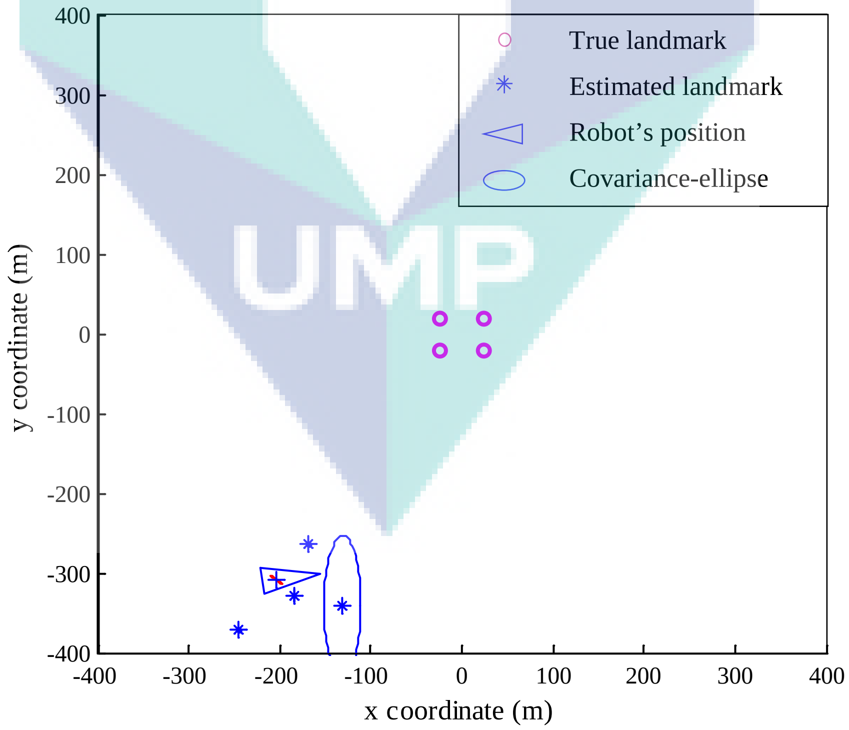

Figure 5.6. Robot unable to estimate the landmarks and its position for the fourth case study.

Therefore, a new strategy needs to be proposed to fully diagonalize the covariance matrix by incorporating the cross-correlation terms into the diagonal elements to find the correct value of ψ_n.

$$P_{(D),k+1} = \begin{bmatrix} P_{\theta\theta} & P_{\theta x} & P_{\theta y} & 0 & 0 & 0 & 0 & 0 \\ P_{x\theta} & P_{xx} & P_{xy} & 0 & 0 & 0 & 0 & 0 \\ P_{y\theta} & P_{yx} & P_{yy} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_3 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_n \end{bmatrix} \tag{5.10}$$

$$P_{(D),k+1} = \begin{bmatrix} P_{\theta\theta} & P_{\theta x} & P_{\theta y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_{x\theta} & P_{xx} & P_{xy} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_{y\theta} & P_{yx} & P_{yy} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{m_{1,x}m_{1,x}} & P_{m_{1,x}m_{1,y}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{m_{1,y}m_{1,x}} & P_{m_{1,y}m_{1,y}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_3 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_n \end{bmatrix} \tag{5.11}$$

5.7 Summary

This chapter presents the analysis of EKF-based SLAM performance under the conditions of a diagonalized covariance. The covariance matrix needs to be simplified to reduce the computational complexity and thus reduce the computational cost. Diagonalization method through eigenvalues could be one of the approaches that can achieve this goal. The simulation results have proven that this technique could be implemented; however more modifications on the algorithm are required to ensure accurate estimation and covariance behavior as discussed in Section 5.6. Besides, the cross-correlation terms should be taken into account since it determines estimation accuracy as discussed in Chapter 4. In the future study, the behavior of each elements of covariance matrix will be analyzed individually in order to define the specific pattern for the development of new algorithm for a successful diagonalization of the covariance matrix. Hence, the performance of the method may be compared to others for benchmarking purposes.

CHAPTER 6

INTERMITTENT MEASUREMENT ANALYSIS

This chapter covers the investigation of the effect of intermittent measurement condition on the EKF-based SLAM performance. Two case studies, a stationary and moving robot, were considered. The nonlinear EKF-based SLAM was analyzed theoretically. The findings demonstrated that the estimation of robot position was still possible even when the measurement data are unavailable. However, the estimation possesses high uncertainties and produce abnormal covariance behavior. In addition, it has been proven that EKF is able to correct the state estimation upon the availability of measurement data, but not the state covariance. Simulation results proved the consistency of the proposed analysis. Parts of this chapter have been published in (Othman and Ahmad, 2013a, b; Othman et al., 2013, and Othman and Ahmad, 2014b). This chapter highlights the work done in accomplishing the third objective of the thesis.

6.1 Introduction

Simultaneous localization and mapping (SLAM) of mobile robot by means of extended Kalman filter requires the availability of continuous data measurement along the process to ensure a successful estimation of the state vector. Extended Kalman filter is a recursive algorithm that uses previous data in completing the iteration. Therefore, the availability of the measurement data is essential in ensuring a successful estimation.

However, the situation of which a loss of measurement occurs should be taken into account since it may happen due to several reasons such as intermittent sensor failure, faulty in the measurement, obstacles that may appear in a dynamic environment, network congestion or accidental loss due to noisy environment or jammed network

(Dong et al., 2012). Such situations are known as intermittent measurement condition. Most of the estimation process in extended Kalman filter-based SLAM is based on an assumption that the measurement data are continuously available for the updating process. However, due to the abovementioned conditions, measurement data may not be available at a certain period of time throughout the estimation process. The influence of this phenomenon on the state estimation values or the state covariance should be investigated since the operator normally relies on these values to evaluate the accuracy of the robot estimation.

Furthermore, the study is also important to predict an actual situation of when an intermittent occurs in a mobile robot SLAM condition. This serves as a basis of the correction and prevention steps such as in controller development, improvement in algorithms and so on. In this chapter, a theoretical study of the extended Kalman filter-based SLAM with intermittent measurement is conducted to examine the estimation behavior of this nonlinear process during that particular condition.

6.2 Related Work

Many studies have been conducted with regards to this issue, which mainly focus on the application of network control plant. Various approaches have been presented in literature to model this phenomenon, for example through binary switching sequence based on Bernoulli process (Sinopoli et al., 2004 and Kluge et al., 2010) or Markovian jumping parameters (Zhan et al., 2009). However, the studies on the intermittent measurement event in robotics system, especially in SLAM are very limited. Hitherto, there are only a few studies performed with regards to this issue specifically on EKF-based SLAM, such as the work of Muraca et al. (Muraca et al., 2008), and Ahmad and Namerikawa (Ahmad and Namerikawa, 2011b, 2013). The former studies were conducted to investigate the effect of missing measurement between on-board sensors and sensors placed in the environment, and they have developed a scanning strategy between those sensors during intermittent measurement. Unfortunately the behavior of state and covariance matrix of the estimation was not discussed or investigated. The latter studies, on the other hand, provide an analysis of EKF-based SLAM during intermittent condition using Fisher Information Matrix (FIM) and measurement innovation characteristics. Both methods have proven that the state estimation of EKF-based SLAM is still possible with some degree of error.

Moreover, Ahmad and Namerikawa (2012b) investigated the behavior of covariance matrix under intermittent measurement by providing statistical bounds of the covariance matrix through analysis of FIM. This chapter attempts to continue the analyses conducted in (Ahmad and Namerikawa, 2012b) by focusing on the behavior of Jacobian matrix, as it was found to influence the estimation under intermittent condition significantly. Jacobian matrix exists due to the linearization process since SLAM is treated as a nonlinear system. Moreover, the study is also a sequel of a preliminary study (Othman and Ahmad, 2013b), in which SLAM was evaluated as a linear system.

6.3 Scope of Analysis

The study begins with the application of the linear behavior on mobile robot SLAM to investigate the intermittent effect under linear condition on two case studies: a stationary and moving robot conditions. The purpose of the study is to investigate the influence of parameters: control input, process noise and measurement matrix on the state and covariance estimation in order to assist the analysis of nonlinear EKF. Then the theoretical analysis on the nonlinear EKF-based SLAM was conducted and the situation was simulated. The mobile robot and the environment model used in the analysis were previously described in Chapter 3. The analysis was performed to represent the possible condition of the system that utilizes the range and bearing sensors. Following sections detail the work performed.

6.4 Assumption of Linear Behavior

In this section, the estimation behavior of SLAM under intermittent condition is evaluated as a linear system. The objective is to investigate its effect on the state estimation and the state covariance and identify the parameter that is significantly influenced by this phenomenon. The analysis and results of linear assumption form the base of the following analysis of EKF-based SLAM under nonlinear characteristic. The study also attempts to demonstrate that if there are some missing measurement data during the robot observation, the estimations of mobile robot pose and landmarks locations become incorrect and resulting in an increase of the state covariance that is determined by the determinant of covariance matrix. Its impact on state covariance matrix is observed and analyzed.

Intermittent measurement has been modelled either as Markov chain or Bernoulli process. To compensate the missing measurements in a model with the Bernoulli process, the update algorithm for Kalman filter, Eq. (3.20) and Eq. (3.21) are modified into

$$\hat{X}^{+}_{k+1} = \hat{X}^{-}_{k+1} + \gamma_{k+1} K_{k+1} \mu_{k+1} \tag{6.1}$$

$$P^{+}_{k+1} = (I - \gamma_{k+1} K_{k+1} H_{k+1}) P^{-}_{k+1} \tag{6.2}$$

where γ_{k+1} is a Bernoulli random variable and has a value of either one or zero (Sinopoli et al., 2004).

Definition 6.1. *Measurement data lost are defined as a condition where measurement data are not successfully retrieved after one sampling time and occurred randomly in mobile robot observations.*

Hence, the abovementioned definition proposes the following proposition:

Proposition 6.2. *If a measurement is not available at time k, then the measurement matrix* $H_k \cong [0]$ *for a linear KF-based SLAM, where* $[0]$ *denotes a zero matrix.*

Proof. The observation process of a mobile robot is represented by the following equation:

$$z_k = \begin{bmatrix} r_i \\ \phi_i \end{bmatrix} = H_k X_k + v_{r_i \; \phi_i} \tag{6.3}$$

In a linear system, measurement matrix H_k represents a relative distance between robot position and landmark location (Dissanayake et al., 2001). In KF-based SLAM of mobile robot, the matrix element normally possesses a value of either one, minus one or zero (Vidal-Calleja et al., 2004a, b). If the measurement is not available, this matrix will indicate a zero matrix since there is no successful observation at the particular moment. □

Note that even if the measurement matrix is zero during the intermittent event, the updated state covariance might not have the same value as the previous state covariance. This is due to the existence of the measurement noise shown in the measurement process, denoted by the Eq. (6.3). Besides, in a nonlinear system, the nonlinear Jacobian transformation during the measurement process also affects the results. This is the theoretical reason why the updated state covariance might not be the same as its previous value. However, if the measurement noise is very small that it can be neglected, then the updated state covariance might have almost the same value as the previous state.

In the next section, the covariance matrix of state estimation behavior is demonstrated and analyzed given this condition partially occurs during mobile robot observation. The analyses are conducted base on two conditions. The first condition indicates that the mobile robot is stationary, with consideration of the existence and non-existence of the process noise. Whereas the second condition designates that the mobile robot is moving. These two conditions are defined to investigate the effect of control input matrix u_k and process noise Q_k (independent variables) on the state estimation $\hat{X}_{k+1}^{+}$ and state covariance matrix P_{k+1}^{+} (dependent variables).

6.4.1 Stationary Robot

For the first case of the study, the robot is stationary and observes one landmark in its environment for *n* times. Since the robot is not moving, there is no control input for the mobile robot's motion, therefore $u_k = 0$. In this scenario, two conditions are observed; with and without the existence of process noise.

6.4.1.1 Absence of Process Noise Condition

Since the mobile robot is stationary, several studies assume that there is no process noise for that particular moment as there is no movement involved (Julier and Uhlmann, 2001). Covariance of process noise Q_k is the summation of the covariance of control noise $\delta\gamma$ and $\delta\omega$. Since the mobile robot is assumed to remain stationary, no control noise is injected into the system, therefore Q_k is assumed zero.

Lemma 6.3. *If the observation is not available at* $1 < k < \infty$ *time, the covariance matrix during an intermittent measurement is larger than the covariance matrix in a normal condition, in which the measurement data are consistently available.*

Proof. The state error covariance matrix is predicted through Eq. (3.14). Since the robot is stationary, the state transition matrix F_{k+1} usually possesses an identity matrix. Thus, priori covariance matrix at time $k+1$ is equal to the posterior covariance matrix at time k since no process noise is added to the system as shown by following equation:

$$\begin{aligned} P_{k+1}^{-} &= F_{k+1} P_k F_{k+1}^{\ T} + Q_{k+1} \\ &= I P_k I^T + 0 \\ &= P_k \end{aligned} \tag{6.4}$$

Moreover, the matrices in the algorithm are positive semidefinite (PSD) matrix (Dissanayake et al., 2001). If the measurement data are consistently available at $1 < k < \infty$ time, the updated covariance at time $k+1$ has smaller value than priori covariance due to the correction done by Kalman filter. The updated state covariance in Eq. (3.21) combined with the information from Eq. (6.4) becomes

$$\begin{aligned} & P_{n(k+1)}^{+} = P_{k+1}^{-} - K_{k+1} H_{k+1} P_{k+1}^{-} \\ & P_{n(k+1)}^{+} \le P_{k+1}^{-} \\ & P_{n(k+1)}^{+} \le P_k \end{aligned} \tag{6.5}$$

If intermittent measurement occurs, there is no observation available, based on the explanation of Definition 6.1, the measurement matrix $H_{k+1} \to [0]$ and $\gamma_{k+1} = 0$ in Eq. (6.2). Under this assumption, posteriori covariance matrix with intermittent measurement is equal to the priori covariance matrix, which is similar to the covariance matrix at time k. Therefore, by integrating the information gained from Eq. (6.5), the covariance matrix with intermittent measurement is larger than that in the normal condition as given by Eq. (6.6).

$$
\begin{aligned}
&P^{+}_{i(k+1)} = P^{-}_{k+1} - \gamma_{k+1} K_{k+1} H_{k+1} P^{-}_{k+1} \\
&P^{+}_{i(k+1)} = P^{-}_{k+1} \\
&P^{+}_{i(k+1)} = P_{k} \\
&P^{+}_{i(k+1)} \geq P^{+}_{n(k+1)}
\end{aligned}
\tag{6.6}
$$

Suppose in the measurement update the covariance is corrected through Kalman gain, but this cannot be done in the present case due to the unavailability of measurement data. During prediction step, the covariance matrix of the robot position P_{rr} and cross covariance between robot and landmark P_{rm} are changed, while covariance for the landmark P_{mm} remains. After the update, all elements in the covariance should be different. However, due to intermittent, the covariance of landmark P_{mm} retains the previous value. Therefore, posteriori covariance matrix is equal to the priori covariance matrix under intermittent condition. Note that subscript i and n in Eq. (6.5) and Eq. (6.6) denote a parameter during intermittent measurement and under normal condition respectively. □

Definition 6.4. *The determinant of the state error covariance matrix is a measure of the volume of the uncertainty ellipsoid associated with the state estimate* (Dissanayake et al., 2001).

