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ND SECOND

GRADE FLUIDS IN A POROUS MEDIUM

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LIST OF SYMBOLS

Roman Letters

A_1	-	first Rivlin-Ericksen tensor
A_2	-	second Rivlin-Ericksen tensor
B	-	Total magnetic field
B_0	-	applied magnetic field
B_0	-	magnitude of applied magnetic field
b	-	body force
C	-	concentration of the fluid
c_p	-	specific heat at constant pressure
$\frac{d}{dt}$	-	material time derivative
div	-	divergence
E	-	electric field
erf	-	error function
erfc	-	complementary error function
\exp	-	exponential function
Gr	-	thermal Grashof number
Gm	-	mass Grashof number
g	-	gravitational acceleration
$H(.)$	-	Heaviside function
I	-	identity tensor
J	-	current density
K	-	dimensionless porosity parameter
k	-	thermal conductivity
L	-	characteristic length
\mathcal{L}	-	Laplace transform
\mathcal{L}^{-1}	-	Inverse Laplace transform

M	-	dimensionless magnetic parameter
Nu	-	Nusselt number
Pr	-	Prandtl number
p	-	scalar pressure
p^*	-	modified pressure gradient
R	-	Darcy's resistance
R	-	radiation parameter
s	-	Laplace transform parameter
Sc	-	Schmidt number
\mathbf{T}	-	Cauchy stress tensor
T	-	temperature of the fluid near the plate
t	-	dimensionless time
t_0	-	characteristic time
F	-	complex velocity
\mathbf{F}	-	body force vector
u	-	velocity in x -direction
v	-	velocity in y -direction
\mathbf{V}	-	velocity vector field
Ω	-	angular velocity vector
Ω	-	constant angular velocity
x	-	dimensionless coordinate axis of the plate
y	-	dimensionless coordinate axis of the plate
z	-	dimensionless coordinate axis normal to the plate

Greek Letters

α_1, α_2	-	material moduli or normal stress moduli
α	-	dimensionless second grade parameter
β	-	volumetric coefficient of thermal expansion
ρ	-	density
σ	-	electrical conductivity
ν	-	kinematic viscosity
μ	-	dynamic viscosity
ω	-	dimensionless rotating parameter
ϕ	-	porosity of the medium
τ	-	dimensionless skin-friction
θ	-	dimensionless heat absorption coefficient
φ	-	porosity of porous medium
$I_0(.)$	-	modified bessel function of order zero
$I_1(.)$	-	modified bessel function of order one

Subscripts

w	-	conditions on the wall
∞	-	free stream condition

Superscript

T	-	transpose operation
$*$	-	dimensional sign

PERPUSTAKAAN UMP



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ND SECOND

GRADE FLUIDS IN A POROUS MEDIUM

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ABSTRACT

In this thesis, the unsteady magnetohydrodynamic (MHD) free convection flows of viscous and second grade fluids past an infinite inclined plate in a porous medium are studied. These viscous and second grade fluids are under the conditions of ramped wall temperature and isothermal plate. Analytic solutions are developed by using Laplace transform technique. The main finding of this thesis is to determine the expressions of exact solutions for velocity, temperature and concentration profiles. All these profiles are graphically plotted for various physical parameters such as radiation, heat absorption, porosity, rotation and second grade parameters. The results show that when temperature decreases, high radiation and heat absorption occurs which consequently decreases the velocity. For larger values of magnetic parameter, the fluid velocity decreases. The velocity is found to increase with increasing values of the porosity parameter. It is also observed that when the second grade parameter increases, the velocity shows an oscillating behavior where the velocity first decreases and then increases. An interesting result for the velocity is observed from the comparison of ramped wall temperature and isothermal. It is found that fluid velocity retarded in the case of ramped wall temperature compared to isothermal case. In limiting cases, the present solutions are reduced in order to compare with existing results. As expected, the results are found identical, verifying the validity of the obtainable solutions. The numerical results of skin-friction, Nusselt number and Sherwood number are also computed and displayed in tables, and also analyzed in details.

ABSTRAK

Dalam tesis ini, aliran tak mantap olakan bebas magnetohidrodinamik (MHD) bendalir likat dan gred kedua yang melintasi plat condong tak terhingga dalam bahantara berliang dikaji. Bendalir likat dan gred kedua ini di bawah syarat suhu tanjakan dinding dan plat isoterma. Penyelesaian analisis dibangunkan dengan menggunakan teknik jelmaan Laplace. Dapatan utama tesis ini adalah untuk penentuan ungkapan penyelesaian tepat bagi profil halaju, profil suhu dan profil kepekatan. Tingkah laku semua profil ini diplot secara grafik dengan parameter fizikal seperti parameter radiasi, parameter penyerapan haba, parameter keliangan, parameter putaran dan parameter gred kedua. Keputusan menunjukkan apabila suhu menurun, radiasi dan penyerapan haba yang tinggi didapati berlaku yang mengakibatkan pengurangan halaju. Untuk nilai parameter magnet yang besar, halaju bendalir berkurangan. Halaju bendalir diperhatikan meningkat apabila parameter keliangan meningkat. Didapati juga peningkatan parameter gred kedua menunjukkan tingkah laku halaju yang berayun di mana pada mulanya halaju berkurangan dan kemudian meningkat. Keputusan yang menarik bagi halaju dapat diperhatikan daripada perbandingan antara suhu tanjakan dinding dan plat isoterma. Suhu tanjakan dinding telah melambatkan halaju bendalir apabila dibandingkan dengan plat isoterma. Dalam kes mengehad, penyelesaian yang diperoleh diturunkan untuk dibandingkan dengan keputusan yang telah diterbitkan. Seperti dijangka, keputusan serupa diperoleh, yang membuktikan kesahihan penyelesaian yang diperoleh. Keputusan berangka untuk geseran kulit, nombor Nusselt dan nombor Sherwood telah juga dihitung dan dipersembahkan dalam bentuk jadual serta dianalisis secara terperinci.

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