A Mixed Integer Linear Programming (MILP) Model for Optimal Design of Water Network

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Abstract- **This work presents the development of a new systematic technique to target fresh water consumption and wastewater generation for systems involving multiple contaminants when all options of water minimization including source elimination, reduction, reuse/recycle, outsourcing and regeneration are considered simultaneously. This problem is formulated as mixed integer linear programming (MILP) and implemented in Generalized Algebraic Modeling System (GAMS). The consideration of process changes will lead to optimal design of minimum water utilization network. The MILP model proposed in this work can be used to simultaneously generate the minimum water targets and design the minimum water network for global water-using operations for buildings and industry. The approach is illustrated by using an industrial involving a chlor-alkali plant. Significant water savings for the industrial case study is achieved, illustrating the effectiveness of the proposed approach.**

Keywords- mathematical model, design, minimum water network, water management hierarchy, MILP

I. INTRODUCTION

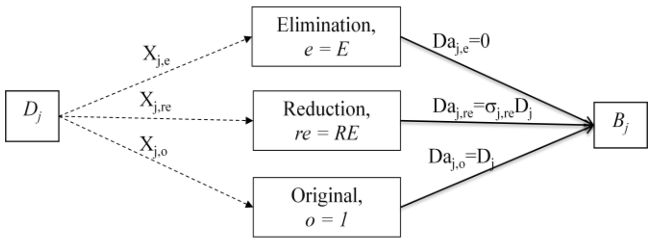
Increasing cost of raw water and effluent treatment, stringent environmental regulations and shortage of raw water have encouraged extensive water conservation efforts through design of maximum water recovery (MWR) networks. In general, water pinch analysis and mathematical modeling approaches have been employed to generate MWR design and maximize opportunities for water reuse and recycling for urban and industrial facilities through water system integration. On the other hand, the minimum water targets can only be achieved when all possible methods are employed to holistically reduce fresh water consumption through elimination, reduction, outsourcing and regeneration. A systematic water reductions technique through water management hierarchy (WMH) was introduced by Wan Alwi and Manan [1] to give new insight in process modification and its application was further demonstrated in Wan Alwi et al. [2]. The process changes are systematically implemented in terms of priority through a clear guidance. However, the tedious graphical and heuristics procedures have limitation when handling large scale and complex problems. In addition, the graphical technique is not applicable for system involving multiple contaminants. Consequently, the development of a new systematic approach to design an optimal water networks by using mathematical programming technique involving multiple contaminants is proposed in this work to overcome the limitations of previous works. The models are capable of predicting which water source should be eliminated or reduced; how much external source is needed; which wastewater source should be reused/recycled, regenerated or discharged, and finally specify the minimum water network configuration to achieve the water targets

