*National Conference in Mechanical Engineering Research and Postgraduate Studies (2nd NCMER 2010)*
*3-4 December 2010, Faculty of Mechanical Engineering, UMP Pekan, Kuantan, Pahang, Malaysia; pp. 852-862*
*ISBN: 978-967-0120-04-1; Editors: M.M. Rahman, M.Y. Taib, A.R. Ismail, A.R. Yusoff, and M.A.M. Romlay*

# ESTIMATION OF FORCED CONVECTION HEAT TRANSFER COEFFICIENT OF NANOFLUIDS USING CONCEPT OF COLBURN ANALOGY

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## ABSTRACT

The concept of Colburn analogy is used in the development of equation for the evaluation of nanofluid forced convection heat transfer coefficients flowing inside a tube. The experiments in the turbulent range of Reynolds numbers with nanofluids of less than 4.0% volume concentration indicate good agreement of the values of friction factor with the Blasius equation. The nanofluid forced convection experimental data available in literature for a wide range of volume concentration, nano materials and particle size is used in the development of the Colburn type equation. The equation developed is in satisfactory agreement with Dittus Boelter and Gnielinski correlations.

*Keywords:* Nanofluids, Colburn Analogy, Heat Transfer Coefficient

## INTRODUCTION

Forced convection heat transfer in a tube has long been recognized as one of the many basic heat transfer problems. The constitutive equation of flow along with the energy equation is solved subject to suitable boundary conditions for obtaining heat transfer coefficient. Analysis for the determination of heat transfer coefficients has been undertaken for a wide range of Prandtl numbers of single phase fluids. With miniaturization the need for transferring high heat fluxes arose, it is observed that the conventional fluids such as water, ethylene glycol, engine oil have limitations to meet the desired performance expectations due to low values of thermal conductivity. Various active and passive methods for enhancement of heat transfer coefficient are pursued to overcome the problem. Passive methods include the use of additives and micron size suspended metallic particles. The use of these particles proved futile due to problems associated with clogging, sedimentation, erosion of pipe lines and large pressure drop. The problem could be overcome with the advent of new techniques for producing smaller particles of nanometer size.

The passive method of heat transfer enhancement can be obtained by the addition of additives to liquids. Solid particles have thermal conductivities several times higher than those of the conventional fluids. Ultra fine solid particles can be used to suspend them uniformly to enhance the thermal conductivity of the fluid. Metallic, non-metallic and polymeric particles can be added to liquids to form slurries. However the usual slurries with suspended particles of the order of millimetres or even micrometers can cause severe problems such as clogging, erosion, etc. associated with higher pressure drop. Furthermore, they suffer from instability and rheological problems. The use of nanometre size particles for use as heat transfer fluid is initiated by a research group at the Argonne National Laboratory. Choi (1995) coined the word 'nano fluids' who observed very high values of thermal conductivity compared to suspended particles of millimetre or micrometer dimension. The nanofluids showed better stability and rheological properties, dramatically higher thermal conductivities with no significant penalty on pressure drop.

The determination of the friction coefficient, *f* and Nusselt number, *Nu* are essential in the determination of forced convection heat transfer coefficient. Therefore, it is very desirable to have a relation between *f* and *Nu* to calculate one when the other is available. Such relations are developed on the basis of the similarity between momentum and heat transfer in boundary layers. The Colburn analogy is applicable for pure fluids and cannot be applied for nanofluids directly. The present work is focused on the development of equation for the estimation of forced convection heat transfer coefficient of nanofluids.

## ANALOGIES BETWEEN MOMENTUM AND HEAT TRANSFER

The study of heat and mass transfer in turbulent flows has been heavily influenced by the formulation of an analogy with the better known law for momentum transfer by Reynolds (1874).

$$St = \frac{C_f}{2} \tag{1}$$

Where;

$St$ Stanton number, $h/(\rho C_p V)$ (2)

$C_f$ Fanning friction factor, $2\tau_{wall}/(\rho V^2)$ (3)

$C_p$ specific heat, J/kg K

$V$ average fluid velocity, m/s

$h$ mean heat transfer coefficient, W/(m$^2$K)

