Numerical Solutions of the Aligned Magnetic Field on the Boundary Layer Flow and Heat Transfer over a Stretching Sheet by using Keller-box Method

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Abstract—In this study, the influence of aligned magnetic field on the steady boundary layer flow and heat transfer over a stretching sheet together with Newtonian heating is considered. The transformed governing nonlinear boundary layer equations in the form of ordinary differential equations are solved numerically by Keller-box method. The numerical solutions of the applied magnetic field with different values of aligned angle, α_1 on the velocity and temperature profile are presented graphically. It is found that, increases in aligned angle retarded the value of fluid's velocity and increases the value of fluid's temperature.
Keywords—boundary layer; heat transfer; aligned magnetic field; stretching sheet

1. INTRODUCTION

The study on fluid flow fluid over a stretching sheet is important in many industrial processes and it has been discover in last few decades. One of them in extrusion processes. The production of sheeting material arises in a number of industrial manufacturing processes and it's includes both metal and polymer sheets (Vajravelu and Mukhopadhyay [1]). The quality of the final product are depends on the rate of heat transfer at the stretching surface. Due to these important applications, Sakiadis [2] first investigated the boundary layer flow on a continuous solid surface moving at constant speed. Sakiadis's theoretical predictions for Newtonian fluids were later verified experimentally by Tsou et al. [3]. Crane [4] was the first who has studied the forced convection boundary layer flow over a stretching sheet. The heat and concentration distribution were obtained by Gupta and Gupta [5] for isothermal moving plate with suction or blowing. The heat transfer for laminar boundary layer of linearly, continuous sheet subject to suction or blowing are studied by Chen and Char [6]. The numerical investigation has been done by Salleh et al. [7] and Qasim et al. [8] for laminar boundary layer flow and heat transfer over a stretching sheet with Newtonian heating using Keller-box method and Runge-Kutta Fehlberg fourth-fifth order method respectively. Very recently, the analytic solutions for Micropolar fluid flow and heat transfer is obtained by Turkyilmazoglu [9] at a permeable stretching sheet.

The study of magnetic field has been carried out by numerous researches due to its important in engineering applications such as Magnetohydrodynamic (MHD) generator, nuclear reactor for liquid-metal cooling and electromagnetic casting. The exact similarity solution was discovered by Pavlov [10] for problem on MHD boundary layer flow of an electrically conducting fluid of uniform transverse magnetic field over an elastic plane surface. In conjunction of this, Chakrabarti and Gupta [11] investigate the temperature distribution in the presence of uniform suction to the stretching sheet. Dutta [12] obtained the exact solution for temperature distribution in an electrically conducting fluid of steady boundary layer flow with uniform magnetic field over a stretching sheet. Meanwhile, the exact similarity transformation of steady electrically conducting fluid with uniform magnetic field for MHD flow past a stretching sheet were obtained by Anderson et al. [13] for inelastic power-law while Anderson [14] for viscoelastic fluid. A numerical analysis of unsteady magnetohydrodynamic for free convective heat and mass transfer in a micropolar fluid over vertical stretching sheet was proposed by Aurangzaib et al. [15]. It is worth to mention here, the above past studies focused only on the transverse magnetic field normal to the plate.

Recently, the effect of aligned angle associated with magnetic field on the boundary layer flow problem has been attracting the researchers' interest. A magnetic field is applied with an acute angle ($0° - 90°$) to the flow region. The influence of aligned magnetic field and radiation are being discussed by Sandeep and Sugunamma [16] for unsteady free convective flow of dissipative fluid past a vertical plate while Raju et al. [17] has been focus on the problem of steady forced convection flow of ferrofluids that moving over a flat plate. An analytical solution was obtained by Kalaivanan et al. [18] for the steady Casson fluid with velocity slip boundary condition past a stretching sheet. The boundary layer flow of nanofluid over exponentially stretching sheet in porous medium was solved numerically by Sulochana et al. [19].