From Eq. (6.5) and Eq. (6.6), the determinant of the state error covariance during intermittent measurement is larger than that in the normal condition:

$$\left\{\det\left(P^{+}_{n(k+1)}\right) \le \det\left(P_k\right)\right\} \quad \cap \quad \left\{\det\left(P^{+}_{i(k+1)}\right) = \det\left(P_k\right)\right\}$$
$$\Rightarrow \det\left(P^{+}_{n(k+1)}\right) \le \det\left(P^{+}_{i(k+1)}\right) \tag{6.7}$$

This suggests that the total uncertainty increases when the measurement data suddenly become unavailable, which indicates an imprecise estimation of current state. Thus, in an intermittent measurement, the mobile robot may incorrectly estimate its current position.

6.4.1.2 Existence of Process Noise Condition

In an actual situation, it is hard to obtain a noise-free system. Although the mobile robot is not moving, the process noise may also exist in the SLAM system, e.g. noises from the environment or encoder attached to the robot. In this section, the analysis is continued with $Q_k \neq 0$ to investigate the impact of process noise on the state covariance.

From Eq. (3.14) the priori covariance matrix at time $k+1$ is no longer equal to the posteriori covariance matrix at time k since the process noise is added to the system. The priori covariance at $k+1$ is larger than the covariance at k.

$$\begin{aligned} P^{-}_{k+1} &= F_{k+1} P_k F_{k+1}{}^{T} + Q_k \\ &= I P_k I^{T} + Q_k \\ P^{-}_{k+1} &> P_k \end{aligned} \tag{6.8}$$

This is true in both cases; normal and intermittent measurement. In the normal condition, the priori covariance matrix is updated using Eq. (3.21) and possesses a smaller value of posteriori covariance matrix. This is true as $k \to \infty$ covariance matrix is decreasing and converging. However, the effect is different when the measurements are not available. Using Eq. (6.2) with $H_{k+1} \to [0]$ and $\gamma_{k+1} = 0$, the covariance matrix is not able to be updated and remains with the value of priori covariance. The covariance matrix accumulates if the measurement is still not available since process noise is injected to the system for each time update (see Eq. (6.8)), as shown below.

$$
\begin{aligned}
P_{i(k+1)}^{+} &= P_{k+1}^{-} - \gamma_{k+1} K_{k+1} H_{k+1} P_{k+1}^{-} \\
P_{i(k+1)}^{+} &= P_{k+1}^{-} = I P_k I^{T} + Q_k
\end{aligned}
\tag{6.9}
$$

The covariance matrix under normal condition normally decreases and converges as $k \to \infty$ while the covariance matrix in an intermittent measurement condition increases as long as measurement is unavailable, and Eq. (6.7) is referred, therefore the determinant of both covariances have a similar trend, thus:

$$
\det\left(P_{n(k+1)}^{+}\right) \leq \det\left(P_{i(k+1)}^{+}\right) \tag{6.10}
$$

This shows that the uncertainty of the state estimation when the data unavailability occurs is higher than that in the normal condition. This indicates erroneous prediction of the robot pose. Therefore, a control strategy is needed to compensate this error.

6.4.2 Moving Robot

The mobile robot moves from a stationary position and observes one landmark in its environment for *n* times. Since the robot is moving, there is a control input applied to the system for the mobile robot's motion, therefore $u_k \neq 0$ and $Q_k \neq 0$. These two parameters are considered during prediction step to predict the priori estimation of the state and its covariance matrix using Eq. (3.13) and Eq. (3.14).

Since the control input is considered only in the prediction of the priori state estimate X_{k+1}^{-}, its effect on the priori covariance matrix P_{k+1}^{-} is not significant. However, the process noise under this condition is possibly larger than that in the stationary condition due to the existence of $\delta\omega$ and $\delta\gamma$. Therefore, the covariance matrix of a moving mobile robot is greater than that in stationary situation as shown in the following equation:

$$P_{k+1}^{-} = F_{k+1} P_k F_{k+1}{}^{T} + Q_k$$

$$\text{If } Q_{k(u)} > Q_{k(s)} \text{ thus } P_{k+1(u)}^{-} > P_{k+1(s)}^{-} \tag{6.11}$$

$$\text{Hence from Eq. (3.21) } P_{k+1(u)}^{+} > P_{k+1(s)}^{+}$$

In Eq. (6.11), the subscript s and u denote a parameter in the case of stationary and moving robot, respectively.

The behavior of covariance matrix in the normal and intermittent measurement of a moving robot is similar to that of a stationary robot. The covariance matrix during the unavailability of the measurement data is higher than that in the normal condition. This can be analogously proven through Eq. (6.8) – Eq. (6.10). Similar case is seen for the determinant of the covariance matrix. The analysis can be proven in a similar way to the first case i.e. in the stationary condition with the existence of process noise, Subsection 6.4.1.2.

From the analyses in the Subsection 6.4.1 and Section 6.4.2, it can be concluded that although the measurements data are not available intermittently during the mobile robot observation, the estimation is still possible, but erroneous. This is proven by the increment of the state error covariance matrix and its determinant in comparison to the normal condition. The analyses also suggest that the measurement matrix H_k significantly influence the performance of KF-based SLAM during an intermittent measurement since the behavior of the covariance matrix may differ if this matrix holds a non-zero matrix. The results and analyses from Section 6.4 are used to compare the effect of intermittent measurement condition under nonlinear EKF-based SLAM.

6.5 Nonlinear Analysis

In this section, the estimation process of nonlinear EKF-based SLAM in an intermittent condition is thoroughly analyzed. The characteristics of the state vector estimation and state covariance matrix (the dependent variables in the study) are investigated. The hypothesis of the study states that the update step is not accomplished since the measurement matrix or Jacobian ∇H_i is assumed to be a zero matrix, similar to that in the linear case analysis, by Proposition 6.2. This assumption is made based on the following statement, under Assumption 6.5. In addition to the analyses under linear condition, it is suggested that the measurement matrix could highly influences the state estimation and covariance matrix if it is a non-zero matrix. Therefore, in the study of nonlinear condition, the Jacobian matrix ∇H_i is defined as an independent variable.

Assumption 6.5. *If the measurement data are intermittently unavailable, EKF might be unable to calculate the difference of the relative measurement between the mobile robot and landmark position,* dx *and* dy*. Therefore, the Jacobian of the observation model* ∇H_i *is assumed to be a zero matrix.*

From Eq. (3.36), let μ_{k+1} represents an innovation in EKF update steps, in which:

$$\mu_{k+1} = z_{k+1} - \nabla H_i \hat{X}_{k+1}^{-} \tag{6.12}$$

If $\nabla H_i \approx [0]$ and z_{k+1} is unavailable, μ_{k+1} will become a zero matrix. Thus, the correction term in Eq. (3.36) will be zero and the estimated state vector will be similar to the predicted state vector $\hat{X}_{k+1}^{+} = \hat{X}_{k+1}^{-}$. This also applies to the estimated error covariance matrix since ∇H_i affects the value of Kalman gain Eq. (3.39). Hence it is assumed that $P_{k+1}^{+} = P_{k+1}^{-}$.

6.5.1 Analysis of Estimation Behavior

As stated in Assumption 6.5, if observation data are unavailable at a specific time k_{lost}, then the estimation at time $k_{lost}+1$ will retain the previous value, $\hat{X}_{k+1}^{+} = \hat{X}_{k+1}^{-}$. However, this assumption is found to be false and will be proven in this subsection.

Theorem 6.6. *Assume that the mobile robot is observing a known landmark B at point A. If the measurement data are not available at* k_{lost}*, the estimated state vector will be larger or smaller than the predicted state vector depending on the value of* $K_{k+1}\mu_{k+1}$*.*

However, the estimated state covariance matrix at this moment will be smaller than the predicted error covariance matrix. Thus, during the time k_{lost}, the estimation is said to be too optimistic.

Proof. As the mobile robot is assumed to have successfully observed the landmark *B* at $k_{lost}-1$, the prediction cycle at k_{lost} may be completed since only the robot position is updated during time update. Hence, the predicted state vector $\hat{X}^{-}_{k_{lost}}$ and the predicted state covariance matrix $P^{-}_{k_{lost}}$ are obtained from Eq. (3.28) and Eq. (3.30):

$$\hat{X}^{-}_{k_{lost}} = [\,\hat{\theta}^{-}_{A_{lost}} \quad \hat{x}^{r-}_{A_{lost}} \quad \hat{y}^{r-}_{A_{lost}} \quad \hat{x}^{-}_{B} \quad \hat{y}^{-}_{B}\,]^{T}$$

$$P^{-}_{k_{lost}} = \begin{bmatrix} P^{-}_{\hat{\theta}^{-}_{A_{lost}}} & * & * & * & * \\ * & P^{-}_{\hat{x}^{r-}_{A_{lost}}} & * & * & * \\ * & * & P^{-}_{\hat{y}^{r-}_{A_{lost}}} & * & * \\ * & * & * & P^{-}_{\hat{x}^{-}_{B}} & * \\ * & * & * & * & P^{-}_{\hat{y}^{-}_{B}} \end{bmatrix} \tag{6.13}$$

where $*$ indicates the cross-correlation terms of error covariance matrix. Since the Jacobian of measurement model in SLAM ∇H_i is evaluated at $\hat{X}^{-}_{k+1}$, using the values of $\hat{X}^{-}_{k_{lost}}$, Jacobian $\nabla H_{A_{lost}}$ at k_{lost} is possible to be attained and may produce a partially observable SLAM (Vidal-Calleja et al., 2004b).

$$\nabla H_{A_{lost}} = \begin{bmatrix} 0 & -\dfrac{dx_A}{r_A} & -\dfrac{dy_A}{r_A} & \dfrac{dx_A}{r_A} & \dfrac{dy_A}{r_A} \\ -1 & \dfrac{dy_A}{r_A{}^2} & -\dfrac{dx_A}{r_A{}^2} & -\dfrac{dy_A}{r_A{}^2} & \dfrac{dx_A}{r_A{}^2} \end{bmatrix}$$

$$\begin{aligned} dx_A &= \hat{x}^{-}_{B} - \hat{x}^{r-}_{A_{lost}} \\ dy_A &= \hat{y}^{-}_{B} - \hat{y}^{r-}_{A_{lost}} \\ r_A &= \sqrt{dx_A{}^2 + dy_A{}^2} \end{aligned} \tag{6.14}$$

This eliminates the initial hypothesis that declares ∇H_i is zero when the measurement data are unavailable.

Since $\hat{X}^-_{k_{lost}}$, $P^-_{k_{lost}}$ and $\nabla H_{A_{lost}}$ are available from the prediction step, and with the fact that the covariance of sensors R_k is normally determined from the beginning and is assumed constant throughout the SLAM process, the Kalman gain $K_{k_{lost}}$ can be obtained from Eq. (3.39). Since no observation data are obtained i.e. $z_{k_{lost}} = [0]$, from Eq. (6.12) the innovation at this moment is $\mu_{k+1} = -\nabla H_i \hat{X}^-_{k+1}$ which is too large compared to the normal innovation when measurement data are available. This condition significantly affects the estimation. Hence, from the update step of EKF-based SLAM, the state vector and state covariance matrix are estimated through Eq. (3.36) and Eq. (3.38).

$$\begin{aligned} \hat{X}^+_{k_{lost}} &= \hat{X}^-_{k_{lost}} + \left\{ \left(K_{k_{lost}} \right) \left(-\nabla H_{A_{lost}} \hat{X}^-_{k_{lost}} \right) \right\} \\ P^+_{k_{lost}} &= P^-_{k_{lost}} - \left\{ K_{k_{lost}} \nabla H_{A_{lost}} P^-_{k_{lost}} \right\} \end{aligned} \tag{6.15}$$

Equation (6.15) proves that the Assumption 6.5 is incorrect. The estimated position of the mobile robot and landmark at k_{lost} may increase or decrease from the predicted positions, depending on the values of $\left\{ \left(K_{k_{lost}} \right) \left(-\nabla H_{A_{lost}} \hat{X}^-_{k_{lost}} \right) \right\}$ and the estimated state covariance matrix is smaller than the predicted state covariance. This indicates that the estimation at k_{lost} is too optimistic. □

Moreover, if the measurement data are still not available at time $k_{lost} + l$, the estimated state vector will become more erroneous. Since $\hat{X}^+_{k_{lost}}$ contains some errors in the estimation, this errors will propagate in the calculation of $\hat{X}^-_{k_{lost}+1}$ and in other parameters throughout the estimation process. The Jacobian at this stage is evaluated at $\hat{X}^-_{k_{lost}+1}$ and therefore will produce incorrect values. Using this false value, the error covariance matrix will be updated and therefore subsequently producing more errors. The statistical behavior of a state covariance matrix during this condition has been

Table 6.1
The parameters defined in the simulation.

Parameter	Symbol	Value
Sampling time (s)	T	0.1
Process noise covariance	Q_k	1×10^{-6}
Measurement noise covariance	R_k	1×10^{-3}
Initial state covariance	P_0	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$
Robot initial position	(x_0^r, y_0^r)	$(0,0)$
Landmarks position	(x_1, y_1)	$(-25,-25)$
	(x_2, y_2)	$(-25,25)$
	(x_3, y_3)	$(25,25)$
	(x_4, y_4)	$(25,-25)$

studied and the covariance bounds have been proposed (Ahmad and Namerikawa, 2012a). As measurement data are still unavailable at $k_{lost}+l$, the error of the estimation will increase.

