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Numerical Solutions on Flow and Heat Transfer of Non-Newtonian Jeffrey Micropolar Fluid

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Abstract

Objectives: The present study investigates the problem of flow and heat transfer on non-Newtonian Jeffrey micropolar fluid numerically. The flow that moving across a stretching sheet has been considered embedded with constant wall temperature. **Methods/Statistical Analysis:** The suitable similarity transformations are used to transform the governing boundary layer equation into ordinary differential equations. This is very important in order to reduce the complexity of the equation. The numerical results are obtained using Keller box method. **Findings:** The procedure to validate the present results has been run and the outcomes obtained are outstanding. The results obtained in graphical form show the parameter Deborah number boost the value of fluid velocity. At near the surface, the larger values of Deborah number led to decrease the distribution of micro rotation of fluid but after $\eta > 1.6$ the trend has changed oppositely. **Application/ Improvements:** The results from this research give advance understanding on the micro rotational effects toward the non-Newtonian fluid flow.

Keywords: Jeffrey Fluid, Micropolar Fluid, Non-Newtonian, Numerical Solution

1. Introduction

The classical Navier Stokes equations (Newtonian) are less valid to represent the current fluid involved in industries nowadays due to its complexity in their properties. The fluids that appropriate and valid to characterize those fluids are non-Newtonian fluid where their viscosity changes when shear is applied. One of the interesting fluid models that lie under this cluster is Jeffrey fluid. The pioneered studied on this fluid has been done by[1], where the problem on crustal flow was investigated. This research has been further discover by few researchers in past few years[2–4] by adopted the numerical as well as analytical method.

Other than this type of fluid, the theory on micropolar fluid where its exhibit micro rotational effects has also gain considerable attention due to its capabilities in predicting fluid behaviour at microscale. The advance studied regarding this idea has been extended in the problem of flow embedded in viscous fluid[5,6]. According to[7] whose proposed micropolar theory, the fluid that's can be classified under this group are like colloidal fluids and biological fluids in thin vessels. Recently, the research on micropolar fluid has been continued by few researchers[8–10]. Instead the study toward different fluid with different effects is getting popular, the geometry which the fluid passing through also has become the hot topic. There are many geometries has been considers for related study. Some of them are circular cylinder, sphere, wedge, inclined sheet and stretching sheet. The different type of geometries led to give different type of equation to be solved at the end.

Up to this time, the combination study on Jeffrey fluid together with microrotational effects is limited.

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Motivated to the significance and important application towards Jeffrey fluid and microrotational effects, this article is aims to investigate the flow of Jeffrey micropolar fluid that passing over a stretching sheet. The theory of boundary layer is adopted and the case of Constant Wall Temperature (CWT) is considered.

2. Mathematical Formulation

In this article, the Jeffrey fluid model is considered where its constitutive equation is different as compare to normal fluid (viscous) since this fluid lies under categorized of non-Newtonian type of fluid. This study is limit to the case of steady two dimensional flows. The fluid characteristic is assumed to be incompressible and flowing past a stretching sheet. This problem is concentrate to the case of constant wall temperature where it denoted to the heating process. The effect of micro rotation is also added into the system of fluid flow. The influence of slip velocity effects has been considered in this investigation. The constitutive equation of Jeffrey fluids are once introduced by[2]

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S} \tag{1}$$

$$\mathbf{S} = \frac{\mu}{1+\lambda_1}(\bar{\dot{\gamma}} + \lambda_2 \bar{\ddot{\gamma}}) \tag{2}$$

Here, $\mathbf{T}$ is defined as Cauchy stress tensor, while $-p\mathbf{I}$ is defined as the indeterminate part of stress. Meanwhile $\mathbf{S}$ is the extra stress tensor, μ is the coefficient of viscosity, λ_1 and λ_2 are the ratio of relaxation to retardation times and the retardation time respectively, $\bar{\dot{\gamma}}$ is the shear rate and dots over the quantities indicate differentiation with respect to time. It is worth to mention that, if $\lambda_1 = \lambda_2 = 0$ the Equation (2) will reduces to the expressions of an incompressible viscous fluid. Under the Boussinesq approximation, the governing equation of momentum, micro rotation and energy can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + \frac{\kappa}{\rho}\frac{\partial N}{\partial y} + \left(\frac{v+\frac{\kappa}{\rho}}{1+\lambda_1}\right)\left[\frac{\partial^2 u}{\partial y^2} + \lambda_2\left(\begin{array}{l} u\frac{\partial^3 u}{\partial x \partial y^2} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \\ \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}) \end{array}\right)\right] \tag{4}$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right) \tag{5}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{6}$$