The aim of the present paper is to investigate the influence of aligned magnetic field on the steady boundary layer flow and heat transfer past a stretching sheet with Newtonian heating. The computation of the problem is done by numerical approach which is called Keller-box method.

2. PROBLEM FORMULATION

The steady, incompressible two dimensional forced convection boundary layer flows over a stretching sheet with Newtonian heating (NH) is considered. An aligned magnetic field with an acute angle α_1 is applied to the flow as in Fig. 1 and the boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma}{\rho} u B_1^2 \sin^2 \alpha_1 \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

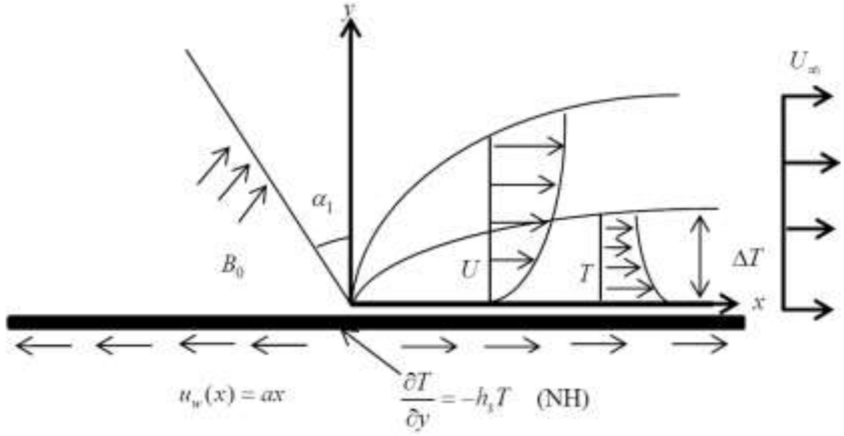

Figure 1: Flow configuration.

with the boundary conditions

$$u = u_w(x) = ax, \quad v = 0, \quad \frac{\partial T}{\partial y} = -h_s T(\text{NH}) \text{ at } y = 0$$
$$u \to 0, \quad T \to T_\infty \text{ as } y \to \infty \tag{4}$$

where $u_w(x)$ is the velocity of the stretching surface with a being a positive constant, α_1 is the inclined angle, σ is electrical conductivity, h_s is the heat transfer parameter, T is the fluid temperature, T_∞ is the ambient temperature, ν is the kinematic viscosity, α is the thermal diffusivity, $B_1 = B_0 x^{-1/2}$ is the transverse magnetic field with B_0 is the magnetic-field strength.

Introducing the similarity transformation on (1)-(3), subject to the boundary conditions (4)

$$\eta = \left(\frac{a}{\nu}\right)^{1/2} y,\ \psi = (a\nu)^{1/2} f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_\infty} \text{ (NH)}, \tag{5}$$

where ψ is the stream function defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, yield

$$f'''(\eta) + f(\eta) f'(\eta) - f'^2(\eta) - M f'(\eta)\sin^2 \alpha_1 = 0 \tag{6}$$

$$\frac{1}{\Pr}\theta''(\eta) + f\theta'(\eta) = 0 \tag{7}$$

with boundary conditions

$$f(0) = 0,\quad f'(0) = 1,\quad \theta'(0) = -\gamma\,[1 + \theta(0)] \text{ (NH)},$$
$$f'(\eta) \to 0,\ \ \theta(\eta) \to 0 \text{ as } \eta \to \infty, \tag{8}$$

where $M = \frac{\sigma B_0^2}{\rho a}$ is the magnetic field parameter, $\Pr = \frac{\nu}{\alpha}$ is the Prandtl number and $\gamma = h_s(\nu / a)^{1/2}$ is the conjugate parameter for Newtonian heating.

