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HYBRID FINITE ELEMENT AND MONTE CARLO ANALYSIS OF CRACKED PIPE

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ABSTRACT

This paper presents a hybrid finite element and Monte Carlo analysis for fracture mechanics analysis of cracked structures. Probabilistic aspect is the main focus which related the nature of crack in material. The methodology involves finite element analysis; statistical models for uncertainty in material properties, crack size, fracture toughness and loads; and standard reliability methods for evaluating probabilistic characteristics of fracture parameter. Hybrid finite element and Monte Carlo analysis can provide the failure probability knowing that there is a crack and that the load can reach accidental values defined in a particular range. The probability of failure caused by uncertainties related to loads and material properties of the structure is estimated using Monte Carlo simulation technique. Therefore the proceeding either to repair the structure or it can be justified that an accident will not occur can be decided. Numerical examples are presented to show that probabilistic methodology based on Monte Carlo simulation provides accurate estimates of failure probability for use in fracture mechanics.

***Keywords**:* Probabilistic Fracture Mechanics, Fracture Mechanics, Finite Element Method.

INTRODUCTION

The performance of an engineered system or product is often affected by unavoidable uncertainties (Apostolakis, 1990). It may be attributed to the inhomogeneous material properties. Probabilistic uncertainty analysis quantifies the effect of input random variables on model outputs. The uncertainties inherent in the loading and the properties of mechanical systems necessitate a probabilistic approach as a realistic and rational platform for both design and analysis. Probability theory determines how the uncertainties in crack size, loads, and material properties, when modelled accurately, affect the integrity of cracked structures. Probabilistic fracture mechanics (PFM) provides a more rational means to describe the actual behaviour and reliability of structures than traditional deterministic methods (Provan, 1987). Several methods with various degrees of complexity that can be used to estimate the reliability or safety index or the probability of failure have been developed or implemented. Many of these methods are applicable when the limit state equations are explicit functions of the random variables involved in a problem. Most of these methods are based on a finite element method (FEM). Although FEM based methods are well developed, research in probabilistic analysis has not been widespread and is only currently gaining attention. The originality of mean value first-order second moment (MVFOSM) method was introduced by Cornell (1969). The MVFOSM method based on a first-order Taylor series approximation of the performance function linearized at the mean values of the random variables. However MVFOSM method has obvious deficiencies such as it uses only the first two moments of random variables instead of the complete distribution information (Haldar and Mahadevan, 2001, Youn and Choi, 2004) and it assumes that the response is normally distributed.

Grigoriu et al. (1990) applied first and second order reliability methods (FORM/SORM) to predict the probability of fracture initiation and a confidence interval of the direction of crack extension. The method can account for random loads, material properties, and crack geometry. However, the randomness in crack geometry was modelled by response surface approximations of stress intensity factor as explicit functions of crack geometry. Furthermore, the usefulness of response surface based methods is limited, since they cannot be applied for general fracture mechanics analysis (Guofeng Chen et al., 2001). This paper presents a computational methodology for probabilistic characterization of fracture initiation in cracked structures. The methodology based on finite element method for deterministic stress analysis, statistical models for loads and material properties and Monte Carlo method for probabilistic analysis. Examples are presented to illustrate the proposed methodology lead to sufficiently close results for the cracked structures. The results from these examples show that the methodology is capable of predicting deterministic and probabilistic characteristic for use in fracture mechanics.

FINITE ELEMENT CALCULATION

In order to perform probabilistic analysis, the finite element analysis needs to be well developed. In this study triangular mesh generation using the advancing front method was used. The mesh finally optimised by smoothing and associated boundary conditions are found by interpolation from the initial geometry conditions, then finally producing the output files. The remeshing algorithms place a rosette of quarter point elements around the crack tip, and then rebuild the mesh around the crack tip. A computer code has been developed using FORTRAN programming language for finite element analysis calculation processes, which is based on displacement control for cracked structure modelling. The important parameter used in linear elastic fracture mechanics are the stress intensity factors in various modes. In this paper, the stress intensity factors during simulation steps were calculated by using the displacement extrapolation method, which shown to be highly accurate. In this paper, the displacement extrapolation method (Phongthanapanich and Dechaumphai, 2004) is used to calculate the stress intensity factors as follows:

$$K_I = \frac{E}{3(1+\upsilon)(1+\kappa)}\sqrt{\frac{2\pi}{L}}\left(4\left(v_b - v_d\right) - \frac{\left(v_c - v_e\right)}{2}\right) \tag{1}$$

$$K_{II} = \frac{E}{3(1+\upsilon)(1+\kappa)}\sqrt{\frac{2\pi}{L}}\left(4\left(u_b - u_d\right) - \frac{\left(u_c - u_e\right)}{2}\right) \tag{2}$$

where E is the modulus of elasticity, v is the Poisson's ratio, κ is the elastic parameter defined by

$$\kappa = \begin{vmatrix} (3-4v) & \text{plane strain} \\ (3-v)/(1+v) & \text{plane stress} \end{vmatrix}$$

and L is the quarter-point element length (Alshoaibi et al. 2007).

For the elastic plastic materials, the crack tip is blunted by plasticity. Then the crack tip opening displacement (CTOA) introduced by A.A. Wells in 1961, used as a material crack parameter. Sutton et al. (2000) developed the CTOA criterion based on a detailed analysis of crack kinking, and assumed that the crack growth occurs when the current CTOA reaches a critical value. In this work, the definition for CTOA given by Kannien (1985) is used

$$CTOA = 2\tan^{-1}\left(\frac{L}{2A}\right) \tag{3}$$

The mesh refinement guided by a characteristic size of each element, predicted according to a given error rate and the degree of the element interpolation function. The error estimation for the simulation is based on stress smoothing. It was a point wise error in stress indicator (ESI) to evaluate the accuracy of the finite element solution.

In general, smaller mesh size gives more accurate finite element approximate solution. However, it leads to a greater computational effort as a results the global stiffness matrix becomes larger. The adaptive remeshing process is carried out to obtain optimal element sizes. This scheme is based on a posteriori error estimator which is obtained from the solution of the previous mesh. The strategy used to refine the mesh during the analysis and the numerical convergence studies are also can be found in Alshoaibi et al. (2007). It has been shown that permissible error of 5% is sufficient to obtain the stress intensity factors with satisfactorily high accuracy. The adaptive mesh refinement is based on a posteriori error estimator which is obtained from the solution from the previous mesh. The error estimator used in this paper is based on stress error norm. The strategy used to refine the mesh during analysis process is adopted from Alshoaibi et al. (2007) as follows:
Determine the error norm for each element

$$\|\mathrm{e}\|^{e} = \int_{\Omega^e} \left(\sigma - \sigma^*\right)^{\mathrm{T}} \left(\sigma - \sigma^*\right) d\,\Omega \tag{4}$$

where σ is the stress field obtained from the finite element calculation and σ^* is the smoothed stress field.
(ii) Determine the average error norm over the whole domain

$$\|\hat{\mathrm{e}}\| = \frac{1}{m}\sum_{e=1}^{m} \int_{\Omega^e} \sigma^{\mathrm{T}} \sigma d\,\Omega \tag{5}$$

where m is the total number of elements in the whole domain.

(iii) Determine a variable, ε_e for each element as

$$\varepsilon_e = \frac{1}{\eta}\frac{\left(\|e\|^e\right)^{1/2}}{\left(\|\hat{e}\|\right)^{1/2}} \tag{6}$$

where η is a percentage that measures the permissible error for each element. If $\varepsilon_e > 1$ the size of the element is reduced and vice versa.

(iv) The new element size is determined as

$$\hat{h}_e = \frac{h_e}{\left(\varepsilon_e\right)^{1/p}} \tag{7}$$

where h_e is the old element size and p is the order of the interpolation shape function.

MONTE CARLO SIMLATION TECHNIQUE

The Monte Carlo simulation technique has five essential elements: (1) the problem in terms of all the random variables are defined; (2) the probabilistic characteristics of all the random variables in terms probability density functions (PDFs) and the corresponding parameters are quantified; (3) the values of these random variables are generated; (4) the problem evaluated deterministically for each set of realizations of all the random variables; (5) probabilistic information from number of simulations, such realization are extracted.