The analogy works well for the region outside the relatively thin layer where from the wall diffusion is important but the exact connection between the wall and the outer region has been elusive (Trinh, 2010b). Thus a number of empirical correlations were proposed to provide useful tools for engineering design. The most widely referred is perhaps the modified Reynolds analogy or Chilton-Colburn analogy (1964).

$$St\,\mathrm{Pr}^{-2/3} = \frac{C_f}{2} \tag{4}$$

$$j = St\,\mathrm{Pr}^{2/3} = \frac{C_f}{2} \tag{5}$$

Where;

$j$ Colburn factor

Pr Prandtl number, $(C_p \mu)/K$ (6)

Though certain semi-theoretical correlations were later derived, (Metzner and Friend, 1958, and Trinh, 1969) the Colburn analogy is often quoted and the j factor is used.

### The Colburn Analogy

The basic mechanism and mathematics of heat, mass and momentum transport are essentially the same. Among other analogies such as Reynolds and Prandtl-Taylor, Colburn was developed to relate heat transfer coefficients with Darcy friction factor '*f*' for flow in a tube.

$$St\,\mathrm{Pr}^{2/3} = \frac{f}{8} \tag{7}$$

Where;

$$f = \frac{2\left(\Delta P\right)d}{L\rho V^{2}} \tag{8}$$

Experimental data with nanofluids in the volume concentration of less than 4.0% is available. Hence it is proposed to develop an equation for the determination of nanofluid heat transfer coefficient in a similar manner.

## EVALUATION OF PROPERTIES

The thermo-physical properties of water and nanofluids such as density, absolute viscosity, specific heat and thermal conductivity are estimated with the regression equations given by Azmi et al. (2010) in Table 1 and Table 2.

Table 1: Properties of water where $T_b$ in $^{o}$C ( $25 \le T_b \le 100$ and $0.07\% \le AD \le 2.75\%$ )

| **Property** | **Regression Equation for water** |
|---|---|
| Density | $\rho_w = 1000 \times \left[1.0 - \frac{\left(T_b - 4.0\right)^2}{119000 + 1365 \times T_b - 4 \times \left(T_b\right)^2}\right]$ |
| Viscosity | $\mu_w = 0.0015 - 3.16325e - 5 \times T_b + 3.04789e - 7 \times \left(T_b\right)^2 - 1.1104e - 9 \times \left(T_b\right)^3$ |
| Thermal conductivity | $k_w = 0.55994 + 0.00216 \times T_b - 1.02749e - 5 \times \left(T_b\right)^2 + 6.72794e - 9 \times \left(T_b\right)^3$ |
| Specific heat | $C_w = 4217.629 - 3.20888 \times T_b + 0.09503 \times \left(T_b\right)^2 - 0.00132 \times \left(T_b\right)^3 + 9.415e - 6 \times \left(T_b\right)^4$<br>$- 2.5479e - 8 \times \left(T_b\right)^5$ |

The thermo-physical properties of water based nanofluids are presented in Table 2.

Table 2: The thermo-physical properties of nanofluids where $T_b$ in °C ( $20 \le T_b \le 70$ and $AD \le 4.2\%$ )

| **Property** | **Regression Equation for nanofluids** |
|---|---|
| Viscosity | $\mu_r = \frac{\mu_{nf}}{\mu_w} = 0.9042 + 0.1245\,[\phi] - 0.08445\left[\frac{T_{nf}}{72}\right] + 0.6436\left[\frac{d_p}{170}\right]$ |
| Thermal conductivity | $k_r = \frac{k_{nf}}{k_w} = 0.9808 + 0.0142\,[\phi] + 0.2718\left[\frac{T_{nf}}{70}\right] - 0.1020\left[\frac{d_p}{150}\right]$ |
| Specific heat | $C_{pr} = \frac{C_{pnf}}{C_{pw}} = 1.036 - 0.0298\,[\phi] - 0.07261\left[\frac{T_{nf}}{70}\right]$ |

## ANALYSIS OF EXISTING DATA

The friction factor and Nusselt number of different water-based nanofluids available in literature (Fotukian and Esfahany, 2010, Sunder and Sharma, 2010, Yu et al., 2009, Williams et al., 2008 and Xuan and Li, 2004) are used in the development of equation valid for both water and nanofluid having base liquid as water.