According to Crane [4] the exact analytic solution for (6) is expressed as

$$f(\eta) = 1 - e^{-\eta},\ f'(\eta) = e^{-\eta},\ \text{and } f''(\eta) = -e^{-\eta} \tag{9a}$$

If $f(\eta)$ is given by exact solution (9), then the exact expression for temperature profile is

$$\theta(\eta) = C_1 \int_\eta^\infty e^{-\Pr\int_\eta^\infty f\, d\eta} d\eta, \tag{9b}$$

where $C_1 = \frac{\gamma(1-\theta(0))}{e^{-\Pr\int_\eta^\infty f\, d\eta}}$

The non-dimensional quantities of physical interest in this problem are the skin friction coefficient C_f and the local Nusselt number Nu_x are defined by

$$C_f \operatorname{Re}_x^{1/2} = f''(0),\ \ Nu_x \operatorname{Re}_x^{-1/2} = \gamma\left(1 + \frac{1}{\theta(0)}\right), \tag{10}$$

where $\operatorname{Re}_x = (ax^2 / \upsilon)$ is the Reynolds number.

3. METHODOLOGY

The finite difference method is also known as Keller-box method was initiated by Keller [20] and further developed by Keller and Cebeci [21, 22]. This numerical approach is completely efficient, accurate and stable for nonlinear boundary layer problem. In this method, the nonlinear governing equations are first transformed to first order system. The first order system is then approximated using central difference. Since the system is nonlinear equation, the Newton's method is applied to linearize the system and finally, the solutions can be solved by block elimination technique. The calculations are executed in Matlab.

A. First Order System

The partial differential equations (6) and (7) subjects to the boundary conditions (8) are reduced to a first order system. For that matter, the new independent variables are introduced as

$$f' = u,\ u' = v,\ \ s' = t \tag{10}$$

where $(')$ is derivative with respect to η. The transformed (6) and (7) can be written as

$$v' + fv - u^2 - M\sin^2 \alpha_1 u = 0 \tag{11}$$

$$\frac{1}{\Pr}t' + ft = 0 \tag{12}$$

and the boundary conditions (8) become

$$\begin{gathered} f(0) = 0,\ \ u(0) = 1,\ \ t(0) = -\gamma[1 + s(0)] \\ u(\eta) \to 0,\ s(\eta) \to 0 \text{ as } \eta \to \infty \end{gathered} \tag{13}$$

B. Finite Difference Scheme

The net rectangle in the η plane is considered as shown in Fig. 2. The net points are denoted as

$$\begin{gathered} \eta_0 = 0,\qquad \eta_j = \eta_{j-1} + h_j,\qquad j = 1,2,...,J, \\ \eta_J \equiv \eta_\infty, \end{gathered} \tag{14}$$

where h_j is the $\Delta\eta_j$-spacing and j is the sequence of numbers that indicate the coordinate location.

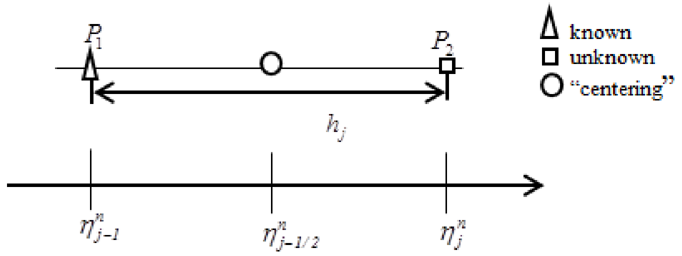

Figure 2: Net rectangle for difference approximations.

The finite difference forms for any points are

$$(\ )^n_{j-1/2} = \frac{1}{2}\left[(\ )^n_j + (\ )^n_{j-1}\right], \tag{15}$$

$$\left(\frac{\partial u}{\partial \eta}\right)^{n-1/2}_{j-1/2} = \frac{u_j^{n-1/2} - u_{j-1}^{n-1/2}}{h_j}, \tag{16}$$