II. MINIMUM WATER NETWORK

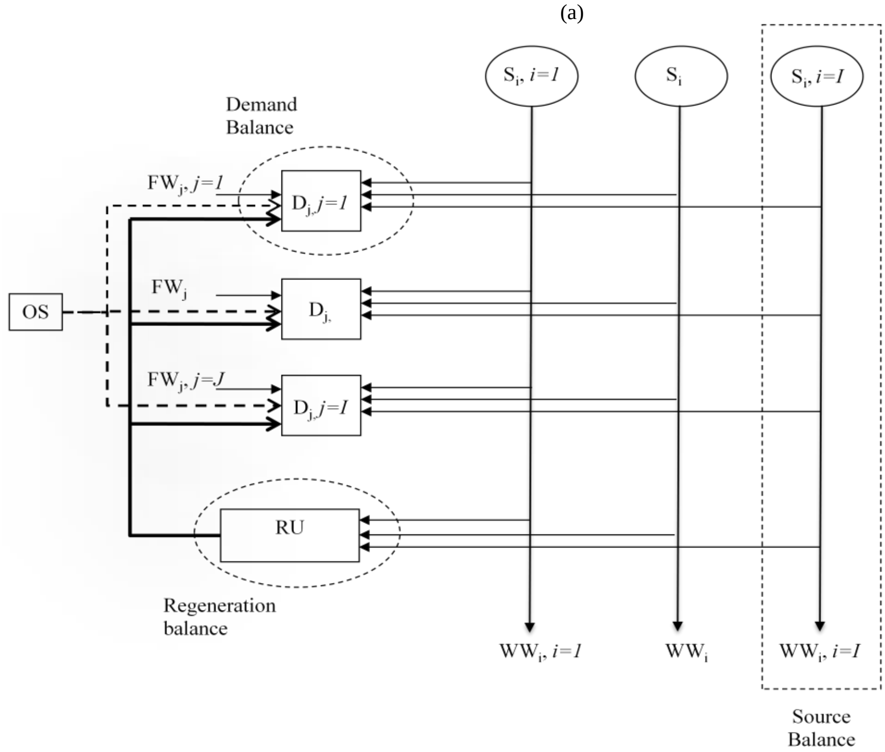
Minimum water network (MWN) design is not only considered reuse and recycling but all conceivable methods to systematically and holistically reduce fresh water consumption through elimination, reduction, reuse/outsourcing, and regeneration [1]. In this work, all the water management schemes are considered simultaneously in order to obtain minimum water targets.

A. Superstructure Representation for Minimum Water Network (MWN)

The representative superstructure includes all possible options for water minimization based on the water management hierarchy (WMH). The MWN superstructure is a combination of superstructures in Figure 1(a) and (b). Fig. 1(a) shows the superstructure on how to obtain the adjusted demand flow rate, B_j when source elimination and reduction are considered. $X_{j,e}$, $X_{j,re}$ and $X_{j,o}$ is a binary or selection variables for the selection of elimination, reduction and original options. While $Da_{j,e}$, $Da_{j,re}$ and $Da_{j,o}$ denotes the flow rate for elimination, reduction or original water demand. The adjusted demand flow rate, B_j is depending on the selections of these options. It is important to note that only one option can be selected at one time. Fig. 1(b) represents all possible connections among water sources, water demands and wastewater discharges as well as outsourcing and regeneration options. For each water operation, the water demand, B_j can be supplied by fresh water, FW_j, outsourced resources, *OS* (e.g rainwater, river and melted snow), reused/recycled water, or regenerated water from regeneration unit, *RU*. While at the water source, A_i, the generated wastewater may be directly discharged to the end-of-pipe treatment, WW_i, or reused in the same or different processes or partially treated in the regeneration unit, *RU* before being reused/recycled. In this case, superstructure of every possible configuration of a water-using network is allowed.



(a)



(b)

Fig. 1 General superstructure for a minimum water utilization network with WMH options that considers both MTB and NMTB operations (a) Water network superstructure to obtain the adjusted demand flow rate, B_j when possible source elimination and reduction are considered. (b) Water network superstructure for maximum water recovery that includes outsourcing and regeneration options.

B. Formulation of MILP Model

In this case, the objective is to minimize fresh water target which leads to minimum wastewater generation. The flow rates and concentrations of water sources and demands can be changed in order to reduce the MWR targets and ultimately attain the MWN benchmark.

Objective function: The objective function can be written as:

$$Min\sum_j FW_j \qquad (1)$$

The minimization of the objective functions in equation (1) is subject to the following constraints.

1) Demand Constraint: Adjusted demand flow rate, B_j is equal to the given demand flow rate after selections of elimination, $D_{j,e}$, reduction, $D_{j,re}$ and original demand flow rate, $D_{j,o}$. Binary variables, $X_{j,e}$, and $X_{j,re}$ are introduced to represent the selection of several possible measures in elimination and reduction levels.

$$\sum_e Da_{j,e}X_{j,e} + \sum_{re} Da_{j,re}X_{j,re} + \sum_o D_j X_{j,o} = B_j \qquad \forall j \in J \qquad (2)$$

Where $D_{j,o}$ is equal to D_j.

