Data-Driven PID Tuning Based on Safe Experimentation Dynamics for Control of Liquid Slosh

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***Abstract*—An introductory research about a data-driven PID tuning for the control of liquid slosh system based on Safe Experimentation Dynamics (SED) is presented in this paper. A performance comparison between the SED and Simultaneous Perturbation Stochastic Approximation (SPSA) based method for data-driven PID tuning is observed and discussed. The performance is evaluated by numerical examples in terms of tracking performance, control input energy and computation time. The simulation results demonstrated that SED based data-driven PID successfully reduced the liquid slosh whilst the desired position of the cart is achieved. In addition, smaller control input energy is used.**

Keywords—Safe Experimentation Dynamics, Data-Driven PID tuning, Slosh Control System.

I. INTRODUCTION

In any moving container contain with liquid should generate the liquid sloshing inside which can affect a system dynamics and performance. As an example, a hazardous overturns of a large liquid cargo from the forces induced by the slosh [1]. Therefore, slosh control is important in suppressing the residual slosh induced by the container motion [2].

Liquid slosh control system is a class of the underactuated mechanical systems. This class of system is well known for the very challenging problem of the controller design due to the small number of input space dimension compared to its controlled variable. In recent decades, various control strategies for slosh control systems have been widely reported [1], [3], [4], [5], [6], [7], [8]. The common ground shared in these reported works are hard to apply in practice due to their control designs derived based on the model of the systems. This is because of the control designs normally impose a huge gap between control theory and real applications due to the unmodeled dynamics problem. In order to vanish this gap, control researchers are now diverting to model-free control approach, since its controller design is based on only the input and output data measurement of actual process plants. In other words, this scheme does not explicitly include any parts or the whole of plant models, which make its control performance independence of the plant model accuracy. Particularly, a model-free proportional-integral-derivative (PID) tuning is one of the promising model-free control strategies due to its simplicity, understandable and more reliable to the industrial usage.

It is crucial to design a PID tuning scheme that perform lesser computation time. For example, the simultaneous perturbation stochastic approximation (SPSA) approach has been widely implemented in the data-driven controller tuning due to its effectiveness in using only two objective functions to update the controller parameters [2], [9], [10]. Nevertheless, there is a probability that the of design parameters in SPSA changed to huge values and abruptly trapped in an unpractical region. In order to solve this weakness, the modified SPSA algorithm is introduced in [11], where the saturation function is used to avoid the unstable solution during the optimization [2]. However, with the introduction of this saturation function, there are additional parameters to be determine in advanced, which may require a lot of effort. Moreover, the saturation function will also limit the searching capability of the algorithm. In order to overcome these problems, the safe experimentation dynamics (SED) [12], [13] based method is seen a promising tool for this view point. It is because the SED algorithm produces stable convergence, less parameters to be used and robust to any disturbance and uncertain dynamics changes during tuning process. Hitherto, there are few works to discuss the application of the SED to model-free PID tuning problems. Therefore, it would be useful to investigate the proposed method competency in tuning the slosh control system.

This research thus implements the SED algorithm in a data-driven PID tuning for slosh control system. The proposed controller effectiveness will be evaluated by numerical examples. Hence, the efficiency of the tuning scheme will be evaluated based on the tracking performance, control input energy and computation time. Furthermore, a comparative assessment between the SED based method and SPSA based method, are discussed.

The reminder of this paper is structured as follows; the data-driven PID controller problem formulation for slosh control tuning is expressed in Section II. While in section III, the safe experimentation dynamics implementation is explained. Section IV presented the simulation results and discussion. Finally, a conclusion of findings is presented in section V.

Notation: $\mathbb{R}$ and $\mathbb{R}_+$ respectively symbolize the set of real numbers and the set of positive real numbers.

II. Problem Formulation

The same control system of PID controller for liquid slosh plants as in [2] is considered as illustrated in Fig. 1, $r(t)$ expressed as the reference, $u(t)$ is the control input, $y(t)$ is the lateral axis of the tank measurement and $\theta(t)$ is the slosh angle measurement. While plant G, represented the liquid slosh system.