Equations (10) – (12) are approximated by using the central difference at mid-point $\eta^n_{j-1/2}$ of the segment P_1P_2. Thus, the following are obtained

$$f_j - f_{j-1} - \frac{h_j}{2}(u_j + u_{j-1}) = 0, \tag{17}$$

$$u_j - u_{j-1} - \frac{h_j}{2}(v_j + v_{j-1}) = 0, \tag{18}$$

$$s_j - s_{j-1} - \frac{h_j}{2}(t_j + t_{j-1}) = 0, \tag{19}$$

$$v_j - v_{j-1} + \frac{h_j}{4}(f_j + f_{j-1})(v_j + v_{j-1}) - \frac{h_j}{4}\left(u_j + u_{j-1}\right)^2 - \frac{h_j}{2} M \sin^2 \alpha_1 (u_j + u_{j-1}) = \left(R_1\right)^{n-1}_{j-1/2}, \tag{20}$$

$$t_j - t_{j-1} + \frac{h_j}{4}\Pr(f_j + f_{j-1})(t_j + t_{j-1}) = \left(R_2\right)^{n-1}_{j-1/2}, \tag{21}$$

where

$$\left(R_1\right)^{n-1}_{j-1/2} = -h_j\left[\frac{v_j - v_{j-1}}{h_j} + f_{j-1/2}v_{j-1/2} - \left(u_{j-1/2}\right)^2 - M\sin^2\alpha_1(u_{j-1/2})\right]^{n-1},$$

$$\left(R_2\right)^{n-1}_{j-1/2} = -h_j\left[\frac{t_j - t_{j-1}}{h_j} + \Pr f_{j-1/2}t_{j-1/2}\right]^{n-1}.$$

Equations (17) - (21) are computed for $j = 1, 2, \ldots J$ at the given n and the boundary conditions (13) become

$$f_0^n = 0,\ u_0^n = 1,\ t_0^n = -\gamma[1 + s_0^n],\ u_J^n = 0, \quad \text{and } s_J^n = 0. \tag{22}$$

C. Newton's Method

The system of nonlinear equations (17) - (21) is linearized by considering the following iterates

$$f_j^{(i+1)} = f_j^{(i)} + \delta f_j^{(i)},\ u_j^{(i+1)} = u_j^{(i)} + \delta u_j^{(i)},\ v_j^{(i+1)} = v_j^{(i)} + \delta v_j^{(i)},$$
$$s_j^{(i+1)} = s_j^{(i)} + \delta s_j^{(i)},\ t_j^{(i+1)} = t_j^{(i)} + \delta t_j^{(i)}. \tag{23}$$

For simplicity, the superscript i from iterates are eliminated and the higher order terms for $\delta f_j^{(i)}, \delta u_j^{(i)}, \delta v_j^{(i)}, \delta s_j^{(i)}, \delta t_j^{(i)}$ are dropped, the system of equations (24) is obtained

$$\delta f_j - \delta f_{j-1} - \frac{1}{2}h_j\left(\delta u_j + \delta u_{j-1}\right) = (r_1)_{j-1/2},$$

$$\delta u_j - \delta u_{j-1} - \frac{1}{2}h_j\left(\delta v_j + \delta v_{j-1}\right) = (r_2)_{j-1/2},$$

$$\delta s_j - \delta s_{j-1} - \frac{1}{2}h_j\left(\delta t_j + \delta t_{j-1}\right) = (r_3)_{j-1/2},$$

$$\left(a_1\right)_j \delta v_j + \left(a_2\right)_j \delta v_{j-1} + \left(a_3\right)_j \delta f_j + \left(a_4\right)_j \delta f_{j-1} + \left(a_5\right)_j \delta u_j + \left(a_6\right)_j \delta u_{j-1} = \left(r_4\right)_{j-1/2},$$

$$\left(b_1\right)_j \delta t_j + \left(b_2\right)_j \delta t_{j-1} + \left(b_3\right)_j \delta f_j + \left(b_4\right)_j \delta f_{j-1} = \left(r_5\right)_{j-1/2},$$

