# DETERMINATION OF CERTAIN THERMO-PHYSICAL PROPERTIES OF NANOFLUIDS

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Report submitted in partial fulfIlment of the requirements for the award of the degree of Bachelor of Mechanical Engineering

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## SUPERVISOR'S DECLARATION

I hereby declare that I have checked this project and in my opinion this project is adequate in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering.

Signature Name of Supervisor: PROFESSOR DR. K.V. SHARMA Position: PROFESSOR Date: 6 DECEMBER 2010

## STUDENT'S DECLARATION

I hereby declare that the work in this report is my own except for quotations and summaries which have been duly acknowledged. The report has not been accepted for any degree and is not concurrently submitted for award of other degree.

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## ABSTRACT

The thesis is about the determination of certain thermo-physical properties of nanofluids. The properties of nanofluids at desired temperature, particle size and concentration can be estimated using the regression equations developed to evaluate the viscosity and thermal conductivity for water based nanofluids. No single equation is available in literature to determine the property of nanofluid at desired temperature, particle size and concentration. Besides, the experimental datas in the existing literature are in discrete manner. The experimental data published in literature is used to obtain regression equation valid for a wide range of operating conditions. Using FORTRAN program for regression analysis, the datas for viscosity and thermal conductivity of water base and nanofluids is used to obtain the regression coefficients. The different models such as Non-Linear regression, Least Square and Linear regression models are considered in the analysis for the determination of coefficients for parameters such as temperature, volume concentration and particle size for water and nanofluids. Finally the model which gives the lowest deviation is selected base on few aspects such as coefficient of equation, maximum and minimum deviation. A graph is plotted using Origin software. Properties of nanofluids can be estimated and the Prandt number can be calculated which will help determine the heat transfer coefficient without conducting any experiment for determining the properties of nanofluids.

## ABSTRAK

Tesis ini membentangkan penentuan sifat termo-fizikal tertentu nanofluids. Sifat nanofluids pada suatu saiz, suhu dan konsentrasi boleh ditentukan dengan menggunakan persamaan regresi yang dihasilkan untuk menilai kelekatan dan konduktiviti terma untuk nanofluids berasaskan air. Tidak ada persamaan tunggal terdapat dalam kajian setakat ini untuk menentukan nanofluid pada saiz, suhu dan konsentrasi. Selain itu, data yang terdapat dalam kajian adalah dalam bentuk berasingan dan tidak lengkap. Dengan data yang lengkap, ia boleh digunakan untuk mendapatkan persamaan regresi untuk keadaan operasi. Menggunakan FORTRAN program untuk regresi analisis, data untuk viskositas dan konduktiviti terma air dan nanofluids digunakan untuk mendapatkan pekali regresi. Model yang berbeza seperti regresi non-linear, regresi kuasa dua terkecil dan regresi linear model dipertimbangkan dalam analisis untuk penentuan pekali untuk parameter seperti suhu, konsentrasi dan saiz untuk air dan nanofluids. Akhirnya model yang memberikan deviasi terendah dipilih berdasarkan beberapa aspek seperti pekali persamaan, maksimum dan minimum deviasi. Graf diplot dengan menggunakan ORIGIN program. Sifat nanofluids boleh dijangka dan Prandt nombor boleh dikira yang akan membantu menentukan pekali perpindahan panas tanpa perlu melakukan apa-apa percubaan apapun untuk menentukan sifat-sifat nanofluids.

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## LIST OF SYMBOLS

$C_p$	Specific heat, J/kg K
k	Thermal conductivity, W/m K
Pr	Prandtl Number
Re	Reynolds number
Т	Temperature, °C
μ	Dynamic viscosity, kg/m <sup>2</sup> s
ρ	Density, kg/m <sup>3</sup>
arphi	Volume concentration of nanoparticles, %
е	Exponential function

## LIST OF ABBREVIATIONS

$AL_2O_3$	Alumina/ Aluminium oxide
CuO	Copper oxide
TiO <sub>2</sub>	Titania / Titanium dioxide
ZnO	Zinc Oxide
SiO <sub>2</sub>	Silica/ Silicon dioxide
b	Mean bulk
bf	Base fluid
cal	Calculation
exp	Experimental
f	Fluid
nf	Nanofluids
r	Ratio
FYP	Final Year Project