where u and v are the velocity components along the x, y axes, u_e is the velocity outside the boundary layer, ν is the kinematic viscosity, κ is a vortex viscosity, ρ is the fluid density, N is the component of micro rotation vector normal to the x, y - axes, j is the micro inertia density, T is the temperature, σ is the electrical conductivity of the fluid and α is given as thermal conductivity.

Following[11,12], the spin gradient γ can be defined as

$$\gamma = \left(\mu + \frac{\kappa}{2}\right)j = \mu\left(1 + \frac{K}{2}\right)j \tag{7}$$

where $K = \frac{\kappa}{\mu}$ is denoted as material parameter, $j = \frac{\nu}{a}$ is a reference length, μ is a dynamic viscosity and a, c are an arbitrary constants.

This problem is solved subjected to boundary conditions

$$\begin{aligned} &u = cx + g\frac{\partial u}{\partial y},\ v = 0,\ N = -n\frac{\partial u}{\partial y},\ T = T_w,\ at\ \ y = 0, \\ &u = u_e \to ax,\ N \to 0,\ \frac{\partial u}{\partial y} \to 0,\ T \to T_\infty,\ as\ \ y \to \infty \end{aligned} \tag{8}$$

Introducing the following similarity transformations

$$\begin{aligned} &\eta = y\sqrt{\frac{c(1+\lambda_1)}{\nu}}, \quad u = cxf'(\eta), \\ &v = -\sqrt{\frac{c\nu}{1+\lambda_1}}f(\eta), \quad N = cx\sqrt{\frac{c(1+\lambda_1)}{\nu}}g(\eta), \\ &\theta = \frac{T - T_\infty}{T_w - T_\infty}. \end{aligned} \tag{9}$$

into Equations (1)-(6) and (8), hence reduces to

$$\begin{aligned} &(1+K)\left[f'''(\eta) + \beta\left(\left(f''(\eta)\right)^2 - f(\eta)f^{(4)}(\eta)\right)\right] \\ &\quad + f(\eta)f''(\eta) + Kg'(\eta) - \left(f'(\eta)\right)^2 + \delta^2 = 0 \end{aligned} \tag{10}$$

$$\left(1+\frac{K}{2}\right)g''(\eta)+fg'(\eta)-f'(\eta)g(\eta)$$
$$-K\left(2g+f''(\eta)\right)=0 \quad (11)$$
$$\theta''+\Pr f(\eta)\theta'=0 \quad (12)$$

subjected to boundary conditions

$$f(0)=0;\ f'(0)=1+kf''(0);\ g(0)=-nf''(0);\ \theta(0)=1,$$
$$f'(\infty)=\delta;\ f''(\infty)=0;\ g(\infty)=0;\ \theta(\infty)=0 \quad (13)$$

where,

$\beta=\lambda_2 c$ (Deborah number),

$\delta=\dfrac{a}{c}$ (Stretching stretch parameter),

$k=g\sqrt{\dfrac{c(1+\lambda_1)}{\nu}}$ (Velocity slip parameter),

$\Pr=\dfrac{\nu}{\alpha}$ (Prandtl number)

3. Results and Discussion

The transformed of system of non-linear ordinary differential Equations (10) to (12) corresponding to boundary Conditions (13) are solved by applying the numerical scheme called Keller-box method. The value of boundary layer thickness has been set at $\eta=10$. The verification on the coding of computation has been run for limiting cases (the current equation is reduce to existing published equation) and the results shows some agreement. The clear outcome of verification process is presented in Table 1 shows It is worth to mention here, the numerical comparison has be done with analytic solutions that proudly presented by[13]. Consequently, the authors believed the present outcomes presented in this article are acceptable and new.

Table 1. Comparison the values $-\theta'(0)$ at K = 0, k = 0, n = 0, β = 0, δ = 1

Pr	In [13]	Present
2	1.12837917	1.128732
5	1.78412412	1.785519
10	2.52313252	2.527078

Figure 1 demonstrated the distribution of fluid flow with various values of Deborah number. It is noticed that, the increasing in values of Deborah number led to increase the velocity of fluid. This trend is similar with those reported in[14]. The value of Prandtl number equal 25 is chosen for computation in order to represent the non-Newtonian fluid.

