Indian Journal of Science and Technology, *Vol 10(7), DOI: 10.17485/ijst/2017/v10i7/111213, February 2017*
ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645

Influence of Aligned Magneto Hydrodynamic of Jeffrey Fluid across a Stretching Sheet

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Abstract

Objectives: Effects of slip velocity and aligned Magneto hydro-dynamic on Jeffrey fluid which passing across a stretching sheet with Newtonian heating boundary condition are investigated. **Methods/Statistical Analysis:** Governing partial differential equations are first transformed into ordinary differential equation by applying the similarity transformations before undergo computation process using y bvp4c in MATLAB. **Findings:** For validation purposes, the present results are comparing with the outcome from previous publications for the case Constant Wall Temperature (CWT) and it shows a very strong agreement on the values of $-\theta'(0)$. From the study, the increasing on the values of magnetic parameter, M drop the fluid velocity for the range value of boundary layer thickness $0 < \eta < 3.5$. After $\eta > 3.5$, the value of fluid velocity is increasing on the larger magnetic parameter. However, the distribution on temperature of fluid shows an opposite trend as compare with fluid velocity. On the range of $0 < \eta < 2.5$, the increasing of Deborah number did not give a strong effect on the fluid velocity but at $\eta > 2.5$ it evidently reduce the velocity of fluid. The fluid temperature is high at smaller values of Deborah number. The distribution on velocity and temperature of fluid presented in this article are strictly asymptotically fulfilling the boundary conditions which then contribute to the adequate of the present results. **Application/Improvements:** The results from this project will enhance the knowledge in non-Newtonian fluid problem via mathematical approach.

Keywords: Heat Transfer, Jeffrey Fluid, Magneto Hydrodynamics, Newtonian Heating

1. Introduction

There is a significant importance of the models of stretching sheet flow because of their implementations in the engineering field such as for liquid coating on photographic films, for extrusion of polymer sheet from expiry, and as a boundary layer along the liquid film concentration procedure. Sakiadis[1] investigated the boundary layer behaviour of the stretching sheet flow model on uninterrupted solid flat surfaces. The study by Erickson et al.[2] is an augmentation of Sakiadis' work where mass transfer is being considered and the heat on an extending surface is being tested with injection or suction. The transfer and flow of heat across a surface that is expanding exponentially and has wall mass suction was mathematically studied by Elbashbeshy[3]. Partha et al.[4] estimated the collective impact of viscous dissipation for the problem of mixed convection on the viscous fluid flow across a stretch exponentially sheet. Nadeem, Hussain[5] analysed the series solutions for the boundary layer flow at the point of stagnation area across a stretching sheet. Besides that, Ishak et al.[6] performed the analysis of transfer of heat which observed in micropolar fluids with inconsistent heat flux on a stretching surface. Some of the latest developments in this field and the related literature can be reviewed in Refs.[7-10]. Since the classical theory of Navier-Stokes does not able to portray all the properties of rheological on complex fluids like blood, polymer solutions, paints, plasma, certain oils, chyme, greases, food in the intestine, nuclear fuel slurries, mercury amalgams, and alloys and metals, the research on non-Newtonian fluids attained significant importance in the recent years. However, there are two major obstacles in the study of non-Newtonian fluid. The first is the occurrence of a

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supplemental non-linear term in the equations of motion that makes the problem more convoluted and the second problem is the absence of a universal non-Newtonian relation that is applicable for all fluids and flows. Due to the complexity of the fluids which ever appear in many industrial applications nowadays, several models of non-Newtonian fluids are introduced and one such model includes is Jeffrey fluid model which is moderately simple and offers the possibility of computation, which led to give a solutions[11-15].

Motivated to the work have been done in literature, this study is focused on the analysis of the Jeffrey fluid flow and heat transfer with the effect of aligned Magneto Hydrodynamics (MHD) across a sheet with Newtonian heating boundary condition via computation. Numerical solutions are obtained by performing the bvp4c solver in MATLAB software and the computation results are then performed in table and graphically. The contributions in this article are on the part where the MHD is applied not limited on transverse to the flow but it's aligned to the fluid flow.

2. Mathematical Model

The model is consider two-dimensional steady laminar flow of a compressible fluid over a continuous stretching sheet where the velocity $u_w(x) = cx + g_c \frac{\partial u}{\partial y}$ and magnetic field moves according to the movement of the fluid together with ambient temperature T_∞ which is uniformly act on fluid. This is due to the surface which is under a permeable stretching condition where the plane y is assumed as zero and the flow being restricted to $y > 0$. The physical geometry of the proposed problem is illustrated in Figure 1.

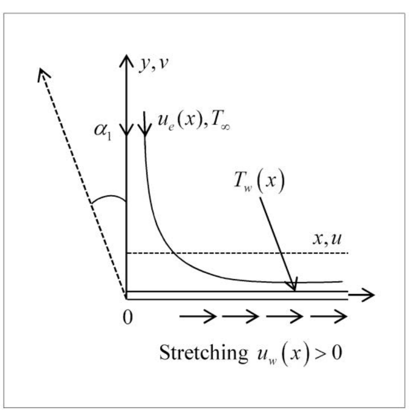

Figure 1. The geometry of physical flow.