## **CHAPTER 1**

#### **INTRODUCTION**

## 1.1 BACKGROUND

Heat transfer is one of the most important industrial process. Throughout any industrial facility, heat must be added, removed, or moved from one process stream to another and it has become a major task for industrial. The enhancement of heating or cooling in an industrial process may create a saving in energy, reduce process time, raise thermal rating and lengthen the working life of equipment. Some processes are even affected qualitatively by the action of enhanced heat transfer. The development of high performance thermal systems for heat transfer enhancement has become popular nowadays. A lot of work has been done to gain an understanding of the heat transfer performance for their practical application to heat transfer enhancement. A wide variety of industrial processes involve the transfer of heat energy. These processes provide a source for energy recovery and process fluid heating/cooling.

The term "nanofluid" is envisioned to describe a solid liquid mixture which consists of nanoparticles and a base liquid, and which is one of the new challenges for thermo-science provided by the nanotechnology. The nanoparticles AL<sub>2</sub>O<sub>3</sub>, CuO, TiO<sub>2</sub>, ZnO and SiO<sub>2</sub> are commonly use in the major research. The base fluid normally using water, mixture water with ethylene glycol and engine oil as cooling agent, but it is depend on the type of application use. However, it is found that the solubility of others base fluid such as ethylene glycol is lower than water at certain temperature although the thermal conductivity is low for ethylene glycol.

The enhancement of heat transfer can be affected by different methods which is gas bubble mixing in the liquid flow and solid particle or liquid drop mixing in the gas flow, surface vibration, heat transfer surface rotation, propeller/coil insert, heat carrier pulsations, flow affected by electrostatic fields, flow suction from the boundary layer. Often it appears highly efficient to use combined method of heat transfer enhancement such as turbulators combined with surface finning, spiral fins simultaneously swirling the flow, twisting devices (e.g. twisted tape) in suspension flow, adding chemical such as additive, surface-adtive agent. (Thomas and James, 1998)

Finally, by adding nanoparticles can enhance heat transfer which have been done by other researchers. The suspended ultrafine particles change transport properties and heat transfer performance of the nanofluids, which exhibits a great potential in enhancing heat transfer compared with the existing show a great potential in increasing heat transfer rates in a variety of application cases. A complete understanding about the heat transfer performance of the nanofluids is necessary for their practical application to heat transfer enhancement.

The emergence of nanofluids with applications for energy transfer is helpful for miniaturization of thermal equipment and help industries to sustain global competition. Nanofluids are engineered colloidal suspensions having metal particles or their oxides of nanometres size in base fluids such as water, ethylene glycol, etc. All properties have a critical length scale below which the physical properties of materials are different from that of the values of the bulk material. Therefore particles below 100nm exhibit properties that are considerable different from those of conventional solids.

Heat transfer performance of the nanofluids is superior to that of the original pure fluid because the suspended ultrafine particles significantly increase the thermal conductivity of the mixture and improve its capability of energy exchange. The enhancement mainly depends upon factors such as the shape of particles, the particles size, the volume fractions of particles in the suspensions, and the thermal properties of particle materials. It exhibit extraordinarily high thermal conductivity which reported that the identity (composition), amount (volume percent), size, and shape of nanoparticles largely determine the extent of this enhancement (Xie et al. 2008). It is known that the suspension of solid particles in a fluid, as originally proposed by Maxwell may increase the thermal conductivity of the base fluid, because thermal conductivity of solids is orders of magnitude higher than that of liquids. However, coarse particles, due to their larger size and greater mass than that of finer ones, are prone to sedimentation, develop resistance to fluid flow, and cause erosion to conduits. (Maxwell., 1873) In generally the thermal conductivities increase in the order gases < liquids < solids. For gases, k depends rather strongly on temperature. For many simple organic liquids, the thermal conductivities are 10 to 100 times larger than those of the corresponding gases at the same temperature, with little effect of pressure in the liquid state. Thermal conductivity and viscosity affect the heat transfer behaviour of nanofluids in opposite ways. As a result, a combination of thermal conductivity enhancement and increment of the viscosity can give either enhancement or deterioration of the heat transfer coefficient.

It is observed that heat transfer enhancement can be achieved by using nanofluids instead of the conventional fluids by others researcher. To estimate the amount of cooling and the parameters influencing this aspect, determination of nanofluid properties such as density, viscosity, thermal conductivity and specific heat is essential. It is intended to study the effect of certain parameters on the thermo-physical properties of nanofluids which has not been undertaken till now.