However, the estimation will be corrected if the measurement data become available again. The prediction and estimation of the mobile robot position is updated based on the range and bearing values obtained. Hence, the EKF will recover the state vector through its normal process as in Eq. (3.28) – Eq. (3.39). This behavior is demonstrated in the simulation analysis.

6.5.2 Simulation Results

The analysis of an extended Kalman filter behavior in the application of mobile robot SLAM in the condition of missing measurement data has been discussed. The aforementioned analysis was then validated against simulation using parameters as listed in Table 6.1. The parameters were selected based on the experimental works conducted by Ahmad and Namerikawa (2011a, 2013). The simulation analyses were

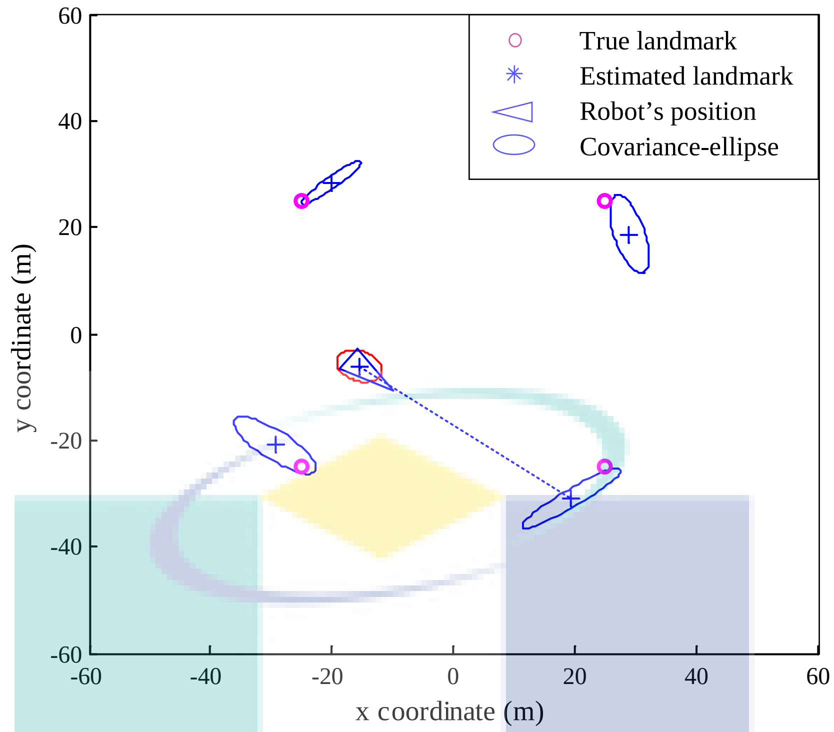

Figure 6.1. The estimation of mobile robot and landmarks position under normal condition with k = 800 s.

conducted based on a SLAM model (HSO, 2013) by incorporating the intermittent occurrence in the model. In the simulation, the mobile robot started moving from its initial position $(0,0)$ and observed four landmarks within its environment. Throughout the observation process, there was a period of time in which the mobile robot did not receive any measurements data, i.e. $z_{k_{lost}} = 0$ for 10 seconds. However, the measurement data were available again afterwards.

Figure 6.1 shows the estimation of the mobile robot's position and landmarks' position under normal condition. The mobile robot moves for 800 seconds and continuously observes the landmarks for every cycle of movement. The position and orientation of the mobile robot in the figure indicates the final state of the mobile robot at k = 800 s. The covariance-ellipses indicate the uncertainties of the estimation, in which smaller ellipse designates better estimation.

The estimation during intermittent condition is shown in Figure 6.2 on page 98. The estimation without intermittent is shown in Figure 6.2 (a), which is within the period of $0 < k < 400$. It is evident that a good estimation was achieved when there is no intermittent occurs. Moreover, a normal covariance behavior is also observed, similar to that seen in Figure 6.1. Figure 6.2 (b) depicts the estimation after the intermittent occurs at $k = 411$s for a period of 10 seconds. This figure indicates that the estimations of both the mobile robot and landmarks position are still possible even when the measurement data are lost for a few seconds, but with a certain degree of error in comparison to that of Figure 6.2 (a).

However, it is observed in Figure 6.2 (c) that the extended Kalman filter is capable in correcting the estimation upon regaining the measurement data. The accuracy of the estimation increased with respect to time. As shown in Figure 6.2 (d), after a period of 300 s, the mobile robot produces an estimation that is close to the actual landmark position. Besides, the final position of mobile robot in Figure 6.2 (d) is similar to that of normal EKF-based SLAM (Figure 6.1). This indicates that the mobile robot was able to localize its position and map the area.

Nonetheless, it can be observed in Figure 6.2 (c) and (d) that the covariance gradually decreases and quickly converges after the occurrence of intermittent with a lower value compared to that in the normal condition, as shown in Figure 6.3 on page 99. This indicates the behavior of an optimistic estimation. This shows that the analysis of the covariance behavior alone is insufficient to observe the actual performance of estimation in this case. This is demonstrated by an absolute error evaluation as shown in Figure 6.4 which clearly shows that the highest estimation error is produced during the period of intermittent, even though the covariance values are small. The figure shows the absolute error between the estimation of x and y coordinates and the true position of the mobile robot under the normal and intermittent condition. The results agree with Ahmad and Namerikawa (2011b) and prove that the position estimations are still possible, but with low accuracy.

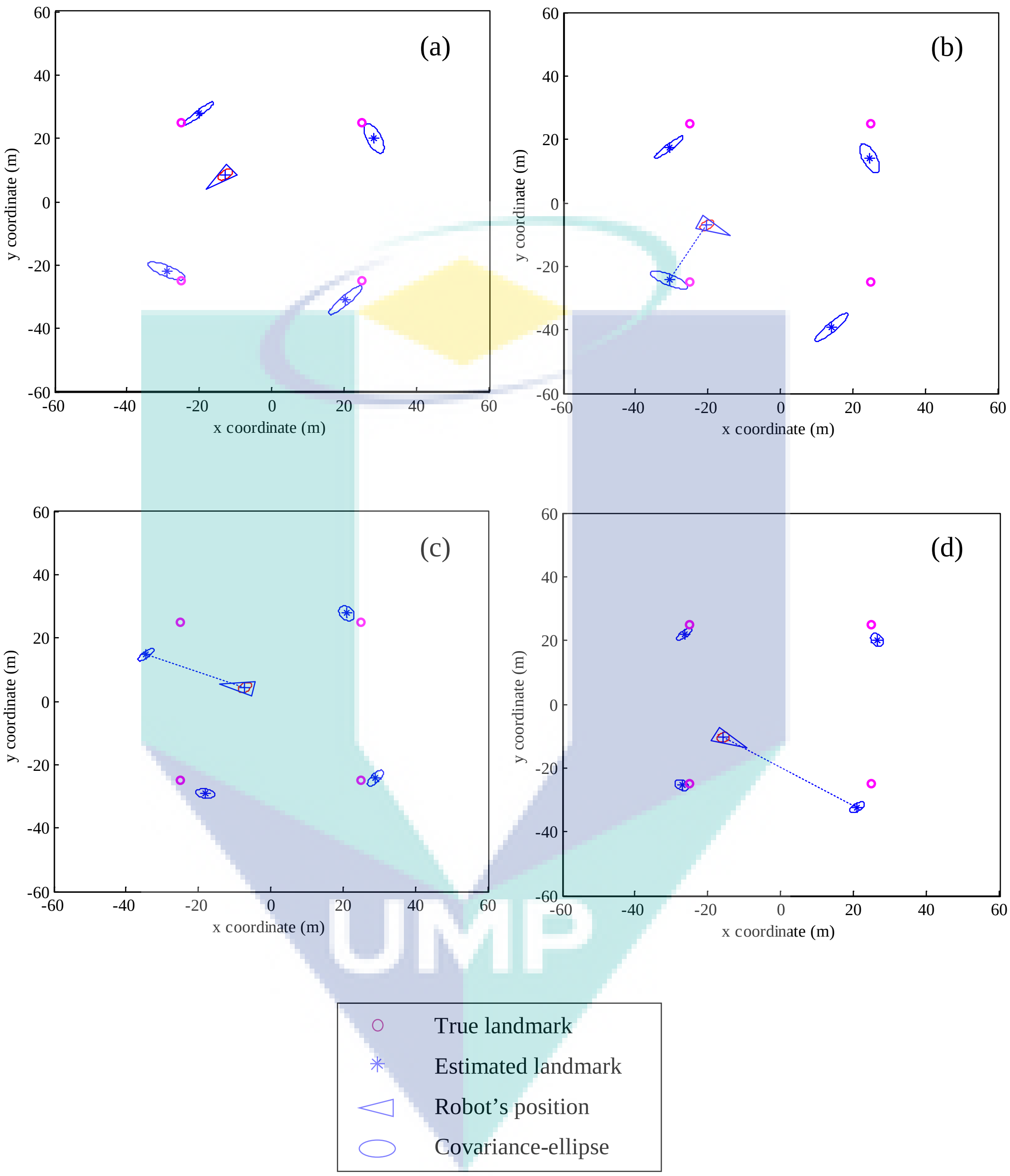

Figure 6.2. The estimation of mobile robot and landmark position with intermittent measurement occurred for 10 s at 401 < k < 411;

(a) before intermittent occurred at k = 400 s
(b) after measurement data unavailable at k = 411 s
(c) measurement data available again at k = 500 s
(d) estimations at k = 800 s

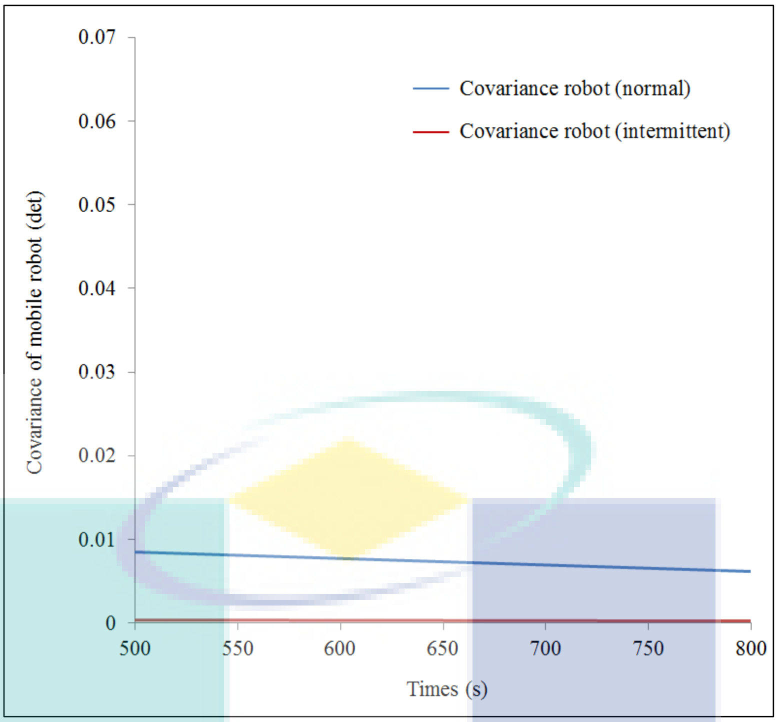

Figure 6.3. Error covariance of the estimation for both conditions.

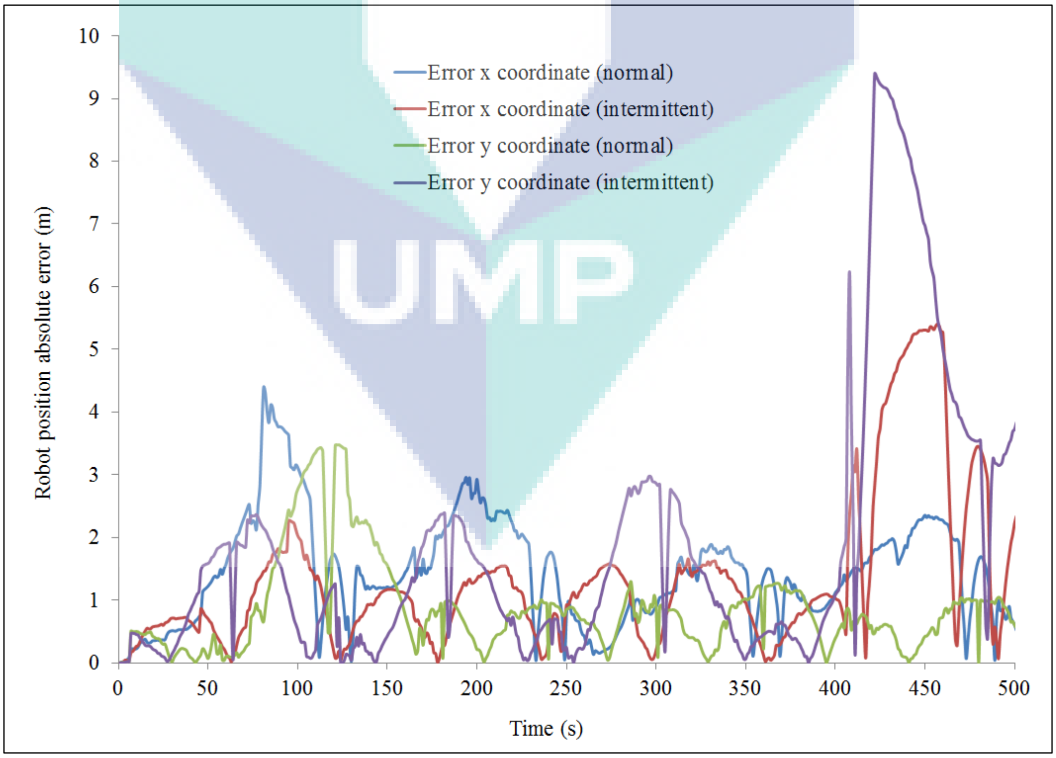

Figure 6.4. Absolute error of robot position, *x* and *y* coordinate under normal and intermittent condition.