The proposed correlations for nanofluid in two different forms are

$$St\,\mathrm{Pr}_w^{2/3} = \frac{f}{8}\left(1 + \phi\,\mathrm{Pr}_{nf}\right)^{0.1083} \tag{9}$$

$$Nu = \frac{f}{8}\left(1 + \phi\,\mathrm{Pr}_{nf}\right)^{0.1083} \mathrm{Re}\,\mathrm{Pr}_w^{\;1/3} \tag{10}$$

Where;

| | |
|---|---|
| $\mathrm{Pr}_w, \mathrm{Pr}_{nf}$ | Prandtl number of water and nanofluid respectively |
| $\phi$ | nanofluid volume concentration, % |
| $f$ | friction factor |
| $C_p$ | specific heat |

The Nusselt number of pure fluids can be estimated with the equations of Gnielinski (1976) and modified Dittus-Boelter (1930) applicable for nanofluids are given respectively as

$$Nu = \frac{\left(\frac{f}{2}\right)(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12.7\left(\frac{f}{2}\right)^{0.5}\left(\mathrm{Pr}^{2/3} - 1\right)}, \text{ where } f = \left(1.58\ln\mathrm{Re} - 3.82\right)^{-2} \tag{11}$$

The equation of Gnielinski applicable in the transition-turbulent Reynolds number range and valid for single phase fluid valid in the range $2300 < \mathrm{Re} < 5 \times 10^6$ and $0.5 < \mathrm{Pr} < 2000$

Another equation applicable for both water and nanofluid is developed given by

$$Nu = 0.0304\,\mathrm{Re}^{0.7853}\,\mathrm{Pr}^{0.4}\left[0.001 + \phi\right]^{0.01398} \tag{12}$$

valid in the range of $10000 < \mathrm{Re} < 25000,\ 2.0 < \mathrm{Pr} < 10$ and $\phi < 3.7\%$. Substituting $\phi = 0$ in Eq. (12) simplifies to

$$Nu = 0.0276\,\mathrm{Re}^{0.7853}\,\mathrm{Pr}^{0.4} \qquad (13)$$

which is in close agreement with the values obtained with Dittus-Boelter equation for water.

## RESULTS AND DISCUSSION

The experimental values of Nusselt number for water and nanofluids is shown in Fig. 1 along with the values estimated with Eq. (10). The good agreement between the values confirms the reliability of the equation proposed. The experimental data of water is shown with values estimated with different equations available in literature in Fig. 2 for wide range of experimental Prandtl numbers. It can be observed that the present equation is able to predict closer to experimental data compared to Gnielinski (1976) and Dittus-Boëlter equations in the range of data presented. The experimental data of Fotukian and Esfahany (2010) with CuO nanofluid is shown in comparison with the proposed equation in Fig. 3. The authors have stated the particle size to vary between 30 and 50nm. The Prandtl number of the nanofluid estimated is observed to vary between 1.4 and 4.7. The Nusselt number estimated from the Eq. (10) for Pr =1.4 under predicts the experimental data as can be seen in Fig. 3.

A comparison of the experimental data of Sundar and Sharma (2010) with values evaluated with Eq. (10) for extreme values of experimental Prandtl numbers is shown in Fig. 4. The authors have conducted experiments with $Al_2O_3$ nanofluid with a particle size of 47nm in the volume concentration range of 0.02 to 0.5%. Experimental data of Yu et al. (2009) with SiC dispersed in water at 3.7% volume concentration in the range $3300 < \mathrm{Re} < 13000$ is shown along with the proposed equation values in Fig. 5. The experimental Nusselt number is shown with values evaluated with Eq. (10) in the ranges of Prandtl number of 4.6 and 7.1 and bulk temperature of 34 and $57^{o}$C.

The variation of Nusselt number with Reynolds number undertaken with Alumina and Zirconia nanofluids at different concentrations obtained by Williams et al. (2008) is shown in Fig. 6 along with the values estimated with Eq. (10) in the temperature range of 26 and $70^{o}$C. Fig. 7 shows a comparison of experimental Nusselt number of Cu in water nanofluid undertaken by Xuan and Li (2003) with values estimated from Eq. (10). The experimental data of water and nanofluid at different concentrations is shown enclosed by the values estimated for Prandtl number range of 5.3 to 10.0. The deviations of a few experimental data points with the proposed equation observed in Figs. 3 to 7 can be due to variation in the property values employed in the analysis.