## **1.2 PROBLEM STATEMENT**

The properties of these water based nanofluids have been presented in the form of tables or equations for a specific concentration at different temperatures or at various concentrations at a specific temperature or at different particle sizes and are therefore discrete in nature. It is same goes with the equation which in discrete manner. It is hard to compare with others researcher and extrude the data for any investigator. The data will be carefully analysed for nanoparticle properties such as density and specific heat which is not available in literature easily. Besides that, there are some limitation such as the experimental data given is not complete. No single equation is available in literature to determine the property of a nanofluid at a desired temperature, particle size and concentration.

#### **1.3 PROJECT OBJECTIVE**

The objective of this project is to determine the property of a nanofluid at a desired temperature, particle size and concentration using a regression equation for evaluation of viscosity and thermal conductivity for water based nanofluids. Characterize the thermo-physical properties and heat transfer performance of nanofluids.

## **1.4 PROJECT SCOPE**

The scopes for this project are to study about the focus of study is to identify the existing literature on nanofluids consolidated for different materials, particle density, particle specific heat, volume concentration of nanofluid, temperature, particle size and any other parameter influencing viscosity. It is proposed to extract the experimental data from various sources, model it as a function of temperature, volume concentration and nanoparticles size using digitizeit software. Regression programs available in books will be used for the analysis and develop an regression equation for the evaluation of viscosity for water based nanofluid. The experimental data presented in the form of equation by the investigator will be used to develop numerical data. The equation will be tested with data not used in the regression analysis. The results is to be presented in the form of graphs using Origin software. Experimental determination of viscosity is also envisaged based on the availability of nanofluids and viscometer.

## **1.5 IMPORTANCE OF RESEARCH**

The initial development of nanofluids is inspire by the industrial technologies which looking for better cooling systems. Nanofluids, which contain nanoparticles dispersed in base fluids, have been proposed as a new kind of heat transfer media because they can improve the heat transport and energy efficiency which have potential applications in the field of heat transfer enhancement. Inherently low thermal conductivity of the liquid cooling media results in equipment limitations, process inefficiencies and reduced thermal limits. Heat rejection requirements are continually increasing for technological devices due to miniaturization or down scaling of the devices. It is now widely accepted that the thermal management in nano-size devices plays a fundamentally critical role in controlling their performance and stability. The problem is acute for internal combustion engines at the macro-scale. There is a great deal of interest with extended-surface thermal control technologies such as the use of fins and microchannels to stretch their limits and to improve the thermal properties of cooling fluids. Hence, the use of nanofluids is a good promising way in future to fulfill the cooling needs.

## **CHAPTER 2**

### LITERATURE REVIEW

## 2.1 INTRODUCTION

Thermo physical properties of the nanofluids are quite essential to predict their heat transfer behavior. It is extremely important in the control for the industrial and energy saving perspectives. There is great industrial interest in nanofluids. Nanoparticles have great potential to improve the thermal transport properties compared to conventional particles fluids suspension, millimetre and micrometer sized particles. In the last decade, nanofluids have gained significant attention due to its enhanced thermal properties. Determining the viscosity of nanofluids is necessary to establishing adequate pumping power in which the convective heat transfer coefficient as Prandtl and Reynolds number are dependent on viscosity.

## 2.2 VISCOSITY OF NANOFLUIDS: THEORITICAL MODELS

Viscosity of nanofluids is less investigated compare to thermal conductivity; however, the rheological properties of liquid suspensions have been studied since Einstein (1906). Einstein's equation can predict the effective viscosity of liquids in the low volume fraction having spherical suspended particles. The equation considers only the liquid particle interaction and is valid for volume concentration of less than 1.0%. The Einstein's equation is given as Eq. 2.1.

$$\frac{\mu_{nf}}{\mu_{bf}} = 1.0 + 1.25\,\varphi \tag{2.1}$$

Brinkman (1952) extended the Einstein's equation up to particle volume concentration of 4.0% given by Eq. 2.2.

$$\frac{\mu_{nf}}{\mu_{bf}} = \frac{1}{(1-\varphi)^{2.5}}$$
(2.2)

With increasing particle volume concentration, the flow around a particle is influenced by the neighboring particles. The Krieger-Dougherty model (1959) is based on the assumption that an equilibrium exist between individual spherical particles and dumbbells that continuously form and dissociate as Eq. 2.3.

$$\frac{\mu_{nf}}{\mu_{bf}} = \left(1 - \frac{\varphi_a}{\varphi_m}\right)^{-\lfloor\eta\rfloor\varphi_m}$$
(2.3)

- -

where  $\varphi_m$  is the maximum concentration at which flow can occur,  $\varphi_a$  the effective volume fraction of aggregates and  $[\eta]$  is the intrinsic viscosity, which for monodisperse systems has a typical value of 2.5.

