EXACT SOLUTIONS FOR SOME TYPES OF NEWTONIAN AND NON-NEWTONIAN FLUIDS

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We hereby declare that we have checked this thesis and in our opinion, this thesis is adequate in term of scope and quality for the award of the degree Doctor of Philosophy in Mathematics.

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I hereby declare that the work in this thesis is based on my original work except for quotations and citation which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

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EXACT SOLUTIONS FOR SOME TYPES OF NEWTONIAN AND NON-NEWTONIAN FLUIDS

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Thesis submitted in fulfilment of the requirements for the awards of the degree of Doctor of Philosophy (Mathematics)

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TO MY BELOVED FAMILY ESPECIALLY MY FATHER AND MOTHER
THANK YOU FOR EVERYTHING
ACKNOWLEDGEMENT

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<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$A$</td>
<td>Acceleration</td>
</tr>
<tr>
<td>$B_0$</td>
<td>Strength of magnetic field</td>
</tr>
<tr>
<td>$B_b$</td>
<td>Imposed magnetic field</td>
</tr>
<tr>
<td>$b$</td>
<td>Body force per unit mass vector</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Induced magnetic field</td>
</tr>
<tr>
<td>$C$</td>
<td>Species concentration</td>
</tr>
<tr>
<td>$C_w$</td>
<td>Species concentration near the plate</td>
</tr>
<tr>
<td>$C_\infty$</td>
<td>Species concentration far away from the plate</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Heat capacity at constant pressure</td>
</tr>
<tr>
<td>$(C_p)_f$</td>
<td>Base fluid heat capacity at constant pressure</td>
</tr>
<tr>
<td>$(C_p)_s$</td>
<td>Solid particle heat capacity at constant pressure</td>
</tr>
<tr>
<td>$(C_p)_{nf}$</td>
<td>Nanofluid heat capacity at constant pressure</td>
</tr>
<tr>
<td>$D$</td>
<td>Mass diffusivity</td>
</tr>
<tr>
<td>$D_f$</td>
<td>Base fluid mass diffusivity</td>
</tr>
<tr>
<td>$D_{nf}$</td>
<td>Nanofluid mass diffusivity</td>
</tr>
<tr>
<td>$E$</td>
<td>Electric current</td>
</tr>
<tr>
<td>$e$</td>
<td>Specific internal energy</td>
</tr>
<tr>
<td>$Gr$</td>
<td>Grashof number</td>
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<tr>
<td>$Gm$</td>
<td>Modified Grashof number</td>
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<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
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<tr>
<td>$H(t)$</td>
<td>Unit step function</td>
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</table>
\( h \)  Heat transfer coefficient
\( \mathbf{I} \)  Body couple per unit mass vector
\( \mathbf{i} \)  Unit vector
\( \mathbf{J} \)  Current density
\( j \)  Microinertia per unit mass
\( K \)  Porosity parameter
\( K_1 \)  Chemical reaction parameter
\( K_f \)  Base fluid thermal conductivity
\( K_s \)  Solid particle thermal conductivity
\( K_{CNT} \)  Carbon nanotubes thermal conductivity
\( K_{nf} \)  Nanofluid thermal conductivity
\( k \)  Thermal conductivity
\( k_i \)  Permeability
\( k^* \)  Mean absorption coefficient
\( M \)  Magnetic parameter
\( M(\cdot) \)  WhittakerM function
\( \mathbf{N} \)  Angular velocity
\( \mathbf{N} \)  Microrotation vector
\( \text{Nu} \)  Nusselt number
\( n \)  Microelement
\( \text{Pr} \)  Prandtl number
\( p \)  Pressure
\( p_y \)  Yield stress
\( Q \)  Heat generation rate per unit volume
\( Q_o \)  Heat generation parameter
\( q \)  Laplace transform parameter
\( q_r \)  Radiative heat flux
\( R \)  Radiation parameter
\( R \)  Darcy's resistance
\( S \)  Dimensionless heat generation parameter
\( Sc \)  Schmidt number
\( Sh \)  Sherwood number
\( T \)  Temperature of the fluid
\( T_\infty \)  Ambient temperature
\( t \)  Time
\( U \)  Amplitude of plate oscillations
\( u \)  Velocity components in \( x \)-direction
\( v \)  Velocity components in \( y \)-direction
\( x \)  Coordinate axis parallel to the plate
\( y \)  Coordinate axis normal to the plate
\( \omega t \)  Phase angle

**Greek symbols**
\( \alpha \)  Casson parameter
\( \alpha_{nf} \)  Nanofluid thermal diffusivity
\( \beta \)  Microrotation parameter
\( \beta_T \)  Volumetric coefficient of thermal expansion
\( \beta_C \)  Volumetric coefficient of mass expansion
\( \chi \)  Spin gradient viscosity coefficient
\( \phi \) Nanoparticle volume fraction

\( \varphi \) Porosity of the medium

\( \zeta \) Similarity variable

\( \gamma \) Newtonian heating parameter

\( \eta \) Spin gradient viscosity parameter

\( \kappa \) Vortex viscosity coefficient

\( \lambda \) Dimensionless chemical reaction parameter

\( \mu \) Dynamic viscosity

\( \mu_B \) Plastic dynamic viscosity

\( \mu_f \) Base fluid dynamic viscosity

\( \mu_{nf} \) Nanofluid dynamic viscosity

\( \nu \) Kinematic viscosity

\( \nu_{nf} \) Nanofluid kinematic viscosity

\( \theta \) Dimensionless temperature

\( \rho \) Fluid density

\( \rho_f \) Base fluid density

\( \rho_s \) Solid particle density

\( \rho_{nf} \) Nanofluid density

\( \rho_{CNT} \) Carbon nanotubes density

\( \sigma \) Electrical conductivity

\( \sigma^* \) Stefan Boltzmann constant

\( \sigma_f \) Base fluid electric conductivity

\( \sigma_s \) Solid particle electric conductivity
\( \sigma_{nf} \)  Nanofluid electric conductivity

\( \sigma_{CNT} \)  Carbon nanotubes electric conductivity

\( \tau \)  Skin friction

\( \tau_{ij} \)  Shear stress

\( \omega \)  Frequency of oscillation

\( \Phi \)  Dimensionless concentration

**Subscripts**

- \( CNT \)  Carbon nanotubes
- \( f \)  Base fluid
- \( nf \)  Nanofluid
- \( s \)  Solid particle
- \( w \)  Condition at wall
- \( \infty \)  Condition at infinity

**Superscripts**

- *  Dimensional variables
- \( p \)  Scalar constant
- tr  Transpose
## LIST OF ABBREVIATIONS

<table>
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<tr>
<th>Abbreviation</th>
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<tbody>
<tr>
<td>CNT</td>
<td>Carbon nanotube</td>
</tr>
<tr>
<td>CBL</td>
<td>Concentration boundary layer</td>
</tr>
<tr>
<td>MBL</td>
<td>Momentum boundary layer</td>
</tr>
<tr>
<td>MHD</td>
<td>Magnetohydrodynamic</td>
</tr>
<tr>
<td>MWCNT</td>
<td>Multi walls carbon nanotube</td>
</tr>
<tr>
<td>NH</td>
<td>Newtonian Heating</td>
</tr>
<tr>
<td>SWCNT</td>
<td>Single wall carbon nanotube</td>
</tr>
<tr>
<td>TBL</td>
<td>Temperature boundary layer</td>
</tr>
<tr>
<td>erf</td>
<td>Error function</td>
</tr>
<tr>
<td>erfc</td>
<td>Complementary error function</td>
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