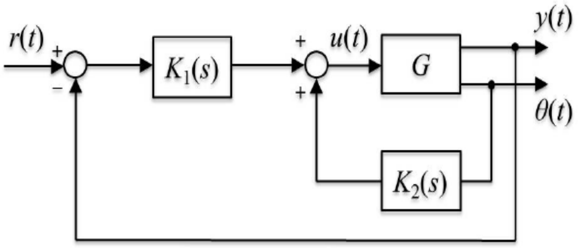

Fig. 1. Liquid Slosh PID Control System[2]

$K_i(s)$, $i = 1,2$ is the controller that given for the PID controller

$$K_i(s) = P_i\left(1 + \frac{1}{I_i s} + \frac{D_i s}{1 + (D_i / N_i)s}\right) \tag{1}$$

where $P_i \in \mathbb{R}$, $I_i \in \mathbb{R}$, $D_i \in \mathbb{R}$ and $N_i \in \mathbb{R}$ is the proportional gain, integral time, derivative time and filter coefficient respectively. Let the performance indexes given as

$$\hat{e} = \int_{t_0}^{t_f} |r(t) - y(t)|^2 dt, \tag{2}$$

$$\hat{\theta} = \int_{t_0}^{t_f} |\theta(t)|^2 dt, \tag{3}$$

$$\hat{u} = \int_{t_0}^{t_f} |u(t)|^2 dt, \tag{4}$$

where the time interval $[t_0, t_f]$ correspond to the period of the performance evaluation, $t_0 \in \{0\} \cup \mathbb{R}_+$ and $t_f \in \mathbb{R}_+$. The objective function is interpreted as follows

$$J(\mathbf{P}, \mathbf{I}, \mathbf{D}, \mathbf{N}) = w_1\hat{e} + w_2\hat{\theta} + w_3\hat{u}, \tag{5}$$

where $\mathbf{P} := [P_1 \quad P_2]^{\mathrm{T}}$, $\mathbf{I} := [I_1 \quad I_2]^{\mathrm{T}}$, $\mathbf{D} := [D_1 \quad D_2]^{\mathrm{T}}$ and $\mathbf{N} := [N_1 \quad N_2]^{\mathrm{T}}$. $w_1 \in \mathbb{R}$, $w_2 \in \mathbb{R}$ and $w_3 \in \mathbb{R}$ are the designated weighting coefficients by the designer and the values are chosen with the same method of the standard Linear Quadratic Regulator (LQR) problem. The expressions $w_1\hat{e}$ and $w_2\hat{\theta}$ in (5) correspond to the tracking error, while $w_3\hat{u}$ expression corresponds to the control input energy. Thus, the optimization problem for data-driven PID controller can be expressed as follows:

Problem 2.1. Find the $K(s)$, a PID controller where the control objective $J(\mathbf{P}, \mathbf{I}, \mathbf{D}, \mathbf{N})$ with respect to **P, I, D**, and **N** is minimizes in corresponded with the measurement data $(u(t), y(t), \theta(t))$, for the PID control system in Fig. 1.

III. Data-Driven PID controller design based on Safe Experimentation Dynamics

A data-driven PID tuning scheme based on SED algorithm [13] is described in this section.

A. Safe Experimentation Dynamics

The optimization problem is considered as

$$\min_{\boldsymbol{p} \in \mathbb{R}^n} f(\boldsymbol{p}) \tag{6}$$

where the objective function is represented by $f : \mathbb{R}^n \to \mathbb{R}$ and $\boldsymbol{p} \in \mathbb{R}^n$ is the design parameter. The optimal solution $\boldsymbol{p}^* \in \mathbb{R}^n$ of optimization problem in (6) is obtain by repetitively updating the design parameter using the SED algorithm.

The update law for the SED algorithm is given by

$$p_i(k+1) = h(\bar{p}_i - K_g r_2) \tag{7}$$

where $k = 0,1,\cdots,$ is the number of iteration, $p_i \in \mathbb{R}$ is the ith element of $\boldsymbol{p} \in \mathbb{R}^n$, $\bar{p}_i \in \mathbb{R}$ is the ith element of $\bar{\boldsymbol{p}} \in \mathbb{R}^n$ and $\bar{\boldsymbol{p}}$ is used to store the current best value of the design parameters, K_g is a scalar that defined the interval size to decide on the random steps on $p_i \in \mathbb{R}$ and $r_2 \in \mathbb{R}$ is a value of random number. The function h in (7) is given as follows

$$h(\cdot) = \begin{cases} p_{\max}, & \bar{p}_i - K_g r_2 > p_{\max} \\ p_{\min}, & \bar{p}_i - K_g r_2 < p_{\min} \end{cases}, \tag{8}$$

where $p_{\max}$ and $p_{\min}$ are the pre-defined maximum and minimum values of design parameter respectively. The step-by-step procedures of SED algorithm are as follows:

Step 1: Determine the value for $p_{\min}$, $p_{\max}$, K_g and E. Then, set $k = 0$ and the initial condition for design parameter is set as $\boldsymbol{p}(0)$ and the objective function be $f(\boldsymbol{p}(0))$. Therefore, as default $\bar{\boldsymbol{p}} = \boldsymbol{p}(0)$ and $\bar{f} = f(\boldsymbol{p}(0))$.

Note that E is a scalar that define the probability to use a new random setting for $\boldsymbol{p}$. Variable $\bar{f}$ is used to store the current best value of the objective function.

Step 2: If the value of $f(\boldsymbol{p}(k)) < \bar{f}$, execute $\bar{\boldsymbol{p}} = \boldsymbol{p}(k)$ and $\bar{f} = f(\boldsymbol{p}(k))$. Otherwise, proceed to step 3.

Step 3: Generate a random number r_1. If $r_1 < E$, generate second random number r_2 and obtain the value for $p_i(k+1)$ by using the update law in (7). Otherwise, $p_i(k+1) = \bar{p}_i$.

Note that, $r_1 \in \mathrm{R}$ is value of random number which is chosen by uniformly distribute between 0 and 1, while r_2 is between $p_{\min}$ and $p_{\max}$.

Step 4: Obtain the objective function $f(p_i(k+1))$.

Step 5: If the pre-stated termination condition is satisfied, the algorithm terminates with the solution $\boldsymbol{p}^* := \arg \min_{\boldsymbol{p} \in \{\boldsymbol{p}(0), \boldsymbol{p}(1), \ldots, \boldsymbol{p}(k+1)\}} f(\boldsymbol{p})$. Otherwise, set $k = k+1$ and continue to step 2.

The pre-stated termination condition is based on the designated maximum number of iteration, $k_{\max}$.

B. Data-Driven PID Controller Design

For this section, the data-driven PID controller design is presented where the design parameter is expressed as:

$$\psi = [P_1\ P_2\ I_1\ I_2\ D_1\ D_2\ N_1\ N_2]^{\mathrm{T}} \in \mathrm{R}^8. \tag{9}$$

In this study, the logarithmic scale is employed to ψ to accelerate the exploration of the design parameter. In particular, we set $\psi_i = 10^{p_i}\ (i = 1, 2, \ldots, 8)$ and the objective function is expressed as $J = [10^{p_1}\ \ 10^{p_2}\ \ \cdots\ \ 10^{p_8}]^{\mathrm{T}}$. Then, the data-driven PID procedures can be described as follows:

Step 1: Let $f(\boldsymbol{p}) := J(\boldsymbol{P}, \boldsymbol{I}, \boldsymbol{D}, \boldsymbol{N})$ and $p_i = \log \psi_i$. Then, determine the maximum iteration number, $k_{\max}$.

Step 2: Perform the SED algorithm in section III-A for the objective function in (5).

Step 3: After reaching $k_{\max}$, attain the optimal output $p^* = \bar{p}(k_{\max})$. Then, apply the $\psi^* = [10^{p_1^*}\ \ 10^{p_2^*}\ \ \cdots\ \ 10^{p_8^*}]^{\mathrm{T}}$ to $K_i(s)$ in the PID control system in Fig. 1.

IV. Simulation Results

The efficiency of performance for the proposed SED based data-driven PID tuning and comparison with SPSA based method are shown in this section.

A. Model of the Liquid Slosh

A rectilinear motion of liquid slosh model in [2], [13] as illustrated in Fig. 2 is considered. Then the system's dynamic equations, which is produced by the Euler-Lagrange equations is expressed in y and θ as follow:

$$M\ddot{y} + ml\cos\theta\ddot{\theta} - ml\dot{\theta}^2\sin\theta = u, \tag{10}$$

$$ml\cos\theta\ddot{y} + ml^2\ddot{\theta} + d\dot{\theta} - mgl\sin\theta = 0, \tag{11}$$

where M, m, l, d and g are respectively the mass of the tank with liquid, mass of the liquid, liquid slosh hypotenuse length, damping coefficient and gravity. Let outputs of the liquid slosh system measurement expressed by $[y(t)\quad \theta(t)]^T = G(u(t))$. Thus, to reduce the slosh angle θ in a moving tank whilst obtaining the desired position y is the control objective of the system.