where

$$
\begin{aligned}
&(a_1)_j = 1 + \frac{1}{2} h_j f_{j-\frac{1}{2}}, \qquad (a_2)_j = (a_1)_j - 2, \\
&(a_3)_j = \frac{1}{2} h_j v_{j-\frac{1}{2}}, \qquad (a_4)_j = (a_3)_j, \\
&(a_5)_j = -h_j u_{j-\frac{1}{2}} - \frac{h_j}{2} M \sin^2 \alpha_1, \quad (a_6)_j = (a_5)_j.
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
&(b_1)_j = 1 + \frac{h_j}{2} \Pr f_{j-\frac{1}{2}}, \quad (b_2)_j = (b_1)_j - 2, \\
&(b_3)_j = -1 + \frac{h_j}{2} \Pr t_{j-\frac{1}{2}}, \quad (b_4)_j = (b_3)_j.
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
&(r_1)_{j-1/2} = f_{j-1} - f_j + h_j u_{j-1/2}, (r_2)_{j-1/2} = u_{j-1} - u_j + h_j v_{j-1/2}, \\
&(r_3)_{j-1/2} = s_{j-1} - s_j + h_j t_{j-1/2}, \\
&(r_4)_{j-1/2} = -v_j + v_{j-1} - h_j \left[f_{j-1/2} v_{j-1/2} - (u_{j-1/2})_{j-\frac{1}{2}} - M \sin^2 \alpha_1 (u_{j-1/2}) \right] + (R_1)_{j-\frac{1}{2}}, \\
&(r_5)_{j-1/2} = -t_j + t_{j-1} - h_j \left[\Pr f_{j-1/2} t_{j-1/2} \right] + (R_2)_{j-1/2}.
\end{aligned} \tag{27}
$$

According to Cebecci and Bradshaw [23], the boundary condition (23) is fully satisfied with no iteration. Therefore, to maintain these correct values in all the iterates, let

$$\delta f_0 = 0,\ \delta u_0 = 0,\ \delta t_0 = 0,\ \delta u_J = 0 \text{ and } \delta s_J = 0. \tag{28}$$

D. Block Elimination Technique

The linearized system of non-linear differential equations (24) has a block- tridiagonal structure which then can be solved by block elimination technique. It can be written in the block matrix form as

$$[A][\delta] = [r] \tag{29}$$

where

$$
\begin{bmatrix}
[A_1] & [C_1] & & & \\
[B_2] & [A_2] & [C_2] & & \\
 & & \ddots & & \\
 & & \ddots & & \\
 & & \ddots & & \\
 & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\
 & & & [B_J] & [A_J]
\end{bmatrix}
\begin{bmatrix}
[\delta_1] \\ [\delta_2] \\ \vdots \\ \vdots \\ \vdots \\ [\delta_{J-1}] \\ [\delta_J]
\end{bmatrix}
=
\begin{bmatrix}
[r_1] \\ [r_2] \\ \vdots \\ \vdots \\ \vdots \\ [r_{J-1}] \\ [r_J]
\end{bmatrix}.
$$

The elements of the matrices are

$$
[A_1] = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 \\
-\frac{1}{2}h_1 & 0 & 0 & -\frac{1}{2}h_1 & 0 \\
0 & -\frac{1}{2}h_1 & 0 & 0 & -\frac{1}{2}h_1 \\
(a_2)_1 & 0 & (a_3)_1 & (a_1)_1 & 0 \\
0 & (b_2)_1 & (b_3)_1 & 0 & (b_1)_1
\end{bmatrix}, \tag{30}
$$

$$
[A_j] = \begin{bmatrix}
-\frac{1}{2}h_j & 0 & 1 & 0 & 0 \\
-1 & 0 & 0 & -\frac{1}{2}h_j & 0 \\
0 & -1 & 0 & 0 & -\frac{1}{2}h_j \\
(a_6)_j & 0 & (a_3)_j & (a_1)_j & 0 \\
0 & 0 & (b_3)_j & 0 & (b_1)_j
\end{bmatrix}, \quad 2 \le j \le J, \tag{31}
$$

$$\left[B_j\right]=\begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2}h_j & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2}h_j \\ 0 & 0 & (a_4)_j & (a_2)_j & 0 \\ 0 & 0 & (b_4)_j & 0 & (b_2)_j \end{bmatrix}, \quad 2 \le j \le J, \tag{32}$$