6.6 Summary

From the analysis and simulation, it can be seen that the mobile robot is still capable of estimating its location and landmarks' position even when the measurement data are not available for a certain period of time. However, the estimation during this particular period is incorrect and the error covariance has decreased and converged quickly. The simulation results show that the EKF is capable of rectifying the estimation upon regaining the measurement data. However, it is impossible for the state error covariance to be corrected, and this will be the focus of the future study. This study is important to predict an actual situation of when an intermittent occurs in a mobile robot SLAM condition, as a basis for the correction and prevention steps such as in the controller development and improvement in algorithms.

CHAPTER 7

SUFFICIENT CONDITION H_∞ FILTER-BASED SLAM

This chapter discusses the lower boundary of the parameter gamma selection in the H_∞ filter-based SLAM. Two distinctive cases of the initial state covariance are analyzed considering an indoor environment to ensure the best solution for SLAM problem exists along with considerations of the process and measurement noises statistical behavior. If the prescribed conditions are not satisfied, then the estimation would exhibit unbounded uncertainties and consequently resulting in erroneous robot and landmarks estimation. The simulation results have shown the reliability and consistency as shown by the theoretical analysis and previous findings. A part of this chapter has been published in (Othman et al., 2015). This chapter discusses the work performed in achieving the fourth objective of the thesis.

7.1 Introduction

The extended Kalman filter (EKF) is often employed in determining the position of mobile robot and landmarks in simultaneous localization and mapping (SLAM). Nonetheless, there are some disadvantages of using EKF, namely, the requirement of Gaussian distribution for the state and noises, as well as the fact that it requires the smallest possible initial state covariance. This has urged researchers to find the alternative ways to counter the aforementioned shortcomings. Therefore, this study is conducted to propose an alternative technique by implementing H_∞ filter in SLAM instead of EKF since the distribution of noises might be unknown in certain condition. To implement the H_∞ filter in SLAM, the parameters of the filter especially γ needs to be properly defined to prevent the finite escape time problem, which has been discussed in Subsection 3.4.2.1. This study proposes a sufficient condition for the estimation purposes.

7.2 Related Work

In this chapter, H_∞ filter-based SLAM performance is further analyzed in extending the previous works (West and Syrmos, 2006, and Ahmad and Namerikawa, 2009b, 2010a, b). One of the earliest applications of this technique on SLAM was reported by West and Syrmos (2006). It has been proven that the H_∞ filter is an alternative solution for SLAM problem in an underwater application. The filter performance has been compared to that of the particle filter and extended Kalman filter. Although the particle filter has produced better estimation, H_∞ filter is deemed the best solution especially when the computational cost and non-Gaussian noise environments are taken into consideration. Further investigations were made on the filter convergence properties as reported by Ahmad and Namerikawa (2009b, 2010a, 2011c) and on the multi robot application (Wencen and Fumin, 2012).

Despite what H_∞ filter could offer in SLAM, the solution can unboundedly increase and exhibit finite escape time problem as reported by Bolzern et al. (1997), and Ahmad and Namerikawa (2009b), which is not the case of the extended Kalman filter. Therefore, to efficiently apply H_∞ filter in SLAM, the filter parameters must be properly designed to achieve the expected performance. Hence, several studies with regards to the filter characteristics have been conducted. Bolzern and Maroni (1999) discovered that H_∞ filter must also satisfy $P_0 = R^{-1}$ to achieve better estimation. A study of both filtering and prediction stages has been proposed and it was found that under the feasibility and sufficient conditions, the filter is able to achieve a stable result. Furthermore, Ahmad and Namerikawa (2010a) proposed the covariance inflation (Vidal-Calleja et al., 2004a, and Andrade-Cetto and Sanfeliu, 2006) and $\gamma -$switching strategy as an additional technique to prevent the occurrence of the finite escape time problem. Experimental results supported their analysis and demonstrated that the methods may alternatively prevent the problem. Motivated by the aforementioned works, further analyses of the H_∞ filter-based SLAM are proposed to gain more insights of the optimal conditions for estimation purposes.

7.3 Scope of Analysis

Based on the preliminary results obtained from the theoretical analysis and simulations (Ahmad and Othman, 2014), it is shown that if some conditions are satisfied, the H_∞ filter provides a better estimation while at the same time refraining the existence of finite escape time in the estimation. The results obtained are also in good agreement with the previous study conducted by Ahmad and Namerikawa (2010b). Nevertheless, it is worth to mention that there is also some trade-off between γ and the design parameters especially between the initial state covariance, process, and measurement noises distributions. Besides, as there are many types of environment available, two conditions of different initial state covariance are examined to understand its effect on SLAM with the consideration of the process and measurement noises distributions. The analyses are conducted to study the filter characteristics in different situations of environmental conditions. The analysis begins with proposing two feasibility conditions for H_∞ filter-based SLAM. Note that the initial state covariance is the dependent variable, whereas process and measurement noises are the independent variables.

7.4 Mathematical Formulation

The H_∞ filter algorithm shows approximately the same structure to the well-known extended Kalman filter (EKF). The presence of γ in the state error covariance owns the essence of H_∞ filter and acts as the main difference to EKF algorithm. The fundamental background of H_∞ filter has been discussed in Subsection 3.4.2. In this section, some of the important characteristics of H_∞ filter in SLAM are further explained.

H_∞ filter theoretically denotes that, for a given $\gamma > 0$, the filter attempts to find a solution for an estimated state $\hat{X}_k$ that satisfies the following criteria:

$$\gamma^2 > \sup \frac{\sum_{k=0}^{N-1} \left\| X_k - \hat{X}_k \right\|_{x_k}^2}{\left\| X_0 - \hat{X}_0 \right\|_{P_0^{-1}}^2 + \sum_{k=0}^{N-1} \left(\left\| w_k \right\|_{Q_k^{-1}}^2 + \left\| v_k \right\|_{R_k^{-1}}^2 \right)} \tag{7.1}$$

where $X_0, X_k \in \mathbb{R}^{3+2m}$ is the robot $(\in \mathbb{R}^3)$ and landmarks $(\in \mathbb{R}^{2m}, m = 1, 2, ..., N)$ states. w_k and v_k are the input noises, and Q_k and R_k are positive semidefinite matrices that define the process and measurement noise based on the specific conditions. The Eq. (7.1) alternatively means that the estimation error to the noise ratio is lower than a certain level of γ. This method also assumes that the noise distributions are statistically bounded. Note that the Eq. (7.1) is equivalent to Eq. (3.49), in which the matrix y_k is substituted with the desired state estimation of mobile robot SLAM, X_k.

From the Eq. (3.53) and under assumption of nonlinear SLAM, the state estimation vector is calculated using

$$\hat{X}_{k+1} = \nabla F_k \hat{X}_k + \nabla F_k K_k \left(z_k - \nabla H_k \hat{X}_k \right) + u_k \tag{7.2}$$

with associated state covariance of the estimation error is given by

$$P_{k+1} = \nabla F_k P_k \left[I_n - \gamma^{-2} P_k + \nabla H_k^T R_k^{-1} \nabla H_k P_k \right]^{-1} \nabla F_k^T + Q_k \tag{7.3}$$

where P_k is the state covariance of the previous state estimation and ∇F_k is the Jacobian transformation of the robot position. The Jacobian transformation is evaluated from the mobile robot model in Eq. (3.5) and landmarks model of Eq. (3.6) at the current state estimate $\hat{X}_k$. For $T = 1$ in a case of stationary landmarks, the following is obtained:

$$\nabla F_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\omega_k \sin\theta_k & 1 & 0 & 0 \\ \omega_k \cos\theta_k & 0 & 1 & 0 \\ 0 & 0 & 0 & I_m \end{bmatrix} \tag{7.4}$$

where I_m is an identity matrix with appropriate dimension with respect to the number of landmarks. Besides, the parameter K_k is defined as follows:

$$K_k = P_k\left(I - \gamma^{-2}P_k + \nabla H_k^T R_k^{-1} \nabla H_k P_k\right)^{-1} \nabla H_k^T R_k^{-1} \tag{7.5}$$

SLAM is a nonlinear system, thus, the Jacobian transformation of measurement between mobile robot position and any *i*-th landmark generally results in following function ∇H_k:

$$\nabla H_k = \begin{bmatrix} 0 & -\frac{dx}{r} & -\frac{dy}{r} & \frac{dx}{r} & \frac{dy}{r} \\ -1 & \frac{dy}{r^2} & -\frac{dx}{r^2} & -\frac{dy}{r^2} & \frac{dx}{r^2} \end{bmatrix} \tag{7.6}$$

where

$$\begin{aligned} dx &= x_i - x_k \\ dy &= y_i - y_k \\ r &= \sqrt{dx^2 + dy^2} \end{aligned} \tag{7.7}$$

7.5 Convergence Analysis

The parameters in H_∞ filter algorithm have a significant effect on the performance of the filter. Unlike the EKF, γ appears in the calculation of the state covariance matrix and parameter K_k; therefore, it should be taken into a serious consideration to ensure an accurate estimation. The presence of γ acts as an attenuator to reduce the system uncertainties during the mobile robot observation and can be adjusted by applying $\gamma-$switching technique (Bolzern et al., 1997). However, in this study, γ is fixed to find the sufficient conditions for a constant γ . If γ continuously changes, the result would be similar to that of Kalman filter behavior instead.

Based on H_∞ filter algorithm, the state covariance matrix of Eq. (7.3) can be generally simplified as

$$P_{k+1} = P_k + Q_k \tag{7.8}$$

Equation (7.8) shows that, in every update, the existence of the process noise Q_k has a significant effect on the state error covariance. The equation is also one of the factors that provide sufficient information in H_∞ filter. Thus, the H_∞ filter is sensitive to the initial state covariance, process, and measurement noises distributions as reported in the literature (Simon, 2001). Taking into account these variables, the analysis contributes adequately to explain the filter performance and consistency. As the H_∞ filter-based SLAM is still considered as new in mobile robot SLAM, the analytical results of the filter convergence are limited. Therefore, a theoretical study on the H_∞ filter convergence is essential.

Motivated by the aforementioned research gap, this study attempts to clarify the effect of initial condition of state covariance with the influence of the process and measurement noise distributions on the H_∞ filter behavior in SLAM. Two case studies were defined:

(i) Robot initial state error covariance is smaller than the landmarks initial state error covariance such that $P_{0rr} \ll P_{0mm}$

(ii) Robot initial state error covariance is greater than or equal to the landmarks initial state error covariance such that $P_{0rr} = P_{0mm}$

The first case shows that the robot has more confidence about its location than the landmark. This case relies on the assumption that robot has adequate proprioceptive sensors for the sensing purposes. The second case defines that both robot and landmarks initial state covariances are unknown. Generally, this is the case in the real SLAM application as usually no prior information is available for reference. Based on these two conditions, a theoretical study and analysis are performed to investigate their influence on SLAM problem. Parallel to the proposed cases, this study also investigates the effect of the process and measurement noises to the estimation.

The performance of H_∞ filter is sensitive and depends on the design parameters such as the process and measurement noises and the initial state covariance. The study continues to describe explicitly that the selection of design parameters should satisfy some conditions to ensure a better performance of the H_∞ filter in comparison to the EKF [see (Ahmad and Namerikawa, 2010a)]. Furthermore, there are certain trade-offs which are necessary between the design parameters to achieve the best solution in H_∞ filter-based SLAM.

7.5.1 Feasibility Conditions

Before presenting the main results, the feasibility conditions for H_∞ filter-based SLAM are proposed. Feasibility conditions are derived to aid the analysis of the two aforementioned case studies. Following definitions are redefined, since the Jacobian matrix is one of the essential parameter, to develop the proposed conditions. Note that Definition 7.1 is similar to Definition 4.2 on page 47.

Definition 7.1. *The Jacobian matrix of a mobile robot observing only one new landmark in its surrounding at point A and makes n observations is given by* (Huang and Dissanayake, 2007)

$$\nabla H_A = \begin{bmatrix} 0 & -\frac{dx_A}{r_A} & -\frac{dy_A}{r_A} & \frac{dx_A}{r_A} & \frac{dy_A}{r_A} \\ -1 & \frac{dy_A}{r_A{}^2} & -\frac{dx_A}{r_A{}^2} & -\frac{dy_A}{r_A{}^2} & \frac{dx_A}{r_A{}^2} \end{bmatrix} = \begin{bmatrix} -e & -A & A \end{bmatrix} \tag{7.9}$$

where

$$e = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad A = \begin{bmatrix} \frac{dx_A}{r_A} & \frac{dy_A}{r_A} \\ -\frac{dy_A}{r_A{}^2} & \frac{dx_A}{r_A{}^2} \end{bmatrix} \tag{7.10}$$

evaluated at the true landmark (x_m, y_m) *and the true robot position* (x_A, y_A) *and its elements are defined by*

$$dx_A = x_m - x_A$$

$$dy_A = y_m - y_A \tag{7.11}$$

$$r_A = \sqrt{dx_A{}^2 + dy_A{}^2}$$

The abovementioned definition is frequently used in this study.

Definition 7.2. *The state vector of mobile robot and landmarks location as described in Eq. (3.2) follows a Gaussian distribution in which it is represented by a mean and covariance of its elements:*

$$X_k \sim \mathcal{N}(x, P) \tag{7.12}$$

The covariance indicates the level of certainty of the mean estimation; the larger the covariance, the larger the uncertainty of the state estimation. Fisher information matrix (FIM) Ω_k specifies the weight of the information contained in a Gaussian distribution. In the case of a mobile robot SLAM, FIM indicates the information obtained by the mobile robot from each observation. FIM is the inverse of the state error covariance. Thus, the information obtained is inversely proportional to the uncertainty:

$$\Omega_k = P_k^{-1} \tag{7.13}$$

Fisher information matrix at time $k + 1$ is the summation of the information matrix at time k and the new information obtained from the observation, described as follows (Huang and Dissanayake, 2007):

$$\Omega_{k+1} = \Omega_k + \nabla H_k^T R_k \nabla H_k \tag{7.14}$$

The Fisher information matrix is used to determine the updated state error covariance of each update in this study. If the mobile robot starts moving from its initial position to point A and makes an observation at that point, then with respect to Definitions 7.1 and 7.2, the FIM yields the following equation:

$$\Omega_{k+1} = \begin{bmatrix} P_{0rr} & 0 \\ 0 & P_{0mm} \end{bmatrix}^{-1} + \begin{bmatrix} -H_A^T \\ A^T \end{bmatrix} R_A^{-1} \begin{bmatrix} -H_A & A \end{bmatrix} - \gamma^{-2} I_n$$

$$= \begin{bmatrix} P_{0_{rr}}^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} I_n & -H_A^T R_A^{-1} A \\ -A^T R_A^{-1} H_A & P_{0_{mm}}^{-1} + A^T R_A^{-1} A - \gamma^{-2} I_n \end{bmatrix} \tag{7.15}$$

with $H_A = \begin{bmatrix} e & A \end{bmatrix}$ and thus from Eq. (7.9) $\nabla H_A = \begin{bmatrix} -H_A & A \end{bmatrix}$. P_{0rr} and P_{0mm} are the initial state error covariance for robot and landmarks, respectively. The landmarks are assumed to be stationary, hence there are no noises affecting the prediction process of the landmarks state. Equation (7.15) is regarded as the feasible condition (Bolzern and Maroni, 1999). This condition is very important as it defines the amount of information available at that specific time. Before presenting the main results, the feasibility conditions for H_∞ filter-based SLAM are proposed.

