EVALUATE FIN PERFORMANCE OF A SPACE RADIATOR TO REMOVE HEAT GENERATED FOR OUTER SPACE APPLICATION

NOR NASHRIQ AZIZI BIN ABD. SHUKOR

BACHELOR OF ENGINEERING
UNIVERSITI MALAYSIA PAHANG

2010

UNIVERSITI MALAYSIA PAHANG

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JUDUL: **EVALUATE FIN PERFORMANCE OF A SPACE RADIATOR TO REMOVE HEAT GENERATED FOR OUTER SPACE APPLICATION**

SESI PENGAJIAN: **2010/2011**

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(TANDATANGAN PENYELIA)

Alamat Tetap:

B2-1-9,
TAMAN AMPANG HILIR,
KAMPUNG PANDAN,
55100 KUALA LUMPUR.
W. PERSEKUTUAN (KL).

PROF. DR. KORADA VISWANATHA SHARMA
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EVALUATE FIN PERFORMANCE OF A SPACE RADIATOR TO REMOVE HEAT GENERATED FOR OUTER SPACE APPLICATION

NOR NASHRIQ AZIZI BIN ABD. SHUKOR

Report submitted in fulfillment of the requirements
for the award of the degree of
Bachelor of Mechanical Engineering

Faculty of Mechanical Engineering
UNIVERSITI MALAYSIA PAHANG

DECEMBER 2010

UNIVERSITI MALAYSIA PAHANG

FACULTY OF MECHANICAL ENGINEERING

I certify that the project entitled "*Evaluate Fin Performance of a Space Radiator to Remove Heat Generated for Outer Space Application*" is written by *Nor Nashriq Azizi Bin Abd. Shukor*. I have examined the final copy of this project and in our opinion; it is fully adequate in terms of scope and quality for the award of the degree of Bachelor of Engineering. I herewith recommend that it be accepted in partial fulfilment of the requirements for the degree of Bachelor of Mechanical Engineering.

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Signature	:
Name of Supervisor	: PROF. DR. KORADA VISWANATHA SHARMA
Position	: LECTURER
Date	: 6 DECEMBER 2010

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Date	: 6 DECEMBER 2010

Dedicated to my beloved parents for their everlasting love, guidance and support in the whole journey of my life

ACKNOWLEDGEMENTS

First and foremost, I would like to express my grateful to ALLAH S.W.T. as for the blessing given that I can finish my project.

In preparing this paper, I have engaged with many people in helping me completing this project. First, I wish to express my sincere appreciation to my supervisor, Prof. Dr. Korada Viswanatha Sharma for his germinal ideas, continuous encouragement, invaluable guidance, advices and motivation. Without his continued support and interest, this thesis would not have been the same as presented here.

In particular, my sincere thankful is also extends to all my colleagues and others who have provided assistance at various occasions. Their views and tips are useful indeed. I cannot find the appropriate words that could properly describe my appreciation for their devotion, support and faith in my ability to attain my goals. I would like to acknowledge their comments and suggestions, which was crucial for the successful completion of this study.

Last but not least I acknowledge without endless love and relentless support from my family, I would not have been here. My father, Abd Shukor Bin Lodin, my mother, Rohayah Binti Samad, all my brothers and sisters that always support, sacrifice, patience, understanding that were inevitable to make this work possible, motivation and encourage me to success. I cannot find the appropriate words that could properly describe my appreciation for their devotion, support and faith in my ability to attain my goals.

Thank you all.

ABSTRACT

Heat is generated in spacecrafts due to air-condition system, electronic and electrical equipment, human beings, etc. The heat generated from these sources must be removed in order to maintain the spacecraft at the required temperatures. Heat rejection to outer space is the area of present work where radiation is the dominant single mode. Extended surfaces are advantageous for this situation for heat dissipation to outer space. To simulate conditions of outer space where convection is not present is difficult and costly. Hence convection cannot be excluded in the testing of space radiator. The present analysis includes convection and irradiation terms. The physical situation considered is a horizontal fin with a rectangular cross-section. One end of the fin is maintained at a constant elevated temperature, and the fin is short and the heat loss from the tip is negligible. Heat is transferred by conduction along the fin and dissipated from the surface via natural convection and radiation. The numerical solution is obtained for the present problem by developing an algorithm where the domain is discretized by Taylor series central difference scheme and have been solved by Jacobi method, which possesses the quality of exceptional accuracy with a few numbers of nodes. The algorithm is computed using FORTRAN Software with certain parameter value. This method of approach helps to estimate its performance under actual working conditions. A comparison is made to published results, and the agreement between the present and previous is very good. Results show that the total heat loss to ambient strongly effected by convection and radiation. In the absence of convection, the efficiency is decrease with the increasing of radiation parameter. It is also show that increasing of radiation parameter will increase the total heat transfer to ambient. Longitudinal heat conduction parameter will increase the amount of heat dissipated to ambient. Because of several limiting assumptions, the results would be used only for preliminary analysis and design particularly when a fin assembly is involved rather than an individual fin.