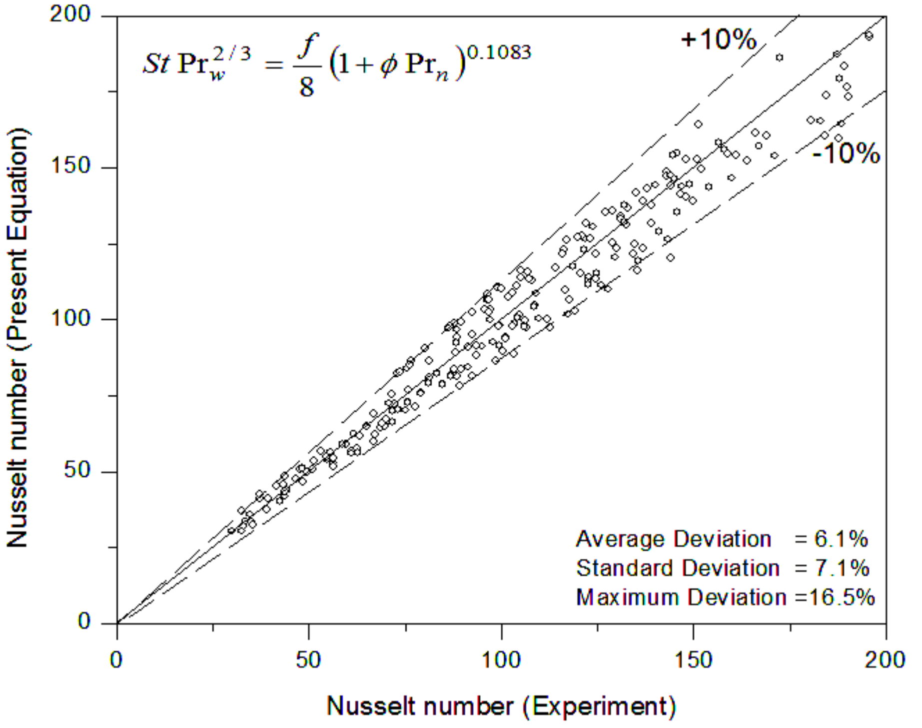

Figure 1: Comparison of experimental data with present equation for Nusselt number