Theorem 7.3. *With the consideration of Eq. (3.5), Eq. (3.6) and Eq. (7.3), the solution of the filter exists if it satisfies the feasibility conditions of* γ *for each case as stated below:*

(i) *if* $P_0 \gg R$, *then* $\gamma^2 > R$;

(ii) *if* $P_0 < R$, *then* $\gamma^2 > P_0$.

Proof. The feasibility conditions are analyzed separately in each case. To investigate these criteria, a one dimensional SLAM (1D SLAM) problem is considered, that is, a robot with a single coordinate system observing landmarks. It has been demonstrated that the behavior of 2D SLAM can be represented by 1D SLAM (Gibbens et al., 2000; Andrade-Cetto and Sanfeliu, 2005; Wijesoma et al., 2005; Perera et al., 2006; Perera et al., 2010). In the following sections, these conditions were implemented: $P_0 > 0$ and $\gamma > 0$.

7.5.1.1 Feasibility Condition 1

As stated in the Theorem 7.3, the first feasibility condition is defined as:
if $P_0 \gg R$, *then* $\gamma^2 > R$.

To ensure convergence, the covariance matrix should possess positive semidefinite (PSD) matrix properties of its elements (Ahmad and Namerikawa, 2010b). Since $\Omega_k = P_k^{-1}$ and $P_0 \gg R_A$, the first element of Eq. (7.15) has the following criteria:

$$P_0^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} > 0, \text{ therefore } P_0^{-1} > 0 \text{ and } H_A^T R_A^{-1} H_A - \gamma^{-2} > 0$$
$$\text{since } P_0 \gg R_A, \text{ hence } H_A^T R_A^{-1} H_A - \gamma^{-2} > P_0^{-1} \tag{7.16}$$

Note that the left and right hand sides of Eq. (7.16) still exhibit a positive semidefinite matrix. Examining the case for 1D SLAM (a mobile robot with a single coordinate, or Monobot) with $H_A = I_n$, from Eq. (7.16), the following equation is achieved:

$$R_A^{-1} - \gamma^{-2} > P_0^{-1} \tag{7.17}$$

Using this new proposed condition and considering that the updated information must be at least a PSD matrix,

$$\begin{aligned} R_A^{-1} - \gamma^{-2} &> 0 \\ R_A^{-1} &> \gamma^{-2} \\ \gamma^2 &> R_A \end{aligned} \tag{7.18}$$

7.5.1.2 Feasibility Condition 2

The second feasibility conditions defined in the Theorem 7.3 is as follows:
$if\ P_0 < R_A\ then\ \gamma^2 > P_0$.

Similar to the analysis in the previous subsection, the first element of FIM is examined to find the consequences of $P_0 < R_A$. Under the same assumption of PSD characteristics as described in Subsection 7.5.1.1 and similar assumption as in Eq. (7.16), the following results are obtained:

$$P_0^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} \ > \ P_0^{-1} - \gamma^{-2} \ > \ 0 \tag{7.19}$$

Hence, for 1D SLAM,

$$\begin{aligned} &\gamma^{-2} < P_0^{-1} \\ &\gamma^2 \ > P_0 \end{aligned} \tag{7.20}$$

Based on the proposed Theorem 7.3, additional conditions are required prior to the implementation stage to ensure that H_∞ filter estimation converges. Besides, the results have aided the previous findings significantly which are identified by Bolzern and Maroni (1999). Note that the initial state covariances for mobile robot and landmarks are considered similar in Theorem 7.3. With the appropriate conditions, the sufficient conditions for H_∞ filter-based SLAM convergence of each case defined previously are investigated. □

7.5.2 Effect of Initial State Covariance

The previous section explains that FIM is used to interpret the H_∞ filter behavior in each update. Since the main difference between H_∞ filter and Kalman filter is due to the existence of γ, in which the characteristics can be explicitly recognized by Eq. (7.15), the equation may be used in analyzing its significance and influence of design parameters on H_∞ filter performance.

7.5.2.1 Case Study 1

For the first case study, the following condition is proposed:

Case 1: $P_{0rr} \ll P_{0mm}$.

If $P_{0rr} \ll P_{0mm}$, the landmarks will contain high uncertainties. Thus, it is appropriate to assume that $P_{0_{mm}}^{-1} \to 0$. Thus, FIM can be expressed as follows:

$$\Omega_{k+1} = \begin{bmatrix} P_{0rr}^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} I_n & -H_A^T R_A^{-1} A \\ -A^T R_A^{-1} H_A & A^T R_A^{-1} A - \gamma^{-2} I_n \end{bmatrix} \tag{7.21}$$

Note that the diagonal elements are the essential elements for the designer to obtain some sufficient conditions in H_∞ filter, since each variable occupies the mobile robot and landmarks uncertainties. Hence, the smaller the values of these variables, the better the state estimation; this is the preferred case. Moreover, Ω must always preserve at least a positive semidefinite matrix in every observation. These are the necessary conditions to ensure a reliable estimation in H_∞ filter-based SLAM. To clearly illustrate this matter, the following proposition is presented. Let $P_{0rr(x)}$, $P_{0rr(y)}$ and $P_{0rr(\theta)}$ define x, y, θ robot initial state covariances.

Proposition 7.4. *For $\gamma > 0$, in a case of a robot that has more confidence about its initial state than the landmarks state, γ selection is affected by the initial state covariance, process, and measurement noises and its selection must satisfy the following properties:*

$$\begin{aligned} &\gamma > \sqrt{\frac{1}{P_{0rr(\theta)}^{-1} + R_A^{-1}}} \\ &\gamma > \sqrt{\frac{R_A(dx^2+dy^2)^2}{P_{0rr(x)}^{-1} R_A (dx^2+dy^2)^2 + dx^4 + dx^2dy^2 + dy^2}} \\ &\gamma > \sqrt{\frac{R_A(dx^2+dy^2)^2}{P_{0rr(y)}^{-1} R_A (dx^2+dy^2)^2 + dy^4 + dy^2dx^2 + dx^2}} \end{aligned} \tag{7.22}$$

Proof. Initially, the diagonal elements are analyzed. This consequently followed by determining the robot and landmarks information during the robot observations, which are represented by $P_{0rr}^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} I_n$ and $A^T R_A^{-1} A - \gamma^{-2} I_n$. The former equation can substantially explain the latter equation. This is shown by the following calculations:

$$
\begin{aligned}
P_{0rr}^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} I_n &= \begin{bmatrix} P_{0rr(\theta)}^{-1} & 0 \\ 0 & P_{0rr(xy)}^{-1} \end{bmatrix} + \begin{bmatrix} e^T \\ A^T \end{bmatrix} R_A^{-1} \begin{bmatrix} e & A \end{bmatrix} - \gamma^{-2} I_n \\
&= \begin{bmatrix} P_{0rr(\theta)}^{-1} + e^T R_A^{-1} e - \gamma^{-2} I_n & e^T R_A^{-1} A \\ A^T R_A^{-1} e & P_{0rr(xy)}^{-1} + A^T R_A^{-1} A - \gamma^{-2} I_n \end{bmatrix}
\end{aligned} \tag{7.23}
$$

where $P_{0rr(\theta)}$ and $P_{0rr(xy)}$ are the initial robot state error covariance about its angle and x, y position, respectively. Note that both diagonal elements must preserve PSD in each observation. In SLAM, the mobile robot heading angle acts as the primary factor that determine the consistency (Huang and Dissanayake, 2007). Hence, it should be analyzed differently with other elements of the state error covariance. As each diagonal matrix element must at least possess a PSD, then, for the robot heading angle covariance $P_{0rr(\theta)}$, the following characteristics are compulsory:

$$
\begin{aligned}
P_{0rr(\theta)}^{-1} + R_A^{-1} - \gamma^{-2} &> 0 \\
\gamma^2 &> \frac{1}{P_{0rr(\theta)}^{-1} + R_A^{-1}}
\end{aligned} \tag{7.24}
$$

The analysis is conducted for the state of robot and landmarks about its x, y estimations, that is, the second diagonal element. By utilizing Definition 7.1 and Eq. (7.15) being referred,

$$
\begin{aligned}
A^T R_A^{-1} A &= \begin{bmatrix} \dfrac{dx_A}{r_A} & -\dfrac{dy_A}{r_A{}^2} \\ \dfrac{dy_A}{r_A} & \dfrac{dx_A}{r_A{}^2} \end{bmatrix} R_A^{-1} \begin{bmatrix} \dfrac{dx_A}{r_A} & \dfrac{dy_A}{r_A} \\ -\dfrac{dy_A}{r_A{}^2} & \dfrac{dx_A}{r_A{}^2} \end{bmatrix} \\
&= \begin{bmatrix} \left(\dfrac{dx_A^2}{r_A^2} + \dfrac{dy_A^2}{r_A^4}\right) R_A^{-1} & \left(\dfrac{dx_A dy_A}{r_A^2} - \dfrac{dy_A dx_A}{r_A^4}\right) R_A^{-1} \\ \left(\dfrac{dy_A dx_A}{r_A^2} - \dfrac{dx_A dy_A}{r_A^4}\right) R_A^{-1} & \left(\dfrac{dy_A^2}{r_A^2} + \dfrac{dx_A^2}{r_A^4}\right) R_A^{-1} \end{bmatrix}
\end{aligned}
\tag{7.25}
$$

Based on this case, it is configurable that the robot has some degree of confidence about its initial location compared to the landmarks condition, which consists of very large initial state covariance. Hence, the inverse of the landmarks initial state covariance is approximately zero. Substituting Eq. (7.25) into the second diagonal term of Eq. (7.23) leads to the following expression:

$$
P_{0rr(x)}^{-1} + \left(\frac{dx_A^2}{r_A^2} + \frac{dy_A^2}{r_A^4}\right) R_A^{-1} - \gamma^{-2} > 0 \tag{7.26}
$$

$$
P_{0rr(y)}^{-1} + \left(\frac{dy_A^2}{r_A^2} + \frac{dx_A^2}{r_A^4}\right) R_A^{-1} - \gamma^{-2} > 0 \tag{7.27}
$$

With a simple algebraic rearrangement to determine γ sufficient conditions, the following is proposed to obtain a better estimation result:

$$
\gamma^2 > \frac{R_A (dx_A^2 + dy_A^2)^2}{P_{0rr(x)}^{-1} R_A (dx_A^2 + dy_A^2)^2 + dx_A^4 + dx_A^2 dy_A^2 + dy_A^2} \tag{7.28}
$$

$$
\gamma^2 > \frac{R_A (dx_A^2 + dy_A^2)^2}{P_{0rr(y)}^{-1} R_A (dx_A^2 + dy_A^2)^2 + dy_A^4 + dy_A^2 dx_A^2 + dx_A^2} \tag{7.29}
$$

This result describes that it is difficult to obtain an appropriate γ due to the nonlinear characteristics of the robot movement and noises. Nevertheless, the estimation also contains the process noise (refer to Eq. (7.8)); it is found that a larger process noise requires a large value of γ. Furthermore, the PSD characteristic is examined in each FIM update. It was found that FIM, which behaves as the information during observations, has provided a proper selection of γ to avoid the finite escape time phenomenon. □

Theorem 7.5. *Note that $\gamma > 0$ and Theorem 7.3 is satisfied. If the mobile robot initial state covariance is very small compared to the initial state covariance of landmarks, then γ is chosen to satisfy the following equations:*

(i) $\gamma > \sqrt{R}$

(ii) $\gamma > \sqrt{P_{0rr}}$

Else, the updated state error covariance exhibits the finite escape time.

Proof. From the properties of PSD, the determinant of the matrix must be positive. This behavior is necessary in the probabilistic SLAM. As a matter of fact, this criterion is used to obtain some typical features for γ selection. The determinant of Eq. (7.21) gives

$$\left(P_{0rr}^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} I_n\right)\left(A^T R_A^{-1} A - \gamma^{-2} I_n\right) - \left(H_A^T R_A^{-1} A A^T R_A^{-1} H_A\right) > 0 \qquad (7.30)$$

However, the above nonlinear equation is difficult to be used in explaining the effect of γ for every respective update. An analysis is proposed in linear 1D SLAM to visualize how γ influences the estimation. In 1D SLAM, the determinant (Eq. (7.30)) eventually becomes as follows:

$$\begin{aligned} \left(P_{0rr}^{-1} + R_A^{-1} - \gamma^{-2}\right)\left(R_A^{-1} - \gamma^{-2}\right) - R_A^{-2} &> 0 \\ P_{0rr}^{-1} R_A^{-1} - \gamma^{-2} P_{0rr}^{-1} - 2\gamma^{-2} R_A^{-1} + \gamma^{-4} &> 0 \qquad (7.31) \\ \gamma^{-4} - \left(P_{0rr}^{-1} + 2R_A^{-1}\right)\gamma^{-2} + P_{0rr}^{-1} R_A^{-1} &> 0 \end{aligned}$$

where $H_A = \begin{bmatrix} -1 & 1 \end{bmatrix}$ and $A = 1$. Furthermore, it is easily recognized that as $P_{0rr}, R_A > 0$, then the following are achieved:

$$\gamma^{-4} - \left(P_{0rr}^{-1} + 2R_A^{-1}\right)\gamma^{-2} + P_{0rr}^{-1}R_A^{-1} < \gamma^{-4} - \left(P_{0rr}^{-1} + R_A^{-1}\right)\gamma^{-2} + P_{0rr}^{-1}R_A^{-1} \tag{7.32}$$

$$\begin{gathered} \gamma^{-4} - \left(P_{0rr}^{-1} + R_A^{-1}\right)\gamma^{-2} + P_{0rr}^{-1}R_A^{-1} = \left(\gamma^{-2} - P_{0rr}^{-1}\right)\left(\gamma^{-2} - R_A^{-1}\right) \\ \left(\gamma^{-2} - P_{0rr}^{-1}\right)\left(\gamma^{-2} - R_A^{-1}\right) > 0 \end{gathered} \tag{7.33}$$

Hence, there exist two different cases with two respective conditions to satisfy Eq. (7.33):

(i) $\gamma > \sqrt{R_A}$ and $\gamma > \sqrt{P_{0rr}}$

(ii) $\gamma < \sqrt{R_A}$ and $\gamma < \sqrt{P_{0rr}}$

However, the above condition (ii) is unlikely to happen. This is related to Eq. (7.15) where this condition can yield a negative definite matrix. Therefore, condition (i) is apparently the only solution in this case. Moreover, the abovementioned analysis explicitly identifies the relationship between γ, initial state covariance, and measurement noise. In addition, the process noise covariance also influences the γ selection as it is included in the calculation of the state covariance matrix.