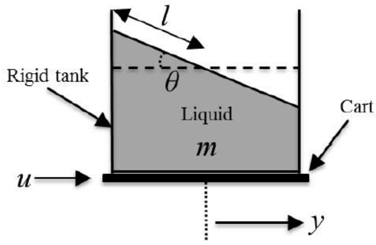

Fig. 2. Liquid Slosh Motion[2]

B. Numerical Example

The parameters for the liquid slosh system in section IV-A is considered as shown in Table I.

TABLE I. LIQUID SLOSH MODEL PARAMETERS

Parameter	Value	Unit
M	6.0	kg
m	1.32	kg
l	0.052126	m
d	3.0490×10-4	kg m^2/s
g	9.81	m/s^2

These parameters are identified by applying a quick-stop experiment as described in [15] and. all the parameters are depended on the tank geometry and, liquid fill ration and characteristics [2]. The given cart position reference is

$$r(t) = \begin{cases} 0, & 0 \le t \le 1, \\ 0.5, & 1 < t \le 20. \end{cases} \tag{12}$$

The parameters of SED algorithm are set as $p_{\max} = 3.0$, $p_{\min} = -2.5$, $E = 0.66$, $K_g = 0.022$ and $k_{\max} = 400$. In the SPSA algorithm, the objective function is executed two times in order to approximate the gradient. Therefore, we set the maximum iterations number for SPSA as $k_{\max} = 200$ for fair comparative assessment with SED. Furthermore, we set $w_1 = 100$, $w_2 = 100$, $w_3 = 5$, $t_0 = 0$ and $t_f = 20$. Next, the SED algorithm's initial condition for $\boldsymbol{p}(0)$ is specified as in

Table II, where a stable closed-loop system is expected during the evaluation period [2].

The objective function responses after 400 iterations for SED while 200 iterations for SPSA based method are shown in Figs. 3 and 4, respectively. It displays that, the objective function is successfully minimized by SED based method after 400 iterations. Since SED algorithm only requires one objective function evaluation per iteration, it requires 400 iterations to obtain equivalent number of objective function evaluation compared with SPSA based method. The optimal design parameters are shown in Table II, while the control performances, in terms of the objective function, norm of error and norm of input, are depicted in Table III. Based on the results, it reveals that the SED based method is capable to minimize the objective function more efficient than SPSA based method. The average computation time taken by SED method also lesser than SPSA method (refer Table III).

Next, the cart position responses are shown in Fig.5 for both methods, where the black colored line indicated the reference signal for the cart position, the blue colored line indicated the initial response of the cart position, while the red and green colored lines were indicated the response after 200 iterations for SPSA and 400 iterations for SED respectively. Based on the graph, the time taken for the cart to settle at the desired position is around 11s for the SED based method and 12s for the SPSA based method. In terms of slosh response as shown in Fig. 6, the SED based method successfully reduces the slosh magnitude. Nevertheless, a significant minimum value of control input energy is used by SED approach as compared to SPSA, as shown in Fig. 7.

TABLE II. DESIGN PARAMETER

ψ	**PID Gain**	***p*** **(0)**	**ψ corresponded to *p* (0) (×10³)**	***p****	**ψ* corresponded to *p**(10³)**
ψ_1	P_1	1.0000	0.0100	0.2056	0.0016
ψ_2	P_2	3.5000	3.1623	2.4360	0.2729
ψ_3	I_1	0.0000	0.0010	0.2542	0.0018
ψ_4	I_2	1.0000	0.0100	0.3102	0.0020
ψ_5	D_1	2.0000	0.1000	0.9516	0.0089
ψ_6	D_2	1.0000	0.0100	-0.4674	0.0003
ψ_7	N_1	0.0000	0.0010	-1.3822	0.0000
ψ_8	N_2	1.0000	0.0100	0.4479	0.0028

