

EXACT SOLUTIONS FOR SOME TYPES OF  
NEWTONIAN AND NON-NEWTONIAN  
FLUIDS

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We hereby declare that we have checked this thesis and in our opinion, this thesis is adequate in term of scope and quality for the award of the degree Doctor of Philosophy in Mathematics.

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### **STUDENT'S DECLARATION**

I hereby declare that the work in this thesis is based on my original work except for quotations and citation which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

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AND NON-NEWTONIAN FLUIDS

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***TO MY BELOVED FAMILY ESPECIALLY MY FATHER AND MOTHER  
THANK YOU FOR EVERYTHING***

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## LIST OF SYMBOLS

$A$	Acceleration
$B_0$	Strength of magnetic field
$\mathbf{B}_0$	Imposed magnetic field
$\mathbf{b}$	Body force per unit mass vector
$\mathbf{b}_1$	Induced magnetic field
$C$	Species concentration
$C_w$	Species concentration near the plate
$C_\infty$	Species concentration far away from the plate
$C_p$	Heat capacity at constant pressure
$(C_p)_f$	Base fluid heat capacity at constant pressure
$(C_p)_s$	Solid particle heat capacity at constant pressure
$(C_p)_{nf}$	Nanofluid heat capacity at constant pressure
$D$	Mass diffusivity
$D_f$	Base fluid mass diffusivity
$D_{nf}$	Nanofluid mass diffusivity
$\mathbf{E}$	Electric current
$e$	Specific internal energy
$Gr$	Grashof number
$Gm$	Modified Grashof number
$\mathbf{g}$	Acceleration due to gravity
$H(t)$	Unit step function

$h_s$	Heat transfer coefficient
$\mathbf{I}$	Body couple per unit mass vector
$\mathbf{i}$	Unit vector
$\mathbf{J}$	Current density
$j$	Microinertia per unit mass
$K$	Porosity parameter
$K_1$	Chemical reaction parameter
$K_f$	Base fluid thermal conductivity
$K_s$	Solid particle thermal conductivity
$K_{CNT}$	Carbon nanotubes thermal conductivity
$K_{nf}$	Nanofluid thermal conductivity
$k$	Thermal conductivity
$k_1$	Permeability
$k^*$	Mean absorption coefficient
$M$	Magnetic parameter
$M(.)$	WhittakerM function
$N$	Angular velocity
$\mathbf{N}$	Microrotation vector
$Nu$	Nusselt number
$n$	Microelement
$Pr$	Prandtl number
$p$	Pressure
$p_y$	Yield stress
$Q$	Heat generation rate per unit volume



$Q_0$	Heat generation parameter
$q$	Laplace transform parameter
$q_r$	Radiative heat flux
$R$	Radiation parameter
$\mathbf{R}$	Darcy's resistance
$S$	Dimensionless heat generation parameter
$Sc$	Schmidt number
$Sh$	Sherwood number
$T$	Temperature of the fluid
$T_\infty$	Ambient temperature
$t$	Time
$U$	Amplitude of plate oscillations
$u$	Velocity components in $x$ -direction
$v$	Velocity components in $y$ -direction
$x$	Coordinate axis parallel to the plate
$y$	Coordinate axis normal to the plate
$\omega t$	Phase angle

### **Greek symbols**

$\alpha$	Casson parameter
$\alpha_{nf}$	Nanofluid thermal diffusivity
$\beta$	Microrotation parameter
$\beta_T$	Volumetric coefficient of thermal expansion
$\beta_C$	Volumetric coefficient of mass expansion
$\chi$	Spin gradient viscosity coefficient

$\phi$	Nanoparticle volume fraction
$\varphi$	Porosity of the medium
$\xi$	Similarity variable
$\gamma$	Newtonian heating parameter
$\eta$	Spin gradient viscosity parameter
$\kappa$	Vortex viscosity coefficient
$\lambda$	Dimensionless chemical reaction parameter
$\mu$	Dynamic viscosity
$\mu_B$	Plastic dynamic viscosity
$\mu_f$	Base fluid dynamic viscosity
$\mu_{nf}$	Nanofluid dynamic viscosity
$\nu$	Kinematic viscosity
$\nu_{nf}$	Nanofluid kinematic viscosity
$\theta$	Dimensionless temperature
$\rho$	Fluid density
$\rho_f$	Base fluid density
$\rho_s$	Solid particle density
$\rho_{nf}$	Nanofluid density
$\rho_{CNT}$	Carbon nanotubes density
$\sigma$	Electrical conductivity
$\sigma^*$	Stefan Boltzmann constant
$\sigma_f$	Base fluid electric conductivity
$\sigma_s$	Solid particle electric conductivity

$\sigma_{nf}$	Nanofluid electric conductivity
$\sigma_{CNT}$	Carbon nanotubes electric conductivity
$\tau$	Skin friction
$\tau_{ij}$	Shear stress
$\omega$	Frequency of oscillation
$\Phi$	Dimensionless concentration

### **Subscripts**

$CNT$	Carbon nanotubes
$f$	Base fluid
$nf$	Nanofluid
$s$	Solid particle
$w$	Condition at wall
$\infty$	Condition at infinity