The results consistently show the same behavior as shown in Theorem 7.3 in which γ must be properly selected considering the environment and system situations; whenever the initial state error covariance for mobile robot is much smaller than landmarks covariance, then γ must be designed according to these two conditions. Note that Theorem 7.5 describes a case where the initial state covariance between the mobile robot and landmarks is different, whereas Theorem 7.3 considers that both initial state covariances have the same values. Furthermore, both feasible conditions proposed by Theorem 7.3 must be satisfied in this case.

In the navigation of mobile robot, the heading angle of mobile robot acts as an important factor to be considered in SLAM (Huang and Dissanayake, 2007). As proposed in Theorem 7.3 and Proposition 7.4, the designer must ensure that those feasibility conditions and Eq. (7.24) are fulfilled to successfully implement the filter. γ is selected by incrementally increasing its value with regards to the value defined in Theorem 7.3 and Proposition 7.4 to obtain the best solution. □

7.5.2.2 Case Study 2

Following condition has been defined for the second case study:

Case 2: $P_{0rr} = P_{0mm}$.

This condition is the appropriate situation of an actual SLAM problem. It is obvious that if a mobile robot is arbitrarily placed in an unknown environment, then it should not have the information of its initial location even though it is equipped with high accuracy sensors. Such situation presumes a uniform distribution of both robot and landmarks belief where both initial state covariances yield very high uncertainties. The following theorem is proposed to indicate H_∞ filter-based SLAM behavior in this particular case.

Theorem 7.6. *Consider that $\gamma > 0$ and Theorem 7.3 is satisfied. There is a γ that provides the best solution in SLAM which satisfies the following, if and only if both robot and landmarks initial state covariances are very large such that robot does not have any prior information about its initial position:*

$$\gamma > \sqrt{\frac{R}{2}} \tag{7.34}$$

Proof. Assume that both robot and landmarks initial state covariances are very large. By referring to the previous case, the determinant of the updated Fisher information matrix of Eq. (7.15) for a robot observing landmarks at point A yields the following:

$$\left(P_{0rr}^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} I_n\right)\left(P_{0mm}^{-1} + A^T R_A^{-1} A - \gamma^{-2} I_n\right) - \left(H_A^T R_A^{-1} A A^T R_A^{-1} H_A\right) > 0 \tag{7.35}$$

To simplify this analysis, the 1D SLAM problem is considered. It is initially known that in this case $P_{0rr} = P_{0mm} = P_0$ and both initial state covariances are very large. From this assumption and similar steps as in Eq. (7.31) – Eq. (7.33), Eq. (7.35) leads to the following equation:

$$
\begin{gathered}
\left(P_0^{-1} + R_A^{-1} - \gamma^{-2}\right)^2 - R_A^{-2} > 0 \\
\gamma^{-4} - 2\gamma^{-2}\left(P_0^{-1} + R_A^{-1}\right) + P_0^{-2} + 2P_0^{-1}R_A^{-1} > 0
\end{gathered}
\tag{7.36}
$$

Consider the aforementioned equation and the fact that $P_0 \gg 0$ and thus $(P_0^{-1} \to 0)$. Hence, by means of factorization, the following equation is obtained:

$$
\begin{gathered}
\gamma^{-4} - 2R_A^{-1}\gamma^{-2} > 0 \\
\gamma^{-2}(\gamma^{-2} - 2R_A^{-1}) > 0
\end{gathered}
\tag{7.37}
$$

Since in the H_∞ filter algorithm $\gamma > 0$, from Eq. (7.37), it appears that the left hand side variables should yield positive values to ensure that the solution of the estimation is available. Therefore, the following relation of γ and measurement noise is obtained:

$$
\begin{gathered}
\gamma^{-2} - 2R_A^{-1} > 0 \\
\gamma > \sqrt{\frac{R_A}{2}}
\end{gathered}
\tag{7.38}
$$

It is worth mentioning that the process noise still slightly affects the estimation if it is too large. If such conditions occur, then γ must be tuned carefully to achieve a desired estimation result. Referring back to the filter algorithm, the H_∞ filter estimation should be the same as EKF if a very large γ is defined. Related to this fact, a condition of γ where H_∞ filter produces a better performance than EKF is proposed. If finite escape time is seen, then γ must be tuned by incrementally increasing its value to obtain a better result. This is the common step in H_∞ filtering, providing similar estimation behavior to EKF. The next section discusses the validation of the theoretical results. □

7.6 Simulation Results

The proposed theoretical results obtained in the previous section were validated against a series of simulations. A small environment which has the parameters as described in Table 7.1 was considered. The parameters were selected based on experiments and analyses conducted in previous works (Ahmad and Namerikawa, 2009b, c, 2010a, 2011c; Ahmad and Othman, 2014). It is assumed that the robot exteroceptive sensors can observe their surrounding in a specific range and the process noises are small such that they can be neglected. The robot was assigned to move in some directions while performing the observations for 1000 s. The landmarks are assumed as point landmarks, stationary, and situated randomly as defined in Subsection 3.2.1. H_∞ filter-based SLAM was compared with the extended Kalman filter-based SLAM with regards to the map construction analysis, state error covariance update, and root-mean-square error (RMSE) evaluation in each case that has been analyzed in the previous section. Note that the process noises are consistently kept small in both cases to eliminate its impact. Moreover, it is also assumed that data association is available at all time.

Figures 7.1 and 7.2 illustrate the simulation results for the proposed feasibility condition stated by Theorem 7.3. To evaluate the reliability of the proposed feasibility conditions, the parameters in Table 7.1 are selected such that they satisfy the defined conditions. Figure 7.1 demonstrates the results of the estimation between H_∞ filter (HF) and extended Kalman filter (EKF) in feasibility condition 1 where the condition of $P_0 \gg R$ is considered. As illustrated in the figure, H_∞ filter shows a better performance than the extended Kalman filter with regards to the mobile robot and landmarks positions. The path of the robot was recorded for 1000 s, in which the mobile robot performed several observations.

The analysis of feasibility condition 2 is depicted in Figure 7.2. The figure indicates that, if the parameters selected correspond to the described condition, that is, $P_0 < R$, then the H_∞ estimation outperforms the extended Kalman filter. According to these results, if both of the feasible conditions are not satisfied, then the estimation diverges and produces erroneous results. Besides, the estimation emphasizes that finite

Table 7.1
Simulation parameters.

Parameter	Symbol	Value
Sampling time (s)	T	0.1
Process noise	Q_k	1×10^{-7}
Feasibility condition		
Feasibility condition 1		
Robot initial state covariance	P_{0rr}	10
Landmark initial state covariance	P_{0mm}	10
Measurement noise (angle)	$R_{k\theta}$	0.002
Measurement noise (distance)	$R_{k(\text{distance})}$	0.002
Selection of Gamma	γ	0.85
Feasibility condition 2		
Robot initial state covariance	P_{0rr}	2
Landmark initial state covariance	P_{0mm}	2
Measurement noise (angle)	$R_{k\theta}$	5
Measurement noise (distance)	$R_{k(\text{distance})}$	5
Selection of Gamma	γ	2.35
Case analysis		
Case 1		
Robot initial state covariance	P_{0rr}	2
Landmark initial state covariance	P_{0mm}	20
Measurement noise (angle)	$R_{k\theta}$	2
Measurement noise (distance)	$R_{k(\text{distance})}$	2
Selection of Gamma	γ	2
Case 2		
Robot initial state covariance	P_{0rr}	5
Landmark initial state covariance	P_{0mm}	5
Measurement noise (angle)	$R_{k\theta}$	2
Measurement noise (distance)	$R_{k(\text{distance})}$	2
Selection of Gamma	γ	2

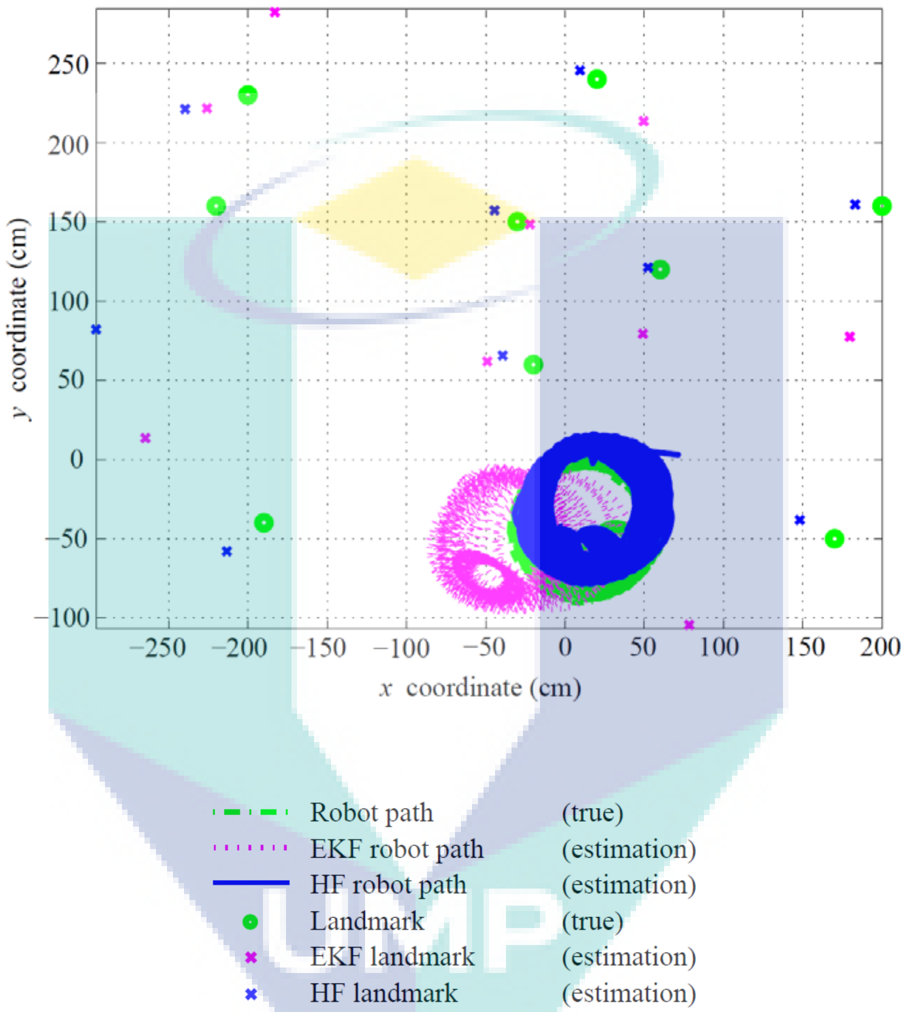

Figure 7.1. Feasibility condition 1: robot localization and map building performance between H_∞ filter and EKF.

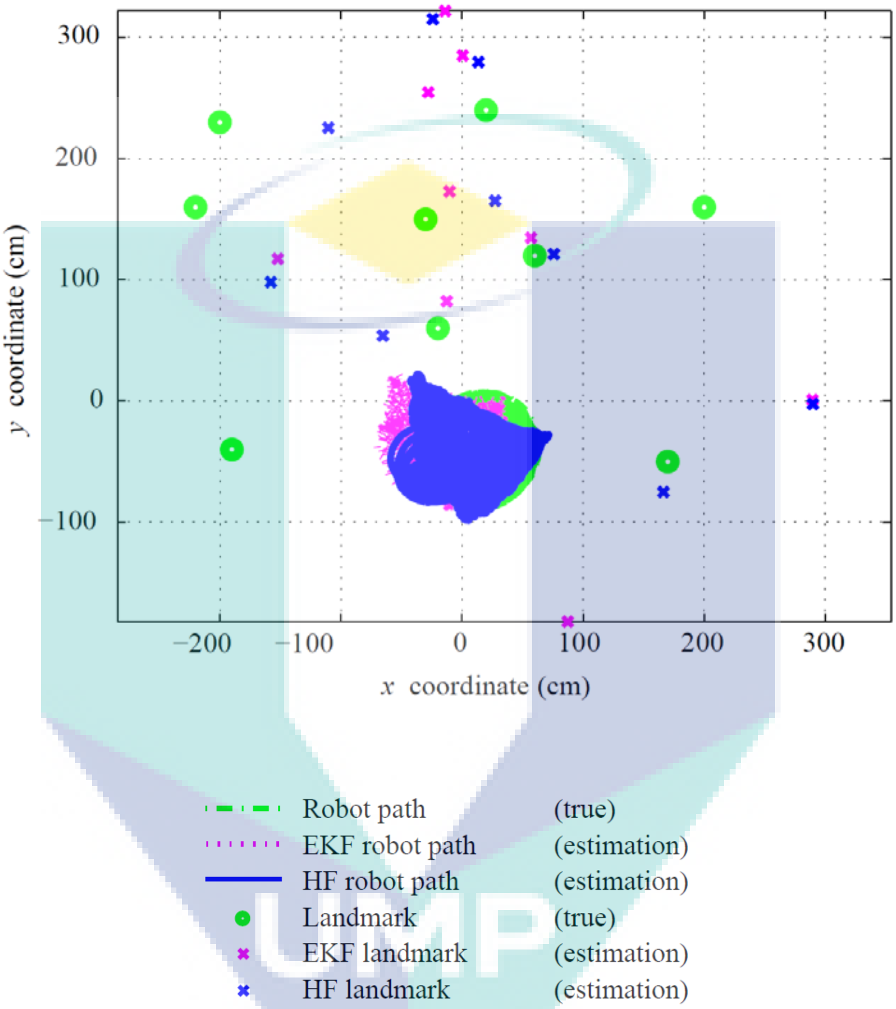

Figure 7.2. Feasibility condition 2: robot localization and map building performance between H_∞ filter and EKF.