2) *Reduction option constraint:* If reduction option is selected, the flow rate for j^{th} demand, $Da_{j,re}$ is reduced by certain percentage, $\sigma_{j,re}$

$$Da_{j,re} = \sigma_{j,re}D_j \qquad \forall j \in J \qquad (3)$$

Substituting $D_{j,re}$ in equation (3) into equation (2) will result to linear constraint (3'). and can be written as below,

$$\sum_e Da_{j,e}X_{j,e} + \sum_{re} \sigma_{j,re}D_j X_{j,re} + \sum_o D_j X_{j,o} = B_j \qquad \forall j \in J \qquad (3')$$

3) *Water Balance for Demand:* The water supplied for each adjusted demand flow rate, B_j is a combination of fresh water, FW_j, potential reused/recycle water, $F_{i,j}$, other resources, $Fos_{os,j}$ (e.g rainwater, river and snow), and

regenerated water from regeneration unit, $F_{r,j}$. The water balance for each demand, B_j is given by:

$$FW_j + \sum_i F_{i,j} + \sum_{os} Fos_{os,j} + \sum_r F_{r,j} = B_j \quad \forall j \in J \tag{4}$$

4) *Water Balance for Source:* The water generated from each source i, A_i is either discharged directly as effluent, WW_i, direct reuse/recycle water from source i to demand j, $F_{i,j}$ or partially treated in regeneration unit, $F_{i,r}$. The water balance for source i is given by:

$$WW_i + \sum_j F_{i,j} + \sum_r F_{i,r} = A_i \quad \forall i \in I \tag{5}$$

5) *Demand Contaminant Load Satisfaction:* Contaminant mass load for adjusted demand j, $B_j Cd_{j,k}^{max}$ is supplied from a *mixed* of contaminant mass load from different sources (e.g fresh water, $FW_j Cw_k$, potential reused/recycle water, $F_{i,j} Cs_{i,k}^{max}$, outsources, $Fos_{os,j} Cos_{j,k}$ or/and regenerated water, $F_{r,j} Cro_{r,k}$). Thus, the contaminant load from all sources must satisfy the contaminant load for demand j .

$$FW_j Cw_k + \sum_i F_{i,j} Cs_{i,k}^{\max} + \sum_{os} Fos_{os,j} Cos_{os,k} + \sum_r F_{r,j} Cro_{r,k} \le B_j Cd_{j,k}^{\max} \quad \forall j \in J \tag{6a}$$

Note that, the regeneration units employed here using centralized wastewater treatment concept and the performance of regeneration units are measured with fixed outlet concentration for all contaminants, $Cro_{r,k}$ or contaminant removal ratio, $RR_{r,k}$.

$$FW_j Cw_k + \sum_i F_{i,j} Cs_{i,k}^{\max} + \sum_{os} Fos_{os,j} Cos_{os,k} + \sum_r F_{r,j} ((1 - RR_{r,k}) Cri_{r,k}) \le B_j Cd_{j,k}^{\max} \quad \forall j \in J \tag{6b}$$

6) *Mass balance on regeneration unit:* The amount of wastewater to be regenerated in the regeneration unit, $F_{i,r}$, depends on the demand for regenerated water, $F_{r,j}$. The total inlet flow rate is equal to the total outlet flow rate for the regeneration unit. Water consumed for regeneration unit cleaning is assumed to be negligible since the cleaning process is only performed once in a while.

$$\sum_i F_{i,r} = \sum_j F_{r,j} \quad \forall r \in R \tag{7}$$

7) *External water sources constraint: The* total external water sources flow rate distributed to demand, $Fos_{os,j}$ must be equal to or lower than maximum design limit, Fos_{os}^{max}

$$\sum_j Fos_{os,j} \le Fos_{os}^{\max} \quad \forall os \in OS \tag{8}$$

8) *Selection of water minimization options:* This constraint is imposed to ensure that only one water minimization options is chosen at one time. Binary variables $X_{j,e}, X_{j,re}$ and $X_{j,o}$ are introduced to represent the water minimization options involving elimination, reduction or original operation respectively.