Formulation of the problem

Consider a cracked structure under uncertain mechanical and geometric characteristics subject to random loads. Denote by $\boldsymbol{X}$ an N-dimensional random vector with components X_1, X_2,…, X_N characterizing uncertainties in the load, crack geometry, and material properties. For example, if the crack size a, elastic modulus E, far field applied stress magnitude σ^∞, and mode I fracture toughness at crack initiation K_{Ic}, are modelled as input random variables, then $\boldsymbol{X} = g(a, E, \sigma^\infty, K_{Ic})$. Let stress intensity factor K, be a relevant crack driving force that can be calculated using standard finite element analysis. Suppose the structure fails when $K > K_{Ic}$. This requirement cannot be satisfied with certainty, since K is dependent on the input vector $\boldsymbol{X}$ which is random, and K_{Ic} itself to be a random variable. The K is evaluated by finite element method which can be expressed in Equation (1).

Quantifying the Probabilistic Characteristics Of Random Variables

Mathematical modelling or representation of a random variable is thus a primary task in any probabilistic formulation, which needs to be conducted systematically. In practice, the choice of probability distribution may be dictated by mathematical convenience or by familiarity with a distribution. In some cases, the physical process may suggest a specific form of distribution. As an example, elastic modulus E is frequently modelled as a Gaussian random variable for DENT specimen. The underlying distribution can be established by conducting some statistical tests known as Goodness-of-fit tests for distribution. The commonly used statistical test for this purpose is the Kolmogorov-Smirnov (K-S) tests. The K-S test compares the observed cumulative frequency and the cumulative density function (CDF) of an assumed theoretical distribution. The data was arranged in increasing order for the first step. Then the maximum difference between the two CDFs of the ordered data estimated by using

$$D_n = \max \left| F_X(x_i) - S_n(x_i) \right| \tag{8}$$

where $F_X(x_i)$ is the theoretical CDF of the assumed distribution at the ith observation of the ordered samples x_i, and $S_n(x_i)$ is the corresponding stepwise CDF of the observed ordered samples. $S_n(x_i)$ can be estimated as

$$S_n(x_i) = \begin{Bmatrix} 0, x < x_1 \\ \dfrac{m}{n}, x_m \le x \le x_{m+1} \\ 1, x \ge x_n \end{Bmatrix} \tag{9}$$

The concept is shown in Figure 2. Mathematically, D_n is a random variable and its distribution depends on the sample size n. The CDF of D_n can be related to the significance level α as

$$P(D_n \le D_n^{\alpha}) = 1 - \alpha \tag{10}$$

Then, according to the K-S test, if the maximum difference D_n is less than or equal to the tabulated value D_n^{α}, the assumed distribution is acceptable at the significance level α.

Generation of random numbers

The N random numbers generated for elastic modulus E according to its probabilistic characteristics and another N random numbers for σ^{∞}, which is uniformly distributed. The generation of random numbers according to a specific distribution is the heart of Monte Carlo simulation. Corresponding to an arbitrary seed value, the generators produced the required number of uniform random numbers between 0 and 1. By changing the seed value, different sets of random numbers can be generated. Depending upon the size of the computer, the random numbers may be repeated. Then, the uniform random numbers u_i between 0 and 1, transformed to random numbers with the appropriate characteristics. In this method, the CDF of the random variable is equated to the generated random number u_i, that is, $F_X(x_i) = u_i$, and the equation is solved for x_i as

$$x_i = F_X^{-1}(u_i) \tag{11}$$

If X is normally distributed, that is, $N(u_X, \sigma_X)$, then $S = (X - \mu_X)/\ \sigma_X$ is a standard normal variate, that is, $N(0, 1)$. It can be shown that

$$u_i = F_X(x_i) = \Phi(s_i) = \Phi\left(\frac{x_i - \mu_X}{\sigma_X}\right) \tag{12}$$

Thus

$$x_i = \mu_X + \sigma_X s_i = \mu_X + \sigma_X \Phi^{-1}(u_i) \tag{13}$$

For Equation (12), the u_i values first need to be transformed to s_i, that is, $s_i = \Phi^{-1}(u_i)$, and Φ^{-1} is the inverse of the CDF of a standard normal variable.