$$\left[C_j\right]=\begin{bmatrix} -\frac{1}{2}h_j & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ (a_5)_j & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad 2 \le j \le J \tag{33}$$

$$[\delta_1]=\begin{bmatrix} \delta v_0 \\ \delta s_0 \\ \delta f_1 \\ \delta v_1 \\ \delta t_1 \end{bmatrix}, \quad [\delta_j]=\begin{bmatrix} \delta u_{j-1} \\ \delta s_{j-1} \\ \delta f_j \\ \delta v_j \\ \delta t_j \end{bmatrix}, \quad 2 \le j \le J \tag{34}$$

and $[r_j]=\begin{bmatrix} (r_1)_{j-1/2} \\ (r_2)_{j-1/2} \\ (r_3)_{j-1/2} \\ (r_4)_{j-1/2} \\ (r_5)_{j-1/2} \end{bmatrix}, \quad 1 \le j \le J.$ (35)

If $\boldsymbol{A}$ is nonsingular, then (29) can be factored as

$\left[A\right]=\left[L\right]\left[U\right]$,

$$=\begin{bmatrix} [\alpha_1] & & & & \\ [B_2] & [\alpha_2] & & & \\ & & \ddots & & \\ & & \ddots & [\alpha_{J-1}] & \\ & & & [B_J] & [\alpha_J] \end{bmatrix}\begin{bmatrix} [I] & [\Gamma_1] & & & \\ & [I] & [\Gamma_2] & & \\ & & \ddots & \ddots & \\ & & & [I] & [\Gamma_{J-1}] \\ & & & & [I] \end{bmatrix} \tag{36}$$

where $[I]$ is the identity matrix of order 5, $\left[\alpha_i\right]$ and $\left[\Gamma_i\right]$ are 5×5 matrices which elements can be expressed as follows

$$\left[\alpha_1\right]=\left[A_1\right], \tag{37}$$

$$\left[A_1\right]\left[\Gamma_1\right]=\left[C_1\right], \tag{38}$$

$$[\alpha_j]=[A_j]-[B_j][\Gamma_{j-1}], \quad j=2,3,\ldots,J, \tag{39}$$

$$[\alpha_j][\Gamma_j]=[C_j], \quad j=2,3,\ldots,J-1. \tag{40}$$

Substituting (36) into (29) yield,

$$\left[L\right]\left[U\right]\left[\delta\right]=\left[r\right]. \tag{41}$$

Equation (41) becomes

$$\left[L\right]\left[W\right]=\left[r\right], \tag{42}$$

by defining

$$\left[U\right]\left[\delta\right]=\left[W\right], \tag{43}$$

where

$$W = \begin{bmatrix} [W_1] \\ [W_2] \\ \vdots \\ [W_{J-1}] \\ [W_J] \end{bmatrix},$$

and $[W_j]$ are 5 × 1 column matrices. From (42), the solution for elements $\boldsymbol{W}$ is obtained

$$[\alpha_1][W_1] = [r_1], \tag{44}$$

$$[\alpha_j][W_j] = [r_j] - [B_j][W_{j-1}], \quad 2 \le j \le J. \tag{45}$$

The forward sweep is performed to find the elements of $\boldsymbol{W}$. Then, the elements δ is calculated in backward sweep by the following relations

$$[\delta_J] = [W_J], \tag{46}$$

$$[\delta_j] = [W_j] - [\Gamma_j][\delta_{j+1}], \quad 1 \le j \le J-1. \tag{47}$$

These calculations are repeated until some convergence criterion is satisfied. Calculations are stopped when

$$\left|\delta v_0^{(i)}\right| < \varepsilon_1, \tag{48}$$

where $\varepsilon_1 = 10^{-7}$ is a too small fixed value.