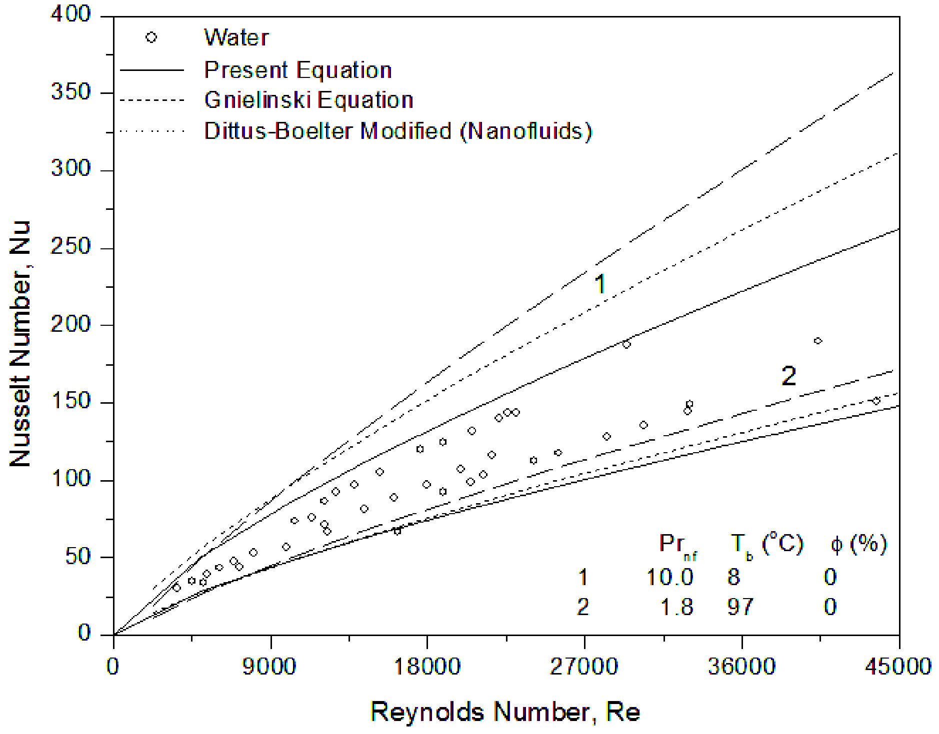

Figure 2: Comparison of experimental data with present equation and other equations for water

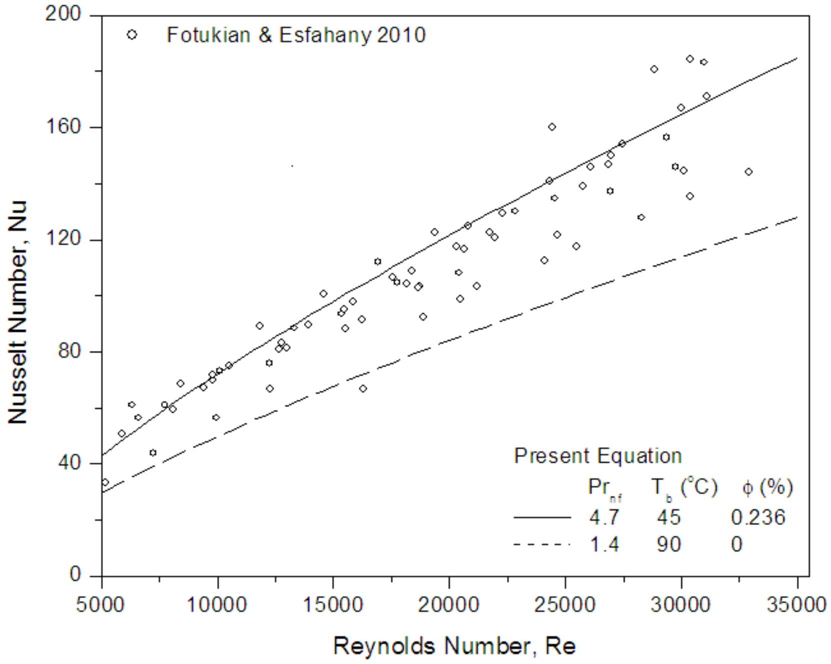

Figure 3: Comparison of experimental Nusselt number with present equation for CuO/water nanofluid

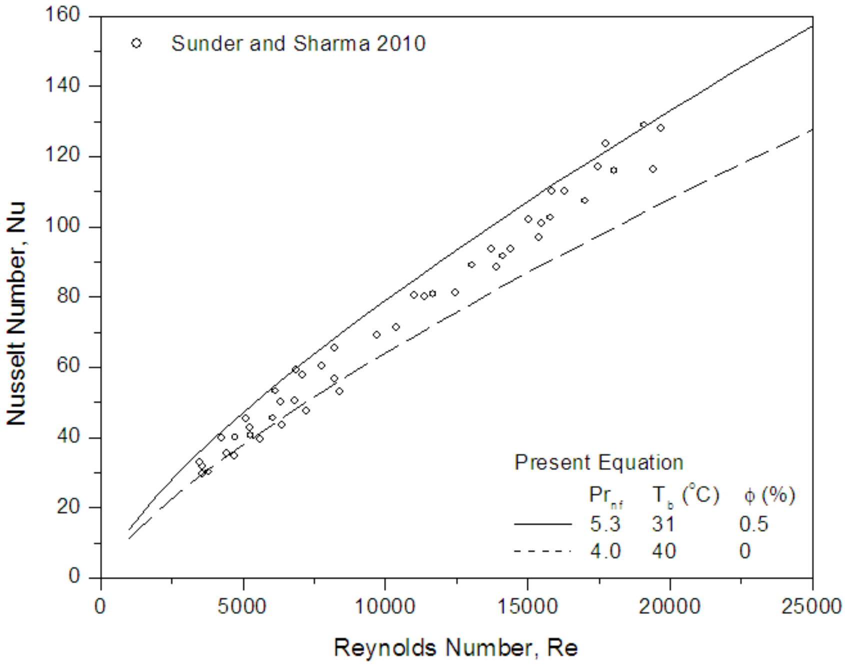

Figure 4: Comparison of experimental Nusselt number with present equation for $Al_2O_3$/water nanofluids