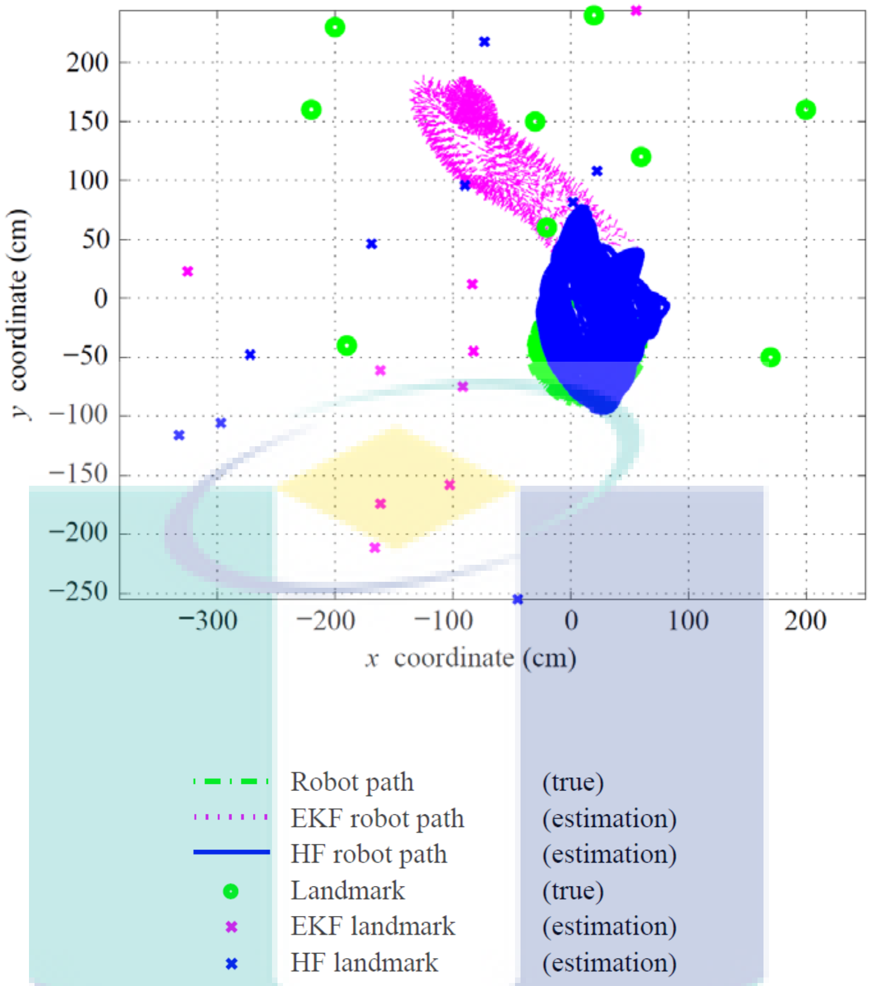

Figure 7.3. Case 1: map construction performance between H_∞ filter and EKF.

escape time might easily occur during mobile robot observation. These findings have been investigated in (Ahmad and Namerikawa, 2010b).

Figures (7.3) – (7.5) illustrate the simulation results of case 1 where the mobile robot has more confidence on its initial position in comparison to the landmarks state covariance; that is, $P_{0rr} \ll P_{0mm}$. It is apparent that the estimation of H_∞ filter outperforms the extended Kalman filter. Figure 7.3 depicts the result of mapping of both filters, while the state error covariances for the estimations are presented in Figure 7.4. RMSE evaluations of the landmarks estimations are presented in Figure 7.5. The mobile robot path estimation as well as the estimation of landmarks position in Figure 7.3 consistently shows that H_∞ filter provides a better estimation than extended Kalman filter. Figure 7.3 clearly depicts the erroneous estimation of extended Kalman filter

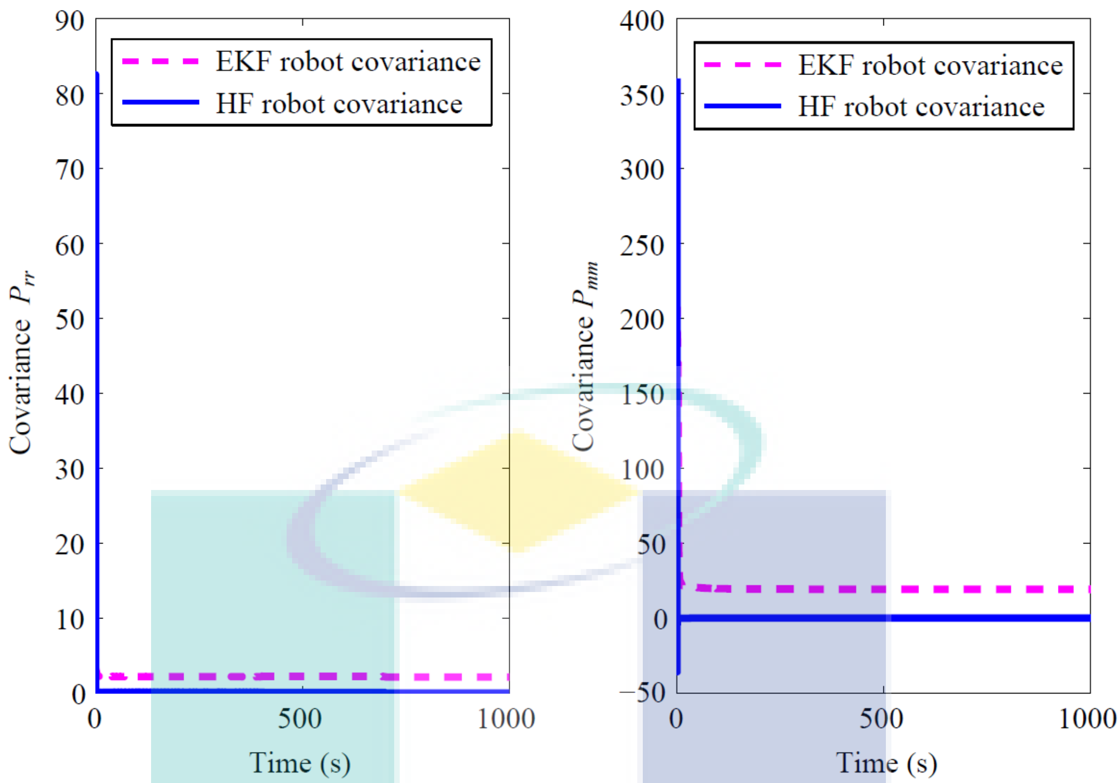

Figure 7.4. Case 1: state error covariance performance between H_∞ filter and EKF.

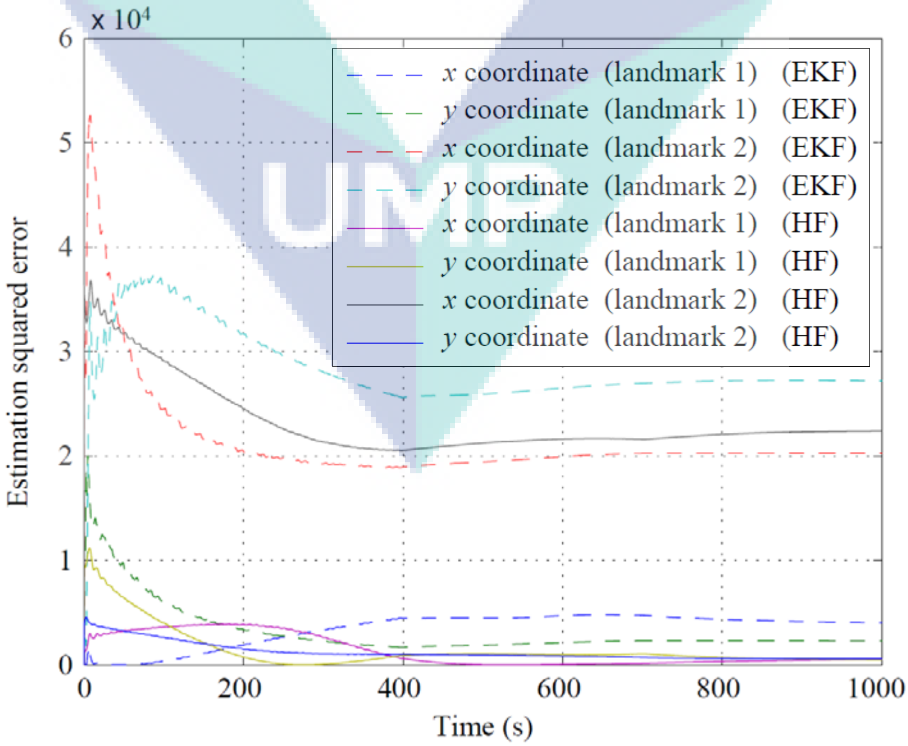

Figure 7.5. Case 1: RMSE performance between H_∞ filter and EKF.

through the path of the mobile robot. The findings are demonstrated in Figure 7.4, in which the state error covariance of extended Kalman filter has a higher value than H_∞ filter for both mobile robot and landmarks position. This denotes that the estimation using the extended Kalman filter possesses higher uncertainties in the conditions described by case 1. Based on the RMSE evaluation for the landmarks position in Figure 7.5, H_∞ filter also exhibits smaller error than extended Kalman filter. However, these results can only be available if and only if the condition of $\gamma > \sqrt{R}$ is satisfied.

Figures (7.6) – (7.8) illustrate the results of case 2 of the initial covariances of $P_{0rr} = P_{0mm} = 5$ for both robot and landmarks states. Similar findings are observed as described in Figures (7.3) – (7.5). The robot could estimate its current path and location with some level of certainty. The uncertainties of estimation proved that H_∞ filter still surpasses the extended Kalman filter performance as shown in Figure 7.7. Moreover, it is evident from Figure 7.8 that the RMSE evaluation of the landmarks position has similar characteristics to that of the case 1 (Figure 7.5). This shows that H_∞ filter can provide a better solution in SLAM problem, if and only if Theorem 7.6 is satisfied in each observation. However, if the proposed conditions are not fulfilled, then the estimation becomes erroneous as explained in the literature (Ahmad and Namerikawa, 2010b); in this case, the extended Kalman filter is superior.

Even though it is imperative to ensure that the conditions specified under Theorems 7.5 and 7.6 are satisfied, note that initial state covariance and process noise might influence the estimation. A larger process noise contributes to a bigger selection of γ as observed from Eq. (7.1). Similar pattern is also seen for the initial state covariance, in which a greater value of initial state covariance of heading angle may lead to an unexpected result (Huang and Dissanayake, 2007). Therefore, the designer should be aware of the importance of the parameter selection in addition to fulfilling the conditions of Theorems 7.3, 7.5, and 7.6.

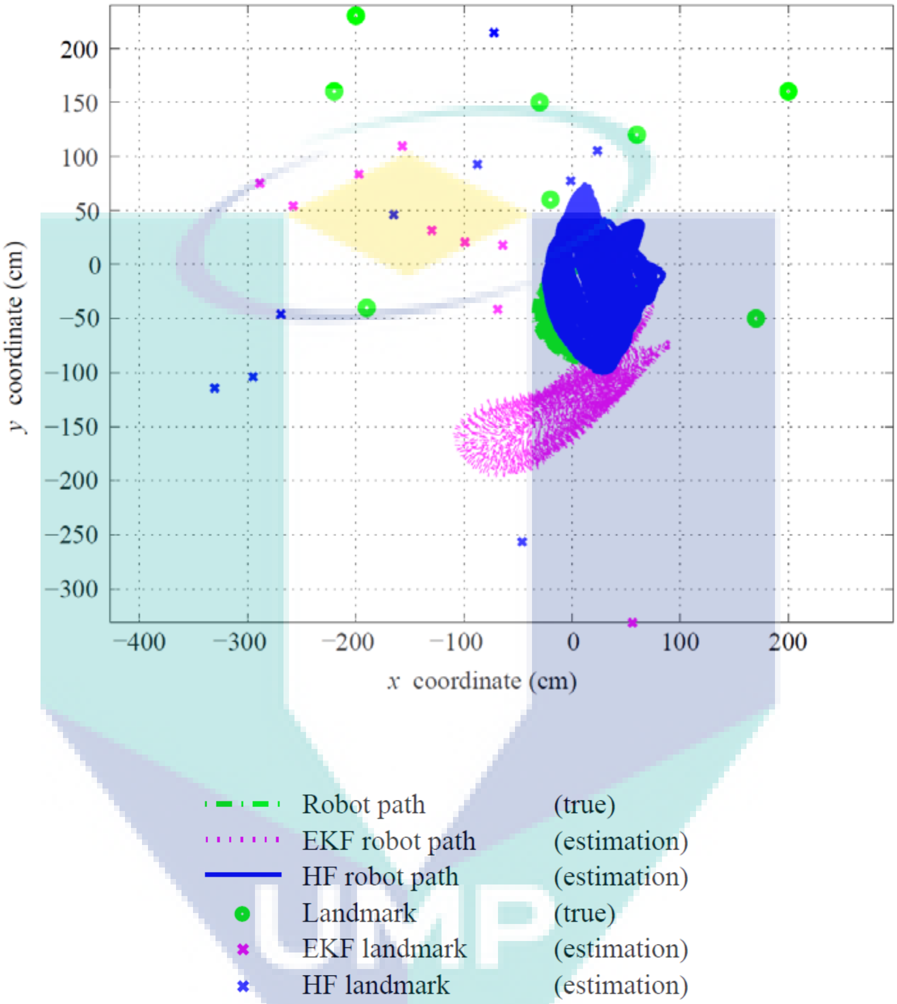

Figure 7.6. Case 2: map construction performance between H_∞ filter and EKF.