Two equal and opposite forces are introduced along the x-axis so that the surface is stretched with velocity $u_w(x)$. It is also assumed that the velocity outside the boundary layer (potential flow) is $u_e(x)$. Under the boundary layer and Bossiness approximations where the flow is assume to non isothermal incompressible flow, the basic equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + \frac{\nu}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial y^2} + \lambda_2\left(u\frac{\partial^3 u}{\partial x \partial y} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right)\right] - \frac{\sigma B_0^2}{\rho}\sin^2(\alpha_1)(u - u_e) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}(T - T_\infty). \tag{3}$$

The boundaries conditions consider in this problem are as follows

$$u = cx + g_c\frac{\partial u}{\partial y},\ v = 0\ ,\ \frac{\partial T}{\partial y} = -h_s T \ \text{ at } \ y = 0$$
$$u \to u_e(x) = ax\,,\ \frac{\partial u}{\partial y} \to 0\ ,\ T \to T_\infty \ \text{ as } \ y \to \infty \tag{4}$$

where a, c (> 0) are constants, u and v are the velocity components in x and y directions, T is the fluid temperature, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, λ_1 is the ratio of relaxation and retardation time and λ_2 is the relaxation time. An applied magnetic field B_0 is enforced along the y-axis with the magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field is assumed to be negligible. The electrical conductivity is represented by σ, ρ is the density, and α_1 is inclined angle. The specific heat under constant pressure condition is denoted by c_p while T_∞ is represent free stream temperature. The thermal diffusivity is representing by α and the gravitational acceleration which acts to the direction downward is denoted by g_c. The dimensionless parameter $k_1 = g_c\sqrt{\frac{c(1+\lambda_1)}{\nu}}$ is thermal conductivity besides h_s is the parameter of heat transfer for the Newtonian heating.

The following transformations is proposed,

$$\eta = y\sqrt{\frac{c(1+\lambda_1)}{\nu}} \quad , \quad u = cxf'(\eta) \quad , \quad v = -\sqrt{\frac{c\nu}{1+\lambda_1}}f(\eta) \quad ,$$

$$\theta = \frac{T - T_\infty}{T_\infty} \tag{5}$$

After applying (5) equations (1-3) are reduced to

$$f''' + ff'' - (f')^2 + \beta\left((f'')^2 - ff^{(4)}\right) - M^2\sin^2(\alpha_1)\left(f' - \delta\right) + \delta^2 = 0 \tag{6}$$

$$\theta'' + \Pr f\theta' + \Pr Q\theta = 0 \tag{7}$$

with respect to boundary conditions

$$f'(0) = 1 + k_1 f''(0) \ , \ f(0) = 0 \ , \ \theta'(0) = -\gamma_0(1+\theta(0)) \ ,$$
$$f''(\infty) = 0 \ , \ \theta(\infty) = 0 \ , \ f'(\infty) = \delta \ . \tag{8}$$

For the case of constant wall temperature, one of the boundary conditions at $\eta = 0$ is $\theta(0) = 1$ (CWT)

where $\beta = \lambda_2 c$ is the Deborah number, $M^2 = \frac{\sigma\beta_0^2}{c\rho}$ is the magnetic parameter, $\delta = \frac{a}{c}$ is the stretching parameter, $\Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Q = \frac{Q_0}{c\rho c_p}$ is heat generation and $\gamma_0 = \frac{h_s}{\sqrt{\frac{c(1+\lambda_1)}{\nu}}}$ is the conjugate parameter.

3. Results and Discussion

The nonlinear ordinary differential equations in (6) and (7) which fulfills the boundary conditions in (8) are numerically solved with the help of bvp4c method for several parameter quantities including the magnetic parameter, M, as will as Deborah number, β. The numerical values were plotted to illustrate the results of the problem. Table 1 shows a comparison between the previously published results from M. Turkyilmazoglu, I. Pop[16] with the present solutions for CWT. Promising results are obtained upon comparison as both results show strong agreement with each other. Effects of the magnetic parameter, M, on velocity and temperature profile are shown in Figure 2 and 3, respectively. Figure 2 exhibits the dimensionless velocity profiles $f'(\eta)$ for different values of dimensionless parameter magnetic where it is clear the bigger values of M lead to condense the velocity of the fluid.

Table 1. Comparative study between the previous and current results

$-\theta'(0)$			
Pr	δ	In[16]	Present
0.5	0.5	0.49039	0.49034
	1	0.56419	0.56415
1	0.5	0.71544	0.71539
	1	0.79788	0.79784
2	0.5	1.0392	1.0391
	1	1.1284	1.1283
5	0.5	1.6886	1.6886
	1	1.7841	1.7841
10	0.5	2.4244	2.4244
	1	2.5231	2.5231

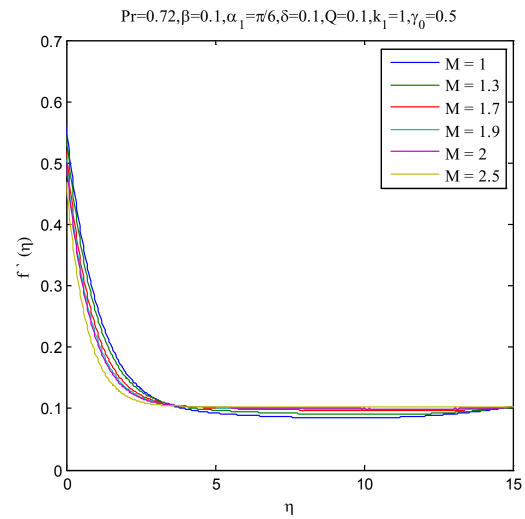

Figure 2. Velocity profile $f'(\eta)$ for some values of M.