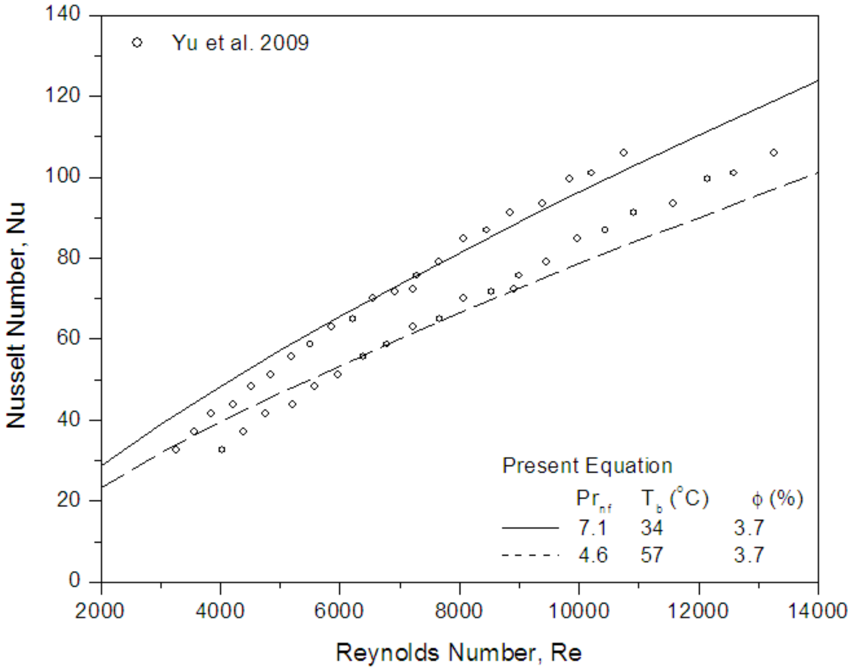

Figure 5: Comparison of experimental Nusselt number with present equation for SiC/water nanofluids

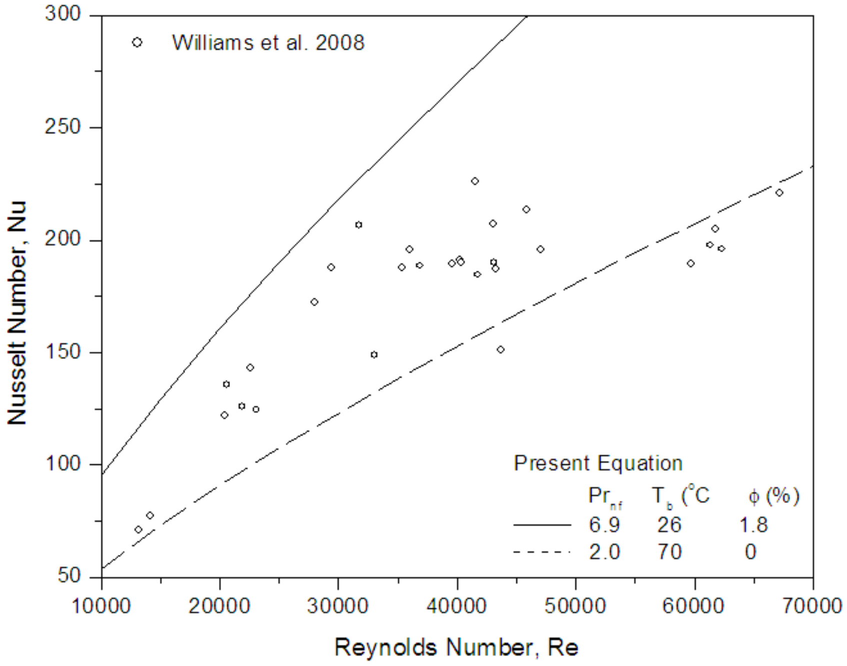

Figure 6: Comparison of experimental Nusselt number with present equation for $Al_2O_3$/water and $ZrO_2$/water nanofluids