TABLE III. STATISCAL RESULTS

Algorithm		**SPSA**	**SED**
J(**P**, **I**, **D**, **N**)	Mean	42.4064	**41.3431**
	Best	41.6587	**41.1790**
	Worst	43.7629	**41.7055**
	Std	0.5079	**0.0967**
Total Norm of Error ($\hat{e}+\hat{\theta}$)	Mean	0.3219	**0.3125**
	Best	0.3102	**0.3075**
	Worst	0.3515	**0.3204**
	Std	0.0086	**0.0028**
Total Norm of Input ($\hat{u}$)	Mean	2.0924	**2.0370**
	Best	1.8211	1.9515
	Worst	2.4319	**2.1366**
	Std	0.1473	**0.0483**
Average Computation Time, s		84.2601	**52.3314**

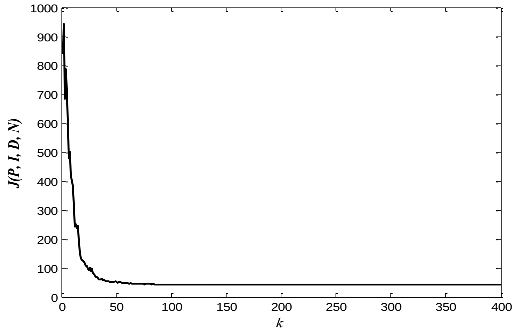

Fig. 3. Response of objective function using SED

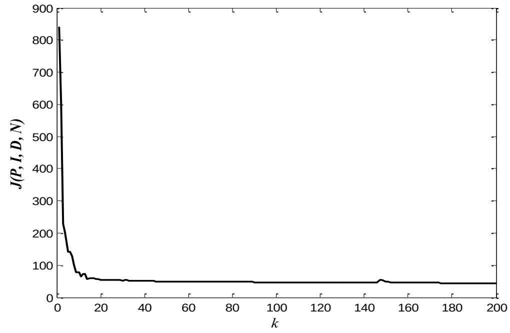

Fig. 4. Response of objective function using SPSA

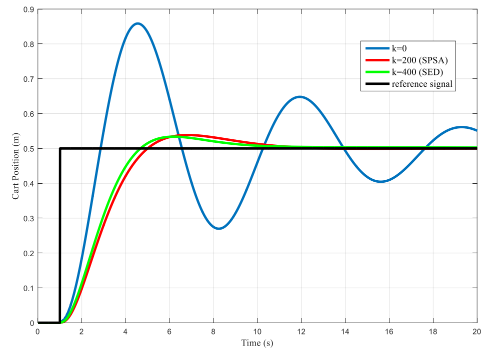

Fig. 5. Cart position responses

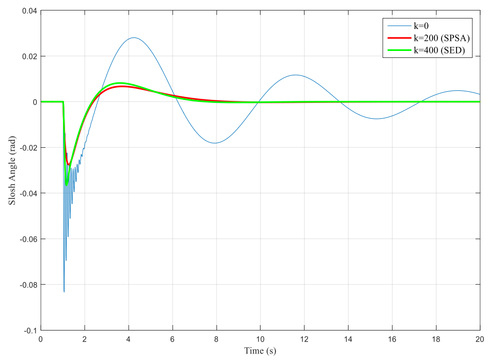

Fig. 6. Slosh angle responses

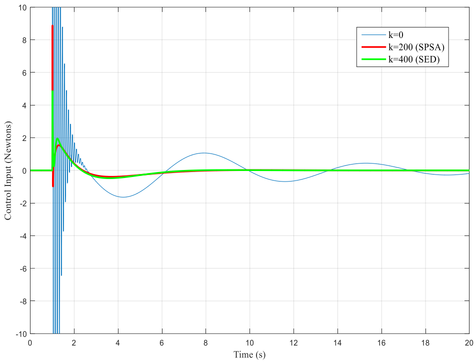

Fig. 7. Control input responses

Remark 4.1. The results obtained in this paper are performed in Matlab/SIMULINK setting by using CPU: Intel Pentium G4400 (3.30GHz), RAM: 4GB.

V. Conclusion

An introductory research on data-driven PID tuning based on SED method and performance comparison with SPSA based method for liquid slosh control has been presented in this paper. The SED method has been verified by applying to the liquid slosh control system as in [2], [13]. In the simulation results, it is demonstrated that the data-driven PID tuning based on SED successfully reduced liquid slosh, whilst reaching the desired cart position, with significant minimum control input energy is used compared to SPSA method.

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