$$\sum_e X_{j,e} + \sum_{re} X_{j,re} + \sum_o X_{j,o} = 1 \quad \forall j \in J \tag{9}$$

9) *MTB constraint:* For MTB operations, the adjusted flow rate of water demand, B_j is equal to the adjusted water source flow rate, A_i.

$$B_j = A_i \quad \forall j \in J \tag{10}$$

10) *NMTB constraint:* If source streams exist for NMTB operations, the adjusted flow rate of water source, A_i, is equal to water source flow rate before implementation of WMH options, S_i.

$$A_i = S_i \quad \forall i \in I \tag{11}$$

11) Non-negativity constraints:

$$FW_j,\ WW_i,\ F_{i,j},\ F_{i,r},\ F_{j,r},\ A_i,\ B_j,\ Da_{j,re} \ge 0 \tag{12}$$

III. CASE STUDY: CHLOR-ALKALI PLANT

Applicability of the proposed approach is illustrated using chlor-alkali plant. In order to achieve optimal solution, GAMS/CPLEX solver was employed to solve the MILP problem. The limiting water data comprises of the overall network water sources and demands streams that include plant uses and domestic uses and are listed in terms of water quality and quantity for the chlor-alkali plant. This problem consists of mass transfer-based and non-mass transfer-based operations. There are fourteen water demands and fifteen water sources. The limiting water data in the system is listed in Table 1 and Tables 2 for water demands and sources, respectively. The fresh water source is available for the water system with the following contaminant levels: Cw_{pH} = $3.16\text{x}10^{-8}$ ppm (pH=7.5), Cw_{TDS} = 40 ppm, $Cw_{hardness}$ = 14 ppm. The various water minimization options were presented in Table 3.

TABLE 1
LIMITING WATER DEMAND DATA FOR CHLOR-ALKALI PLANT.

D_j	Demand	Flow rate (t/hr)	pH	TDS (ppm)	Hardness (ppm)
D_1	Washing at filling station and road tanker for NaOH	0.40	7.5	65	17.1
D_2	Washing at filling station and road tanker for HCl	0.40	7.5	65	17.1
D_3	Demineralized filter backwash	0.19	7.5	65	17.1
D_4	Demineralized ion exchange regeneration (after acid injection)	0.27	7.5	40	17.1
D_5	Demineralized ion exchange regeneration (after caustic injection)	0.27	7.5	40	17.1
D_6	Scrubber	4.00	7.5	100	17.1
D_7	Laboratory uses	1.04	7.5	65	14.0
D_8	Cooling tower make-up water	8.33	7.5	100	14.0
D_9	Carbon filter inlet	13.56	7.5	60	14.0
D_{10}	Toilet flushing	0.08	7.5	100	17.1
D_{11}	Toilet pipes	0.10	7.5	65	17.1
D_{12}	Office cleaning	0.05	7.5	65	17.1
D_{13}	Wash basin	0.01	7.5	65	17.1
D_{14}	Ablution	0.12	7.5	65	17.1

TABLE 2
LIMITING WATER SOURCE DATA FOR CHLOR-ALKALI PLANT.