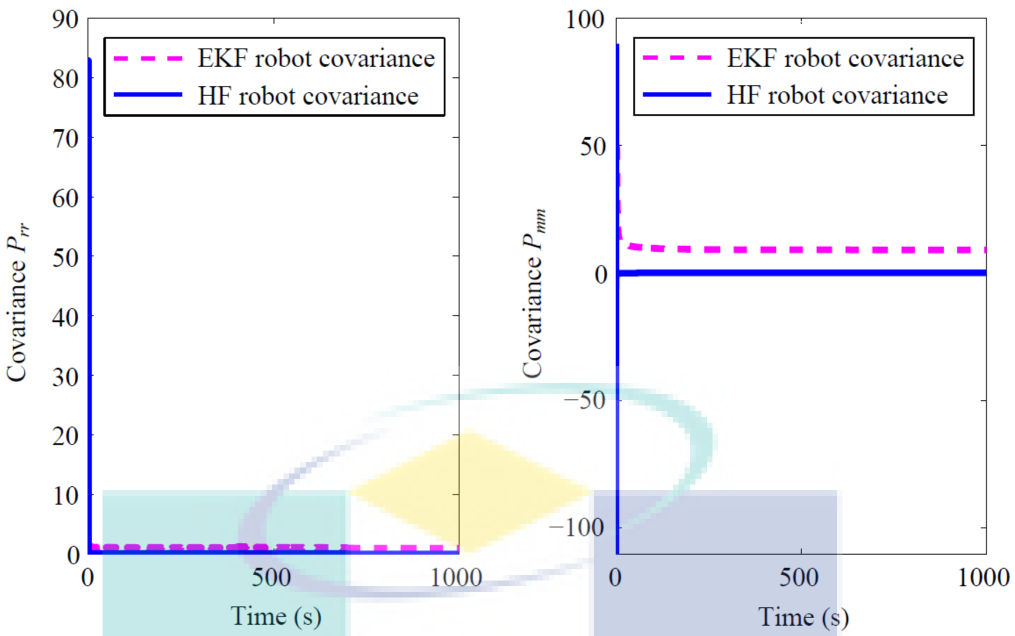

Figure 7.7. Case 2: state error covariance performance between H_∞ filter and EKF.

It is observed that the H_∞ filter estimations are superior to the extended Kalman filter even when it comes to the Gaussian noise environment with an appropriate selection of γ and other parameters design. Moreover, the results support the findings of (Ahmad and Namerikawa, 2009b) as the state error covariance update converges almost to zero in the estimation. In addition, it is also apparent that the extended Kalman filter estimation becomes more inconsistent as the initial state covariance becomes larger (Huang and Dissanayake, 2007). This is in contrast to the extended Kalman filter, where even if the initial state covariance has a much bigger value, the H_∞ filter still preserves a better estimation. To conclude, H_∞ filter based SLAM is one of the alternative solutions in SLAM especially for bigger initial state covariance and non-Gaussian noise environment.

7.7 Summary

This study demonstrates that the H_∞ filter may be considered as one of the best alternatives to overcome the navigation issues in SLAM especially in an environment with unknown noise characteristics. It is evident from the two case studies that the measurement noise must be less than γ^2 for a system with small process noise. Extra

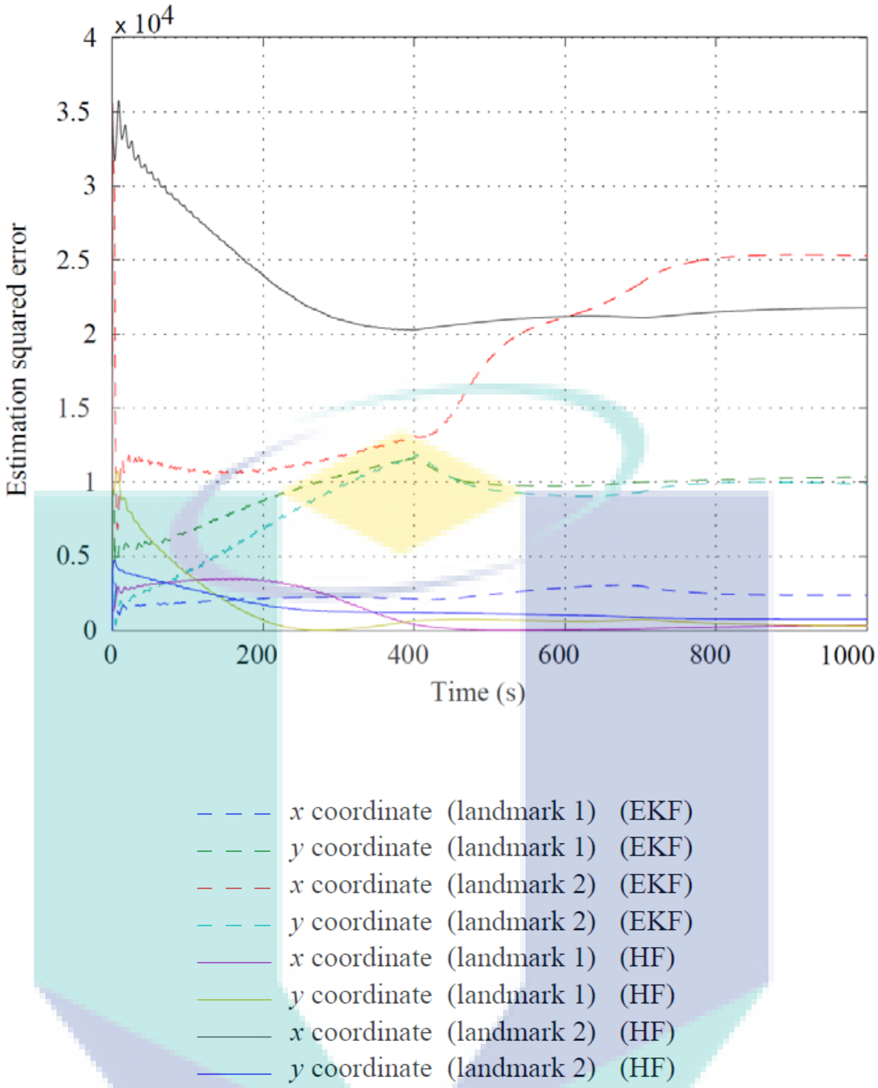

Figure 7.8. Case 2: RMSE performance between H_∞ filter and EKF.

attention should be given by the designers if both the initial state covariance and process noise are large, which consequently demands a bigger γ selection for the whole system to operate efficiently. However, to sufficiently achieve an expected performance in H_∞ filter, the designer must ensure that the aforementioned conditions are satisfied in their system design, taking into consideration the conditions of initial state covariance, process, and measurement noises distributions.

CHAPTER 8

CONCLUSION

This thesis attempts to investigate the behavior of estimator or observer in simultaneous localization and mapping of mobile robot under specific conditions. A thorough theoretical analysis on each issue is important as a guideline for the operators or users in developing a controller or other control strategies for the mobile robot SLAM. In this chapter, the contributions of this thesis for each research problems are summarized. Furthermore, a set of future directions for extending this work is also suggested. A detailed conclusion of each research problem is written at the end of each respective chapter.

8.1 Summary of Contributions

The contributions of this thesis can be summarized as follows:

8.1.1 The Impact of Cross-correlation Elements

In Chapter 4, the impact of cross-correlation elements of state covariance matrix on the estimation accuracy was investigated. To date, the mathematical analysis conducted is the first analysis that has been presented in such form. The results were in a good agreement with other studies that were conducted by means of experimental works or covariance ellipse analysis.

Moreover, the analysis was conducted based on the conditions that the mobile robot and the positions of the landmarks are located on the negative side of the global coordinate frame. In reality, the position of mobile robot and landmarks are not always located on the positive side and initially the mobile robot has no knowledge of its position. Due to these reasons, the analysis was proposed under the said condition in order to examine the effect of the negative position on the estimation, covariance behavior, and the Jacobian of the measurement matrix. This study accomplished the first objective of the thesis.

8.1.2 Diagonalization of State Covariance Matrix

In Chapter 5, a method to diagonalize the state covariance matrix through eigenvalues approach was presented. The diagonalization process attempts to simplify the structure of the state covariance matrix in order to reduce the computational cost in EKF-based SLAM. The study suggested that diagonalization method through eigenvalues might be one of the approaches to achieve this goal. The simulation results prove that this technique could be implemented, however more modification on the algorithm should be done to ensure the estimation and covariance behavior are correct. Besides, the cross-correlation terms should be taken into account since it is important to ensure the estimation accuracy. This is a novel approach that focuses on the EKF-based nonlinear SLAM. The analyses conducted conclude the second objective of the thesis.

8.1.3 Intermittent Measurement Condition in SLAM

In Chapter 6, the analysis of mobile robot SLAM under an intermittent measurement condition was conducted. The analysis provides important information, in which the state estimation of mobile robot position and landmarks are still possible and can be corrected by extended Kalman filter in the situation of sudden unavailability of the measurement data. This finding proves that the extended Kalman filter is capable in handling uncertainties in EKF-based SLAM. However, the state covariance matrix was unable to be corrected and it was too optimistic. The state covariance matrix remains lower than the actual values. Such behavior of the covariance matrix and state estimation under occurrence of sudden unavailability of measurement data has never been reported before. This work completes the third objective of the thesis.

8.1.4 H_∞ Filter-based SLAM

The requirement of Gaussian distribution characteristics for the process and measurement noises, as well as the need for the smallest possible initial state covariance matrix limits the Kalman filter to be successfully operated in various types of environment. Therefore, H_∞ filter could be one of the alternatives to be employed in the mobile robot SLAM within an environment with unknown distribution. However, H_∞ filter requires a proper tuning of the parameters, especially γ in order to obtain best performance. Therefore in Chapter 7, a guideline in choosing the right value of γ based on predefined conditions was provided. The lower boundary of the γ selection was defined depending on the value of initial state covariance matrix of the mobile robot and the landmarks. This concludes the final objective of the thesis.

8.2 Future Research Directions

8.2.1 The Impact of Cross-correlation Elements

The simulation results in Chapter 4 showed that the position of the mobile robot during its observation with the reference to its initial position has significant effect on the updated state covariance. The changes of movement from positive to negative direction may influence the findings. Particularly, this occurs due to the calculation during the mobile robot measurement model, which may influence the Jacobian matrix. This has been discovered through simulation analysis, and therefore requires further investigation and mathematical proves to corroborate the finding.

8.2.2 Diagonalization of State Covariance Matrix

Diagonalization technique through eigenvalues approach could be implemented in diagonalizing state covariance matrix in EKF-based SLAM. However, the cross-correlation terms should be integrated into the diagonal elements. As a future work, a technique to integrate the cross-correlation elements in diagonalization process should be introduced, or through a numerical analysis method. In the future, this work will be extended by numerically analyzing each element of the state covariance matrix to identify a specific pattern that generates the elements. Hence, a new equation for a diagonal state covariance matrix could be introduced.

8.2.3 Intermittent Measurement Condition in SLAM

The study has proven that the state estimation can be corrected by the extended Kalman filter as the measurement data become available. However, the state covariance matrix remains uncorrected. This finding should be investigated further to find the reason behind the inability of covariance matrix to be fixed. More analysis should be conducted on the derivation of covariance matrix in EKF-based SLAM in the intermittent measurement. Hence, a possible solution towards the problem may be suggested.

8.2.4 H_∞ Filter-based SLAM

In Chapter 7 the lower boundary of the γ selection was defined based on the specified conditions. However, it is hypothesized that there exists an upper boundary of the γ selection, in which should also be determined. Generally, if the selected γ is too large, the H_∞ filter will behave similarly as the Kalman filter. The value of γ needs to be properly selected based on the environment conditions of the system. In mobile robot SLAM, the conditions are always changing, depending on the type of onboard sensors, the structure of the ground plane, and the behavior of the landmarks. As a future work, the upper boundary of the γ for the conditions specified in Chapter 7 will be proposed.

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APPENDIX

A LIST OF PUBLICATIONS

A.1 Journals

1) "Sufficient Condition for Estimation in Designing H_∞ Filter-Based SLAM," *Mathematical Problems in Engineering*, vol. 2015, Article ID 238131, 14 pages, Hindawi Publishing, 2015. DOI:10.1155/2015/238131.

   (ISI-indexed, 2014 Impact Factor = 0.762)

2) "The Impact of Cross-correlation on Mobile Robot Localization," *International Journal of Control, Automation, and Systems*, vol. 13, no. 5, Springer, pp. 1251-1261, 2015. DOI:10.1007/s12555-014-0076-6.

   (ISI-indexed, 2014 Impact Factor = 0.954)

A.2 Conference Proceedings

1) "Analysis of Intermittent Measurement for KF-based SLAM," 2nd International Conference on Electrical, Control and Computer Engineering (InECCE 2013), Malaysia (Pahang), August 2013.

2) "The Importance of Cross-correlation and its Effect to Mobile Robot Localization," 2nd International Conference on Electrical, Control and Computer Engineering (InECCE 2013), Malaysia (Pahang), August 2013.

3) "The Analysis of Covariance Matrix for Kalman Filter based SLAM with Intermittent Measurement", 2013 International Conference on Systems, Control and Informatics (SCI2013), Italy (Venice), September 2013.

4) “The Effect of Intermittent Measurement in Simultaneous Localization and Mapping”, 2013 IEEE International Conference on Control System, Computing and Engineering (ICCSCE 2013), Malaysia (Penang), November 2013.

5) “Estimation Behavior of Intermittent Measurement in EKF Based SLAM,” IEEE Symposium on Industrial Electronics & Applications (ISIEA 2014), Malaysia (Kota Kinabalu), September 2014.

6) “An Approach to Reduce Computational Cost for Localization Problem,” Colloquium on Robotics, Unmanned Systems and Cybernetics 2014 (CRUSC 2014), Malaysia (Pahang), November 2014.

7) “Analyzing the Mobile Robot Localization Performance in Partially Observable Conditions,” IEEE Symposium on Industrial Electronics & Applications (ISIEA 2014), Malaysia (Kota Kinabalu), September 2014.

8) “An Analysis of γ Effects to H_∞ Filter-based Localization,” Colloquium on Robotics, Unmanned Systems and Cybernetics 2014 (CRUSC 2014), Malaysia (Pahang), November 2014.