ABSTRAK

Haba didalam kapal angkasa telah dihasilkan oleh sistem penghawa dingin, peralatan elektronik dan elektrik, radiasi dari manusia dan sebagainya. Haba yang agak tinggi ini harus dikeluarkan untuk mengekalkan suhu yang diperlukan oleh kapal angkasa itu sendiri. Penyingkiran haba ke luar angkasa adalah subjek utama didalam kajian ini di mana radiasi adalah satu satunya cara untuk haba disingkirkan dari kapal angkasa. Sirip (permukaan lebihan) telah digunakan untuk meningkatkan penyingkiran haba ke luar angkasa. Untuk mensimulasikan keadaan ruangan luar di mana perolakan haba tidak berlaku adalah sukar dan mahal. Oleh itu, perolakan haba tidak boleh dikecualikan dalam menguji sistem penyejukan kapal angkasa. Analisis yang dilakukan ini merangkumi perolakan dan radiasi. Analisis dilakukan keatas satu sirip berbentuk segi empat. Pangkal sirip ditetapkan pada suhu yang tinggi dan malar dan sirip ini adalah pendek, oleh itu kehilangan haba dari hujungnya boleh diabaikan. Haba dipindahkan oleh konduksi sepanjang sirip dan disingkirkan dari permukaan melalui perolakan dan radiasi. Penyelesaian berangka yang diperolehi untuk analisis ini telah dikembangkan dari sebuah algoritma yang didiskritisasi oleh *Taylor series central difference scheme* dan telah diselesaikan dengan kaedah Jacobi yang persis dengan beberapa nod. Algoritma ini telah dihitung menggunakan perisian FORTRAN dengan beberapa nilai tertentu. Kaedah ini membantu untuk menilai prestasi sistem penyejukan kapal angkasa di angkasa lepas. Perbandingan keputusan analisis dengan keputusan yang telah diterbitkan adalah sangat hamper. Keputusan kajian menunjukkan bahawa haba yang disingkirkan ke persekitaran sangat dipengaruhi oleh perolakan dan radiasi. Ketika ketiadaan perolakan, kecekapan sirip menurun apabila radiasi meningkat. Keputusan juga menunjukan peningkatan radiasi akan meningkatkan jumlah haba yang disingkirkan. Nisbah sirip-cecair akan meningkatkan jumlah haba yang disingkirkan. Disebabkan beberapa permudahan analisis, hasilnya akan digunakan hanya untuk analisis awal terutama pada kombinasi sirip yang banyak.

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LIST OF SYMBOLS

q_{cond}	Heat transfer by conduction, $kA_c\Delta T / L$
q_{conv}	Heat transfer by convection, $hA_s\Delta T$
q_{rad}	Heat transfer by radiation, $\sigma\varepsilon(T_s^4-T_\infty^4)$
q''	Heat Flux
Q_{fin}	Total Energy at Fin
Q_{fluid}	Total Energy in Fluid
T_{out}	Temperature Out From Fin
T_{in}	Temperature In To Fin
T_s	Surface Temperature
T_b	Base Temperature
T_∞	Ambient Temperature
T_f	Fluid Temperature
T_{fi}	Fluid Temperature, in
T_{fo}	Fluid Temperature, out
^oC	Degree Celsius
k	Thermal Conductivity
h	Convection Heat Transfer Coefficient
E	Surface Emissive Power
σ	Stefan-Boltzmann Constant
ε	Emissivity
A	Area
A_C	Cross Sectional Area
A_S	Surface Area

L	Length of Fin
P	Perimeter
H	Length of Tube
G	Irradiation
C_p	Specific heat capacity at constant pressure
m	Mass
Bi	Biot Number, hL_c / k
δ	Thickness of Fin
δ_c	Thickness of Fin at Cold Side
δ_h	Thickness of Fin at Hot Side
N_P	Profile Parameter, δ_c / δ_h
N_R	Radiation Parameter, $[2\varepsilon\sigma L^2(T_b\text{-}T_\infty)^3]$ / $k\delta_h$
N_C	Convection Parameter, h / $\sigma\varepsilon(T_b\text{-}T_\infty)^3$
N_G	Irradiation Parameter, αG / $\sigma\varepsilon(T_b\text{-}T_\infty)^4$
N_F	Longitudinal Heat Conduction Parameter, $2k_p\delta_h H$ / mc_pL
θ	Dimensionless Temperature Ratio Term, $(T\text{-}T_\infty)$ / $(T_b\text{-}T_\infty)$
θ_b	Dimensionless Fluid Temperature, $(T_{fi}\text{-}T_{fo})$ / $(T_{fi}\text{-}T_\infty)$
ψ	Dimensionless Temperature Ratio Term, T_∞ / $(T_b\text{-}T_\infty)$
ξ	Dimensionless Distance, x / L
x	Distance along the Fin
z	Distance along the Tube

LIST OF ABBREVIATIONS

UMP	Universiti Malaysia Pahang
FKM	Fakulti Kejuruteraan Mekanikal
NASA	National Aeronautics and Space Administration
ISS	International Space Station
ANSI	American National Standards Institute
SI	The International System of Units
FORCE	Fortran Compiler and Editor

CHAPTER 1

INTRODUCTION

1.1