S_i	Source	Flow rate (t/hr)	pH	TDS (ppm)	Hardness (ppm)
S_1	Washing at filling station and road tanker for NaOH	0.40	10.8	30360	14
S_2	Washing at filling station and road tanker for HCl	0.40	2.5	704	16
S_3	Demineralized filter backwash	0.19	7.4	75	20
S_4	Demineralized ion exchange regeneration (after acid injection)	0.27	1.2	3300	14
S_5	Demineralized ion exchange regeneration (after caustic injection)	0.27	9.3	60	14
S_6	Scrubber	4.00	0.3	528	40
S_7	Laboratory uses	1.04	8.3	400	100
S_8	Cooling tower blow down	0.49	6.9	3300	147
S_9	Brine filter backwash	0.50	10.6	6579	14
S_{10}	Brine ion exchange regeneration (brine displacement)	0.63	10.2	526	0
S_{11}	Brine ion exchange regeneration after acid injection	0.49	0.02	396	0
S_{12}	Brine ion exchange regeneration after caustic injection	0.62	13.6	1254	0
S_{13}	Wash basin	0.01	7.7	60	20
S_{14}	Ablution	0.12	7.7	60	20
S_{15}	Evaporation condensate	0.01	11.1	76	0

TABLE 3
VARIOUS WATER MINIMIZATION OPTIONS FOR CHLOR-ALKALI PLANT

WMH	Strategy
Elimination	D_{10}: Change 12 liter flushing toilet to a modern composting toilet
Reduction	D_6: Reduce current fresh water usage at HCl scrubber D_8: Replace chemical used for water treatment with new polymer chemical at cooling water system D_{10}: Option 1: Change 12 litre flushing toilet to dual flush toilet Option 2: Change 12 litre flushing toilet to vacuum toilet D_{14}: Change normal ablution tap to laminar flow tap
Reuse	Total water reuse
External water sources	Rainwater harvesting [Fos_{os}^{max}= 0.21 t/day, (Cos_{TDS} =16 ppm, $Cos_{hardness}$ = 5 ppm and pH = 7.5).
Regeneration	Wastewater regeneration (Cro_{TDS} = 30 ppm, $Cro_{hardness}$ = 2 ppm and pH = 7.5).

Solving equation (1) with the constraints in equations (2)-(12) yielded an optimal solution for designing the minimum water network. The minimum fresh water and wastewater flow rates are 18.51 t/hr and 0 t/hr respectively. This corresponds to 35.8% fresh water and 100% wastewater savings.

IV. CONCLUSION

A new systematic approach to target fresh water consumption and wastewater generation for systems involving multiple contaminants when all options of water minimization are considered simultaneously has been developed. The MILP model is able to holistically determine water source to be eliminated or reduced, the amount of external water source needed, which wastewater source should be reused/recycled, regenerated or discharged. The model is also able to specify the minimum water network configuration to minimize fresh water consumption. The approach has been successfully implemented on a chlor-alkali plant case study.

