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UNSTE

ND SECOND

GRADE FLUIDS IN A POROUS MEDIUM

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## LIST OF SYMBOLS

### Roman Letters

$A_1$	-	first Rivilin-Ericksen tensor
$A_2$	-	second Rivilin-Ericksen tensor
$B$	-	Total magnetic field
$B_0$	-	applied magnetic field
$B_0$	-	magnitude of applied magnetic field
$b$	-	body force
$C$	-	concentration of the fluid
$c_p$	-	specific heat at constant pressure
$\frac{d}{dt}$	-	material time derivative
<b>div</b>	-	divergence
$E$	-	electric field
erf	-	error function
erfc	-	complementary error function
exp	-	exponential function
$Gr$	-	thermal Grashof number
$Gm$	-	mass Grashof number
$g$	-	gravitational acceleration
$H(.)$	-	Heaviside function
$I$	-	identity tensor
$J$	-	current density
$K$	-	dimensionless porosity parameter
$k$	-	thermal conductivity
$L$	-	characteristic length
$\mathcal{L}$	-	Laplace transform
$\mathcal{L}^{-1}$	-	Inverse Laplace transform

$M$	-	dimensionless magnetic parameter
$Nu$	-	Nusselt number
$Pr$	-	Prandtl number
$p$	-	scalar pressure
$p^*$	-	modified pressure gradient
$\mathbf{R}$	-	Darcy's resistance
$R$	-	radiation parameter
$s$	-	Laplace transform parameter
$Sc$	-	Schmidt number
$\mathbf{T}$	-	Cauchy stress tensor
$T$	-	temperature of the fluid near the plate
$t$	-	dimensionless time
$t_0$	-	characteristic time
$F$	-	complex velocity
$\mathbf{F}$	-	body force vector
$u$	-	velocity in $x$ -direction
$v$	-	velocity in $y$ -direction
$\mathbf{V}$	-	velocity vector field
$\boldsymbol{\Omega}$	-	angular velocity vector
$\Omega$	-	constant angular velocity
$x$	-	dimensionless coordinate axis of the plate
$y$	-	dimensionless coordinate axis of the plate
$z$	-	dimensionless coordinate axis normal to the plate

**Greek Letters**

$\alpha_1, \alpha_2$	-	material moduli or normal stress moduli
$\alpha$	-	dimensionless second grade parameter
$\beta$	-	volumetric coefficient of thermal expansion
$\rho$	-	density
$\sigma$	-	electrical conductivity
$\nu$	-	kinematic viscosity
$\mu$	-	dynamic viscosity
$\omega$	-	dimensionless rotating parameter
$\phi$	-	porosity of the medium
$\tau$	-	dimensionless skin-friction
$\theta$	-	dimensionless heat absorption coefficient
$\varphi$	-	porosity of porous medium
$I_0(\cdot)$	-	modified bessel function of order zero
$I_1(\cdot)$	-	modified bessel function of order one

**Subscripts**

$w$	-	conditions on the wall
$\infty$	-	free stream condition

**Superscript**

$T$	-	transpose operation
$*$	-	dimensional sign