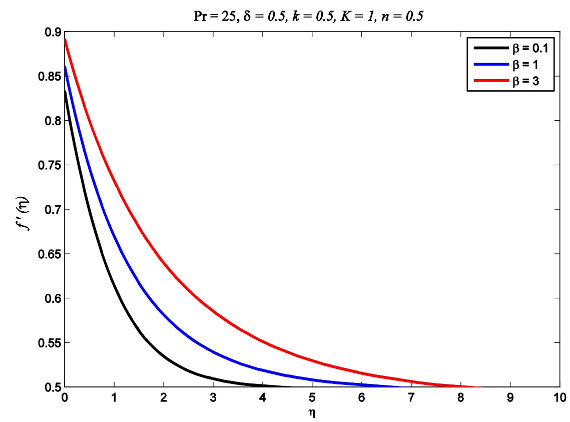

Figure 1. Distribution of velocity of fluid.

Figure 2 captured the profile of micro rotation of fluid for different values of Deborah number. It is observed that, at near the surface ($\eta<1.6$) the profile of micro rotation decrease as the values of parameter Deborah number increased but the trend is opposite after boundary layer thickness increase (bigger η). It is happen because at smaller values of Deborah number, the fluid tends to behave like viscous fluid.

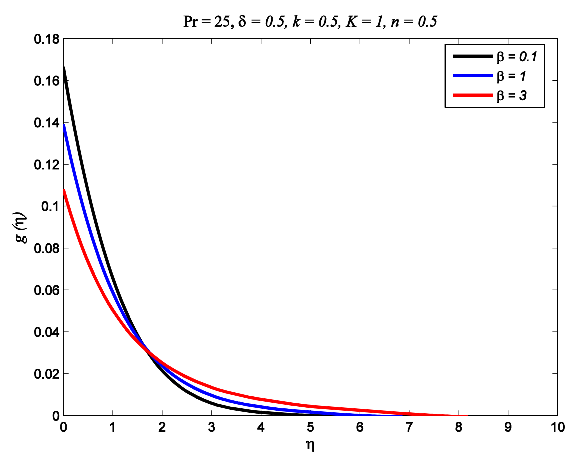

Figure 2. Distribution of microrotation of fluid.

The behavior on temperature distribution of fluid against the Prandtl number has been illustrated in Figure 3. The trend show the bigger of Prandtl number reduced the temperature of fluid. It is the fact that, the bigger of Prandtl number have the weaker diffusivity. From those figure, it is noticed that the results fulfill the boundary condition completely.

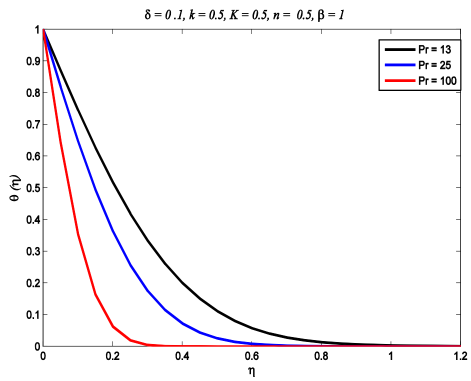

Figure 3. Distribution of temperature of fluid.

4. Conclusion

In this studied, we have considered the problem of Jeffrey non-Newtonian fluid embedded with micro rotation effects that passing through a stretching sheet with constant wall temperature boundary condition. From the results presented in this article, it can be concluded as follows,

- As parameter Deborah number increased, it led to increase the velocity of fluid and decrease the micro rotation of fluid near the surface. At far from the surface, the bigger of Deborah number boost the micro rotation of fluid.
- The bigger of Prandtl number led to decrease the temperature of fluid.

The International Conference on Fluids and Chemical Engineering (FluidsChE 2017) is the second in series with complete information on the official website[15] and organized by The Center of Excellence for Advanced Research in Fluid Flow (CARIFF)[16]. The publications on chemical engineering allied fields have been published as a special note in volume 3[17]. Host being University Malaysia Pahang[18] is the parent governing body for this conference.

5. Acknowledgement

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