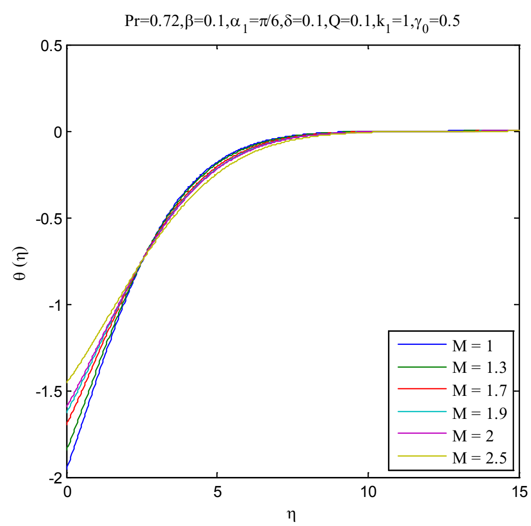

Figure 3. Temperature profile $\theta(\eta)$ for some values of M.

Figure 3 illustrates that the increase in M leads to the increase in temperature profile. Physically, the changes on the velocity as well as temperature of the fluid are due to the fact that the magnetic field contributes the external forces which led to disturb the fluid flow.

The fluid velocity and temperature at different quantity of β are illustrated in Figure 4 and 5, respectively, where the velocity profile increases while the temperature profile decreases with increasing values of β.

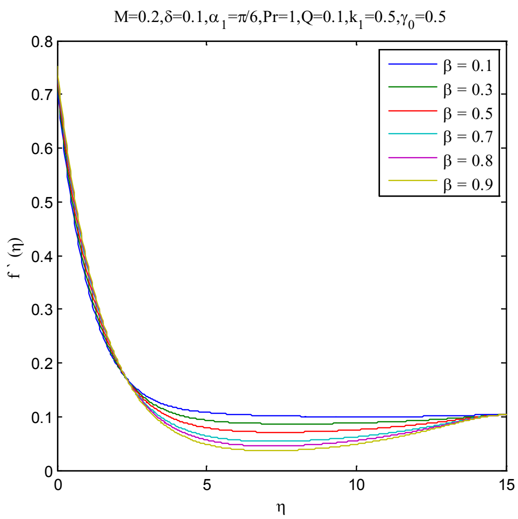

Figure 4. Velocity profile $f'(\eta)$ for some value of β.

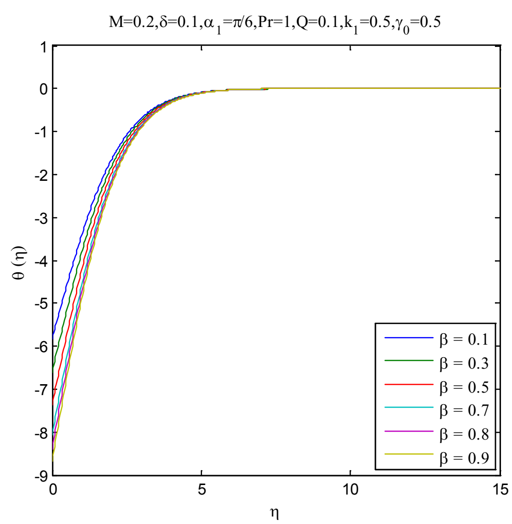

Figure 5. Temperature profile $\theta(\eta)$ for some values o β.

4. Conclusion

A numerical study on Jeffrey fluid across a stretching sheet with the condition velocity slip and Newtonian heating together with aligned Magneto Hydrodynamics (MHD) is presented. Using similarity variable, the ordinary differential equations representing the momentum and energy equations are obtained and then solved with the help of numerical bvp4c solver from Matlab. The parameter involved in this study significantly affect the flow and heat transfer. The distribution of fluid velocity as well as temperature are strictly asymptotically approach the boundary conditions. The International Conference on Fluids and Chemical Engineering (FluidsChE 2017) is the second in series with complete information on the official website[17] and organized by The Center of Excellence for Advanced Research in Fluid Flow (CARIFF)[18]. The publications on chemical engineering allied fields have been published as a special note in volume 3[19]. Host being University Malaysia Pahang[20] is the parent governing body for this conference.

5. Acknowledgement

This manuscript is received financial supports from Universiti Malaysia Pahang, under (RDU150101 & RDU160330) and Department of Mathematics, College of Education for Pure Sciences, University of Karbala, and Karbala, Iraq.

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