If the random variable X is lognormally distributed with parameters λ_x and ζ_x, then the ith random number x_i according to the lognormal distribution can be generated as

$$u_i = \Phi\left(\frac{\ln x_i - \lambda_X}{\zeta_X}\right) \tag{14}$$

Evaluation of the Problem

The N generated random numbers for each of the random variables in the problem gave N sets of random numbers, each set representing a realization of the problem. The generated sample points for the output or response, then used to calculate the probability of failure considering various performance criteria. The accuracy of the evaluation will increase as the number of simulations M, increases. The DENT and cracked pipe specimens are considered to carry comprehensively evaluate the modelling of uncertainty by the developed program in the forthcoming section.

Probability of Failure

Consider the limit state represented by $\boldsymbol{X} = g(a, E, \sigma^{\infty}, K_{Ic})$ corresponding to a failure mode for a structure. With all the random variables assumed to be statistically independent, the Monte Carlo simulation approach consists of drawing samples of the variables according to their PDFs and then feeding them into the mathematical model $g()$. The samples thus obtained gave the probabilistic characteristics of the response random variable $\boldsymbol{X}$. It is known that if the value of K is over than K_{Ic}, it indicates failure. Let M_f be the number of simulation cycles when K is over than K_{Ic} and let M be the total number of simulation cycles. Therefore, an estimate of the probability of failure P_f can be expressed as

$$P_f = \frac{M_f}{M} \tag{15}$$

RESULTS AND DISCUSSION

The pipes of nuclear plants undergo great thermal and mechanical cycles which can lead to initiation and propagation of cracks. When a crack is observed, the problem is to know whether it is suitable to repair the structure as a priority or if it can be justified that an accident will not occur. Therefore, the probabilistic analysis can provide the failure probability knowing that there is a crack and that the load can reach accidental values defined in a particular range. Figure 1 shows an axisymmetrically cracked pipe under internal pressure and axial tension. Due to the boundary conditions at the pipe ends, the applied hydraulic pressure induces, beside the radial pressure, longitudinal tension forces.

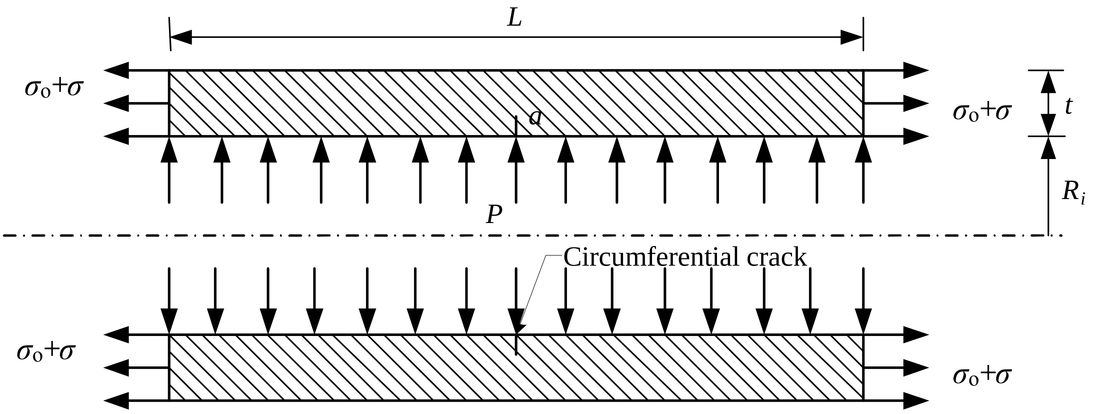

Figure 1: Axisymmetrically cracked pipe

The system variables are described as follows:

- a, the crack length (15 mm)
- L, the pipe length (1000 mm)
- P, the internal pressure (15.5 MPa)
- R_i, the inner radius (393.5 mm)
- t, the thickness (62.5 mm)
- σ, the applied tensile stress (varying from 100 up to 200 MPa). It represents the load effect which could accidentally increase, knowing that the nominal value is around 100 MPa.
- σ_0, the stress due to the end effects, given by

$$\sigma_0 = P \frac{R_i^2}{(R_i + t)^2 - R_i^2}$$

Figure 2 depicts an adaptive finite element mesh of cracked pipe. A half model was used to take advantage of the symmetry. Table 1 lists the means, COV and probability distributions of elastic modulus, crack tip opening angle, applied tensile stress and yield strength. The Poisson's ratio of $v = 0.3$ was assumed to be deterministic.

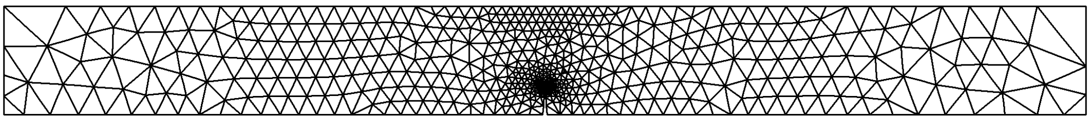

Figure 2: An adaptive mesh for cracked pipe

Figure 3 shows the comparisons of the probability of failure, P_f using present study method and published results done by Pendola et al. (2000) for the cracked pipe. The continuous lines in Figure 3 represent the values of P_f obtained from combinations of ANSYS-RYFES software. The circle points in Figure 3 indicate the P_f from this study involving elastic-plastic analysis. The P_f values from this study are comparatively closer to the Pendola et al. (2000) solution.

Table 1: Statistical properties of random input for cracked pipe

Random variable	Mean	COV	Probability distribution
Elastic modulus, E	175.5GPa	0.05	Lognormal
Crack tip opening angle, CTOA	5.25^{o}	0.15	Gaussian
Applied tensile stress, σ	150MPa	0.19	Gaussian
Yield strength, σ_y	260.5MPa	0.05	Lognormal

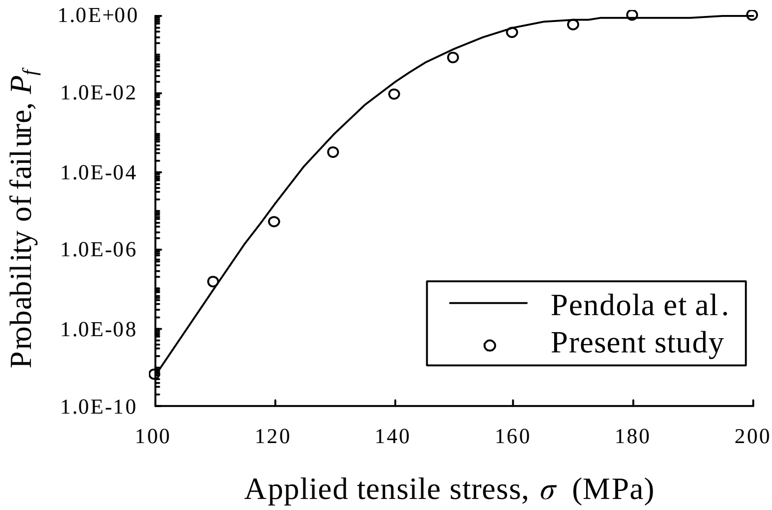

Figure 3: Failure probability of cracked pipe by Pendola et al. (2000) and present study.

CONCLUSION

The probabilistic method has been presented for fracture mechanics analysis of cracked structures. The numerical example presented in this paper is derived on linear-elastic and elastic-plastic fracture mechanics based failure criterion. The methodology involves development of finite element analysis codes, statistical models for uncertainty and probabilistic analyses using Monte Carlo simulation. The numerical implementations lead to sufficiently close results and attest the quality of the solution of the cracked model. The calculation of P_f is equivalent to the other methods obtained by relevant researchers. The results from these examples indicate that the methodology is capable of determining accurate probabilistic analyses in fracture mechanics.

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