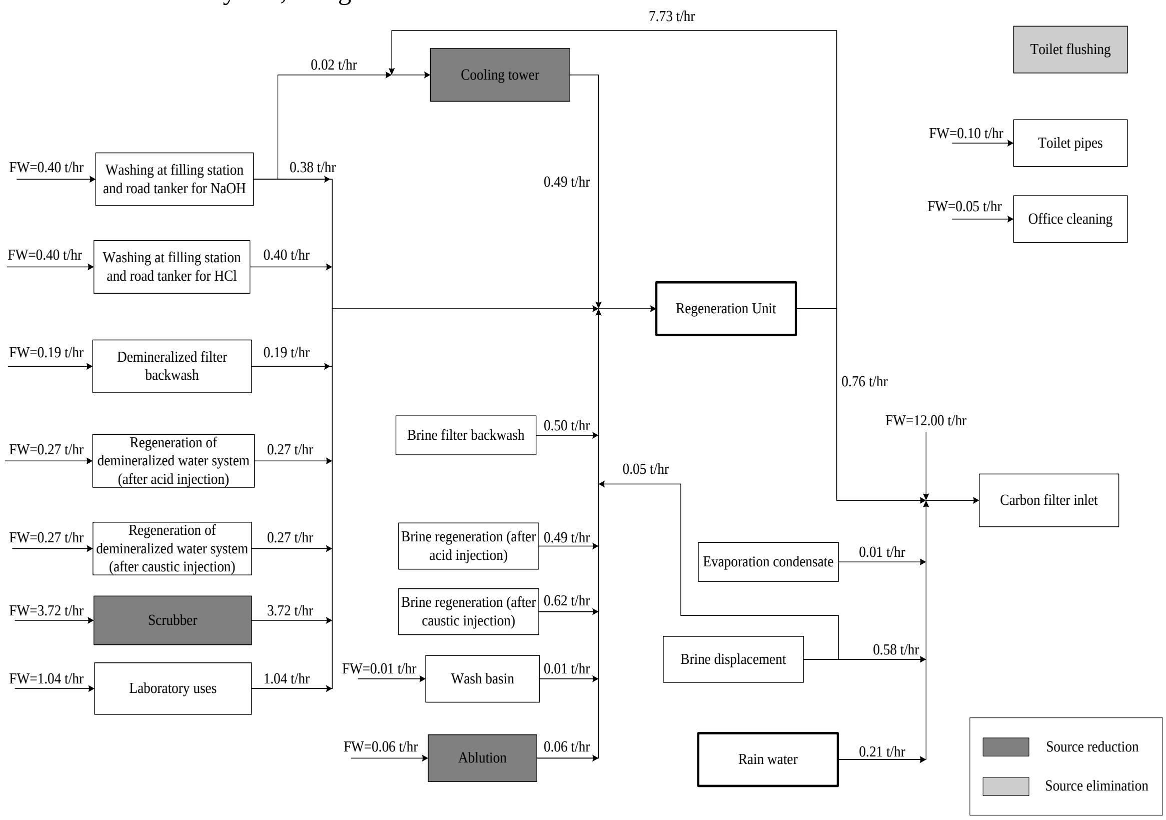


Fig. 2 Optimal water network design for chlor-alkali plant

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NOMENCLATURE

Subscripts

i	Index for water source
j	Index for water demand
k	Index for water contaminant
r	Index for regeneration unit
E	Index for water elimination option
Re	Index for water reduction option
O	Index for original water demand
Os	Index for external water sources

Parameters

$Cs_{i,k}^{max}$	maximum concentration of contaminant *k* from water source *i*
$Cd_{j,k}^{max}$	maximum concentration of contaminant *k* in demand *j*
Cw_k	fresh water concentration of contaminant *k*
$Cos_{os,k}$	outsource concentration of contaminant *k*
$Cro_{r,k}$	outlet concentration of contaminant k from regeneration unit *r*
S_i	flow rate of water source *i*
D_j	flow rate of water demand *j*
Fos_{os}^{max}	maximum flow rate of outsource *os*
$Da_{j,e}$	flow rate of elimination option *e* for demand *j*
$\sigma_{j,re}$	percentage of water reduction *re* for demand *j*

Continuous Variables

FW_j	fresh water supplied to demand *j*
$F_{i,j}$	water flow rate from source *i* to demand *j*
WW_i	unused portion of water source *i* (waste)
$Fos_{os,j}$	outsource flow rate *os* to demand *j*
$F_{i,r}$	water flow rate from source *i* to regeneration unit *r*
$F_{r,j}$	water flow rate from regeneration unit *r* to demand *j*
A_i	adjusted flow rate of water source *i*
B_j	adjusted flow rate of water demand *j*

$Da_{j,re}$ flow rate of reduction option *e* for demand *j*
$Da_{j,o}$ original flow rate *o* for demand *j*
$Cri_{r,k}$ inlet concentration of contaminant k to regeneration unit *r*

Binary Variables
$X_{j,e}$ Selection of elimination options *e* for demand *j*
$X_{j,re}$ Selection of reduction options *re* for demand *j*
$X_{j,o}$ Selection of original flow rate *o* for demand *j*

Acronyms
GAMS Generalized Algebraic Modeling System
MILP mixed integer linear programming
MTB mass transfer-based
MWR maximum water recovery
MWN minimum water network
NMTB non-mass transfer-based
TDS total dissolved solid
WMH water management hierarchy