Frankel and Acrivos (1967) derive a suspension viscosity model by using an asymptotic mathematical analysis of governing equations of change. According to the Frankel and Acrivos model, the effective Newtonian viscosity of a suspension of uniform solid spheres is shown as Eq. 2.4.

$$\frac{\mu_{nf}}{\mu_{bf}} = \frac{9}{8} \times \left[ \frac{\left(\frac{\varphi}{\Phi_{m}}\right)^{\frac{1}{3}}}{1 - \left(\frac{\varphi}{\Phi_{m}}\right)^{\frac{1}{3}}} \right]$$
(2.4)

The most important assumption in this model is that the suspension exhibits Newtonian behavior. Based on the comparisons with experimental data, Frankel and Acrivos found that the equation is valid for  $\frac{\varphi}{\Phi_m}$  greater than 0.5 in which the randomly packed spheres,  $\Phi_m$  is 0.64.

Lundgren (1972) has proposed the following equation under the form of Taylor series as shown Eq. 2.5.

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\varphi + 6.25\varphi^2 + O(\varphi^3)$$
(2.5)

It is obvious that if the terms  $O(\varphi^2)$  or higher are neglected, the above formula reduces to that of Einstein. Batchelor (1977) studied the effect of these hydrodynamic interactions or the Brownian motion on viscosity of suspensions and developed a relation valid for particle volume concentration up to 10.0% as Eq. 2.6.

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\,\varphi + 6.5\,\varphi^2 \tag{2.6}$$

Graham (1981) has proposed a generalized form of the Frankel and Acrivos (1967) formula which is agrees well with Einstein's for small  $\varphi$ . The Graham formula is shown as Eq. 2.7.

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\varphi + 4.5 \times \left[\frac{1}{\left(\frac{h}{d_p}\right) \times \left(2 + \frac{h}{d_p}\right) \times \left(1 + \frac{h}{d_p}\right)^2}\right]$$
(2.7)

where  $\mu_{nf}$  is the viscosity of the nanofluids,  $\mu_{bf}$  is the viscosity of the base fluid,  $\Phi_m$  is the maximum solid volume fraction, where  $d_p$  is the particle radius, h is the interparticle spacing and  $\varphi$  is the nanoparticles volume fraction of the suspended solutes or particles consider only nanoparticles volume concentration on nanofluids viscosity prediction.

#### 2.3 VISCOSITY OF NANOFLUIDS: EXPERIMETAL OBSERVATIONS

From the theoretical point of view, there is a new challenge to the researchers in fluid dynamics and heat transfer to understand various properties of nanofluids. There exist very few established theoretical formulas, some theoretical models such as Einstein model have been modified and use to predict the effective viscosity of nanofluids.

A simple expression was proposed by Kitano et al. (1981) involving  $\varphi_m$  was also used to predict the viscosity of two phase mixture by Eq. 2.8.

$$\frac{\mu_{nf}}{\mu_{bf}} = \left(1 - \frac{\varphi}{\varphi_m}\right)^{-2}$$
(2.8)

Chen et al. (2007) do an argument that the packing density may be assumed to change with radial position as aggregates do not have constant packing throughout the structure. In such a situation,  $a_a$  may be taken as  $\varphi_a = \varphi \left(\frac{a_a}{a}\right)^{3-D}$ , where,  $a_a$  is the radii of aggregates and *a* is primary nanoparticles. The term D is fractal index, which for nanoparticles has a typical value of 1.8. In respect with this, Chen et al. (2007) has modified the Krieger–Dougherty equation as shown by Eq. 2.9.

$$\frac{\mu_{nf}}{\mu_{bf}} = \left[1 - \frac{\varphi_a}{\varphi_m} \left(\frac{a_a}{a}\right)^{1.2}\right]^{-[n] \varphi_m}$$
(2.9)