### **Superscripts**

*	Dimensional variables
$p$	Scalar constant
tr	Transpose

## LIST OF ABBREVIATIONS

CNT	Carbon nanotube
CBL	Concentration boundary layer
MBL	Momentum boundary layer
MHD	Magnetohydrodynamic
MWCNT	Multi walls carbon nanotube
NH	Newtonian Heating
SWCNT	Single wall carbon nanotube
TBL	Temperature boundary layer
erf	Error function
erfc	Complementary error function

EXACT SOLUTIONS FOR SOME TYPES OF NEWTONIAN  
AND NON-NEWTONIAN FLUIDS

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## ABSTRACT

It is well known that many manufacturing processes in industry nowadays involve Newtonian and non-Newtonian fluids. In nature, water and air are examples of Newtonian fluid. Whereas apple sauce, drilling muds, ketchup sauce, shampoo, blood and many others belongs to the class of non-Newtonian fluids. Due to the variety of such fluids it is very difficult to suggest a single constitutive equation which can describe the rheological behavior of all these fluids. Therefore, several models of Newtonian and non-Newtonian fluids have been proposed. In this thesis, the unsteady flows of Casson and micropolar fluids are considered over an oscillating vertical plate whereas magnetohydrodynamic flow of a nanofluid over an accelerated vertical plate and linear stretching sheet are studied. All these problems are modelled using the fundamental equations of fluid dynamics. The proposed model of each problem depends on a system of governing equations along with imposed initial and boundary conditions. Appropriate non-dimensional variables are introduced to reduce the governing equations along with imposed conditions into dimensionless forms. Exact solutions of each problem are obtained by using the Laplace transform method. Moreover, the classical solutions corresponding to the Stokes first problem and Newtonian fluid are also obtained as a special case. The exact solutions of velocity, temperature and concentration are plotted graphically and discussed for different parameters. Results show that velocity decreases significantly with an increasing of Casson parameter but it increases when Grashof number and Newtonian heating parameter are increased. Further, in the case of micropolar fluid, angular velocity increases near the plate and decreases away from the plate due to an increase in viscosity parameter. It is found that temperature increases due to suspension of different nanoparticles as well as carbon nanotubes water based nanofluids. The exact solutions obtained in the present study are very important not only because they are solutions of some fundamental flows, but also due to the fact that they serve as accuracy standards for approximate methods, whether numerical, asymptotic or experimental.

## ABSTRAK

Telah diketahui umum bahawa kebanyakan industri proses pembuatan hari ini melibatkan bendalir Newtonan dan tak Newtonan. Secara semulajadi, udara dan air adalah contoh-contoh bendalir Newtonan. Manakala sos epal, komposit penggerudian, sos tomato, syampu, darah dan lain-lainnya tergolong dalam kelas bendalir tak Newtonan. Disebabkan oleh kepelbagaian bendalir, adalah sangat sukar untuk mencadangkan satu persamaan jujuk yang boleh menerangkan ciri-ciri reologi bagi semua bendalir ini. Oleh itu, beberapa model bendalir Newtonan dan tak Newtonan telah dicadangkan. Dalam tesis ini, aliran tak mantap bagi bendalir Casson dan mikrokutub telah dipertimbangkan ke atas plat tegak berayun manakala aliran nanobendalir magnetohidrodinamik ke atas plat tegak memecut dan helaian meregang linear telah dikaji. Semua masalah ini dimodelkan menggunakan persamaan asas bagi dinamik bendalir. Model yang dicadangkan bagi setiap masalah bergantung kepada sistem persamaan menakluk bersama dengan syarat awal dan syarat sempadan yang ditetapkan. Pembolehubah tak bermatra yang sesuai diperkenalkan untuk menurunkan persamaan menakluk dan syarat yang ditetapkan kepada bentuk tak bermatra. Penyelesaian tepat bagi setiap masalah diperolehi dengan menggunakan kaedah penjelmaan Laplace. Selain itu, penyelesaian klasik yang sepadan dengan masalah pertama Stoke dan bendalir Newtonan juga diperolehi sebagai kes khas. Penyelesaian tepat bagi halaju, suhu dan kepekatan diplotkan secara grafik dan dibincangkan dengan parameter yang berbeza. Keputusan kajian menunjukkan bahawa halaju berkurangan dengan ketara dengan peningkatan parameter Casson tetapi meningkat apabila nombor Grashof dan parameter pemanas Newtonan bertambah. Selanjutnya, bagi kes bendalir mikrokutub, halaju bersudut meningkat menghampiri plat dan menyusut menjauhi plat dengan peningkatan parameter kelikatan. Didapati bahawa suhu meningkat disebabkan oleh sebatian nanopartikel yang berbeza termasuklah nanotub karbon iaitu nanobendalir berasaskan air. Penyelesaian tepat yang diperolehi dalam kajian ini amat penting bukan sahaja kerana ia merupakan penyelesaian kepada beberapa aliran asas, tetapi juga disebabkan oleh ketepatan piawai bagi kaedah anggaran, sama ada secara berangka, asimptot atau secara eksperimen.

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