4. Results and Discussion

The system of non-linear ordinary differential equations (6) and (7) with respect to boundary conditions (8) are solved numerically using the Keller-box method and the results are presented in tabular as well as graphical form. To validate the present results the comparison with previously published results and exact solution has been made for various values of Prandtl number by neglecting the magnetic field $(M = 0)$ and aligned angle $\alpha_1 = 0$ at $\gamma = 1$ (see Table 1). It is noticed from Table 1, the present results are in a good agreement with the previously published results. Therefore, the authors confident the numerical results obtained are accurate and precise for the present problem.

Table 1: Comparative results

Pr	$\boldsymbol{\theta(0)}$			
	Salleh et al. [7]	**Exact equation (9b)**	**Turkyilmazoglu [9]**	**Present**
3	6.02577	6.051585	6.05158546	6.025776
5	1.76594	1.760395	1.76039543	1.765941
7	1.13511	1.116815	1.11681524	1.135107
10	0.76531	0.764524	0.76452369	0.765312

The current results presented in this paper are limited to distribution of velocity and temperature for various values of aligned angle only at fixed values of $\Pr = 1$, $\gamma = 0.5$ and $M = 1$. Fig. 3 and 4 illustrated the effects of aligned angle α_1 on distribution of velocity and temperature of fluid respectively. At $\alpha_1 = 0$ the flow is in the absence of magnetic field while $\alpha_1 = \pi/2$ indicating the transverse magnetic field on the flow region. It is clearly shown from Fig.3 that increasing value of aligned angle $\alpha_1 = 0, \pi/6, \pi/4, \pi/3, \pi/2$ lead to increase the fluid's temperature profile. Meanwhile, the fluid's velocity decrease with the increasing values of aligned angle. Physically, the increasing in values of α_1 strengthens the applied magnetic field in the flow region which lead to the enhancement of Lorentz forced.

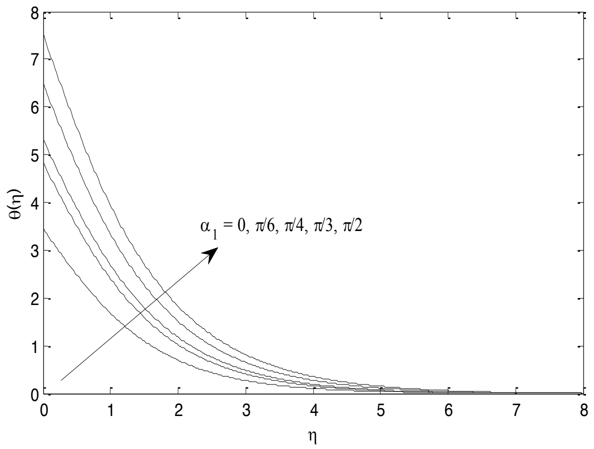

Figure 3: Temperature profile for various value of α_1

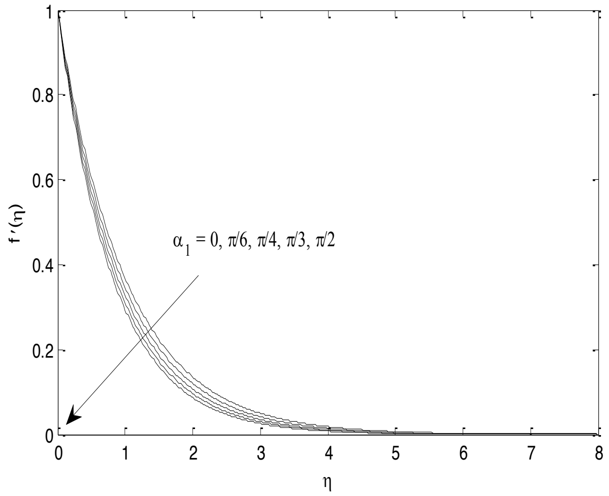

Figure 4: Velocity profile for various value of α_1

5. CONCLUSION

The numerical analysis on the effect of aligned magnetic field of a Newtonian Fluid flow over a stretching sheet with Newtonian heating is discussed in the present study. It is observed that increase in aligned angle enhances the magnetic field which leads to decline the fluid's velocity and increase the fluid's temperature.

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