Nguyen et al. (2008) has proposed a formula for calculating viscosity of nanofluids for 47 and 36 nm particle-sizes and particle volume fractions of 1 and 4% with particle volume concentration,  $\varphi$  and temperature, *T* respectively as shown by Eq. 2.10, Eq. 2.11, Eq. 2.12 and Eq. 2.13.

$$\mu_r = \frac{\mu_{nf}}{\mu_{bf}} = 0.904 \ e^{0.1483 \ \phi}$$
(2.10)

$$\mu_r = \frac{\mu_{nf}}{\mu_{bf}} = 1 + 0.025 \ \phi + 0.015 \ \phi^2 \tag{2.11}$$

$$\mu_r = \frac{\mu_{nf}}{\mu_{bf}} = 1.125 - 0.0007 \ T \tag{2.12}$$

$$\mu_r = \frac{\mu_{nf}}{\mu_{bf}} = 2.1275 - 0.0215 \ T + 0.0002 \ T^2$$
(2.13)

Masoumi et al. (2009) proposed a new analytical model to predict nanofluids viscosity by considering the Brownian motion of nanoparticles and showed its applicability for Al203–water system. The correlation for the effective viscosity of nanofluids is shown as Eq. 2.14.

$$\mu_{nf} = \mu_{bf} + \frac{\rho_N V_B d_N^2}{72 \, C \, \delta} \tag{2.14}$$

In the above expression,  $V_B$ ,  $\rho_N$  and  $d_N$  are Brownian velocity, density and diameter of the nanoparticles respectively. *C* is a correction factor and  $\delta$  represents the distance between the centers of the particles which the equation shown as Eq. 2.15.

$$\delta = \sqrt[3]{\frac{\pi}{6\varphi}} d_N \tag{2.15}$$

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Abu-Nada et al. (2010) performed a two-dimensional regression on experimental data of Nguyen et al. (2007) and developed the following relation including temperature T and volume fraction  $\varphi$  as shown in Eq. 2.16 and Eq. 2.17.

$$\mu_{Al_2O_3} = \exp \left(3.003 - 0.04203 \ T - 0.5445 \ \varphi + 0.0002553 \ T^2 - 0.0534 \ \varphi^2 - 1.622 \ \varphi^{-1}\right)$$
(2.16)

$$\mu_{CuO} = -0.6967 + \left(\frac{15.937}{T}\right) + 1.238\varphi + \left(\frac{1356.14}{T^2}\right) - .259\varphi^2 - 30.88\left(\frac{\varphi}{T}\right) - \left(\frac{1965274}{T^3}\right) + .01593\varphi^3 + 4.38206\left(\frac{\varphi^2}{T}\right) + 147.573\left(\frac{\varphi}{T}\right)$$
(2.17)

Massimo Corcione (2011) proposed an equation for the nanofluids effective dynamic viscosity,  $\mu_{eff}$  normalized by the dynamic viscosity of the base liquid,  $\mu_f$  is derived from a wide variety of experimental data available in the literature. These experimental data relative to nanofluids consisting of alumina, titania, silica and copper nanoparticles with a diameter ranging between 25 nm and 200 nm which suspended in water, ethylene glycol, propylene glycol or ethanol. The best-fit of the selected data enumerated mean empirical correlation with standard deviation of error 1.84% and the equation is shown as Eq. 2.18.

$$\frac{\mu_{eff}}{\mu_f} = \frac{1}{1 - 34.87 \ \left(d_p / d_f\right)^{-0.3} \varphi^{-0.3}}$$
(2.18)

where  $d_f$  is the equivalent diameter of a base fluid molecule, given by Eq. 2.19.

$$d_f = 0.1 \left(\frac{6M}{N\pi\rho_{f0}}\right)^{1/3}$$
(2.19)

in which M is the molecular weight of the base fluid, N is the Avogadro number, and  $\rho_{f0}$  is the mass density of the base fluid calculated at temperature  $T_o = 293$  K.

#### 2.3.1 Viscosity of Nanofluids as a Function of Volume Concentration

Pumping power requirements and convective heat transfer coefficients of fluids depend strongly on the Reynolds and Prandtl number which in turn are highly influenced by viscosity. Thus, accurate determination of viscosity of fluids is important in thermal application. The Einstein (1906) equation is commonly used to predict the effective viscosity of suspensions containing a low volume fraction of particles (usually