

# A New Structure for Scaling Functions System with Dyadic Intervals

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**Abstract.** A scaling functions system is a series of subspaces  $\{V_j\}_{j \in \mathbb{Z}}$  that are embedded and spanned by a group of scaling basis functions  $\{\phi_{j,k}\}$ . To fully grasp how to construct this system using a unique function  $\phi(x) \in L^2(I_{j,k})$  when  $\{I_{j,k}\}$  is the Dyadic intervals set, its structure is studied. The Dyadic intervals structure show us that is no any intersection appear between their sub intervals at the different scaling and shifting values. This property introduced a new way to prove the orthonormality of this system by using the supported intervals of the step functions. A new scaling relation  $P_k$  called the scaling filter is defined on Dyadic intervals, is used to characterize this system. This filter allows for analyzing  $L^2(I_{j,k})$  and other spaces by multi-resolution analysis, as well as it provides some of the requisite conditions. To explain the structure of this system, the clarity examples are given.