APPLIED CALCULUS
FOR ENGINEERING & SCIENCE STUDENTS
This module is designed for one-semester calculus course aimed at students majoring in science and engineering. A wide variety of topics, examples and problems are provided. This module can be used as an aided tool in teaching and learning multivariable calculus.

This module consists of four chapters. Students are introduced to polar coordinates and vectors in Chapter 1. This chapter includes the 3-dimensional graph sketching and polar curve, allowing for a source of rich illustrations and exercises. The next chapter concentrate on vector-valued functions including discussions on arc length, curvature and motion in space. Chapter 3 covers partial derivative involving functions with more than one variable. The final chapter covers multiple integrals and the applications.

This text also provides a complete set of teaching materials for courses in multivariable calculus that incorporates recent curricular and pedagogical developments in teaching and learning of calculus. An effort has been made to present the material clearly and at a level of sophistication appropriate for the audience. With the knowledge that any book can always be improved, we welcome corrections, constructive criticism, and suggestions from every reader.
PREFACE

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