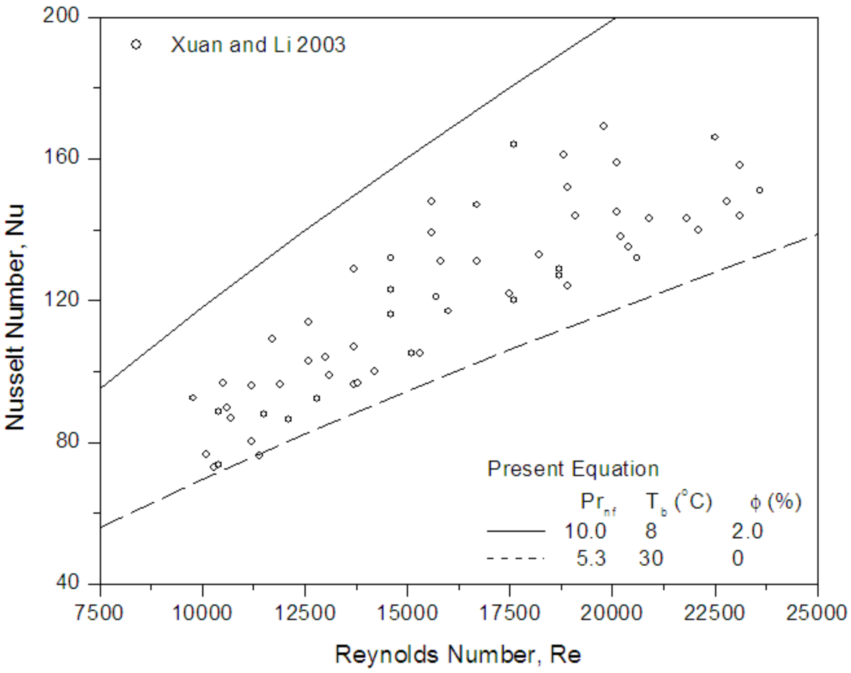

Figure 7: Comparison of experimental Nusselt number with present equation for Cu/water nanofluids

## CONCLUSION

The equation for the estimation of nanofluid forced convection Nusselt number is developed in structure similar to Colburn equation and is given as $St\,\mathrm{Pr}_w^{2/3} = \frac{f}{8}\left(1+\phi\,\mathrm{Pr}_{nf}\right)^{0.1083}$. The equation is valid for water based nanofluids for $\phi \le 3.7\%$, $10000 \le \mathrm{Re} \le 70000$ and $1.4 \le \mathrm{Pr} \le 10.0$. The Nusselt number estimated with the equation predict values close to experimental data for different nanofluids tested.

## ACKNOWLEDGEMENT

The financial support by Universiti Malaysia Pahang is gratefully acknowledged wide RDU number RDU0903100 and RDU100349.

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## Nomenclature

| | |
|---|---|
| $C_f$ | Fanning friction factor |
| $C_p$ | specific heat, J/ (K.kg) |
| $d$ | diameter, m |
| $f$ | Darcy friction factor |
| $h$ | mean heat transfer coefficient, W/(m$^2$.K) |
| $j$ | Colburn factor |
| $k$ | thermal conductivity, J/ (K.m) |
| $L$ | length, m |
| $Nu$ | Nusselt number |
| Pr | Prandtl number |
| $\Delta P$ | pressure difference |
| $Re$ | Reynolds number |
| $St$ | Stanton number |
| $T$ | temperature, $^{o}$C |
| $V$ | velocity, m/s |

### *Greek symbols*

| | |
|---|---|
| $\phi$ | volume concentration of nanofluids, % |
| $\mu$ | absolute viscosity, kg/(m.s) |
| $\rho$ | density, kg/m$^3$ |
| $\tau$ | shear stress |

### *Subscripts*

| | |
|---|---|
| $b$ | bulk |
| $nf$ | nanofluid |
| $p$ | nanoparticle |
| $r$ | ratio |
| $w$ | water |

### *Abbreviation*

| | |
|---|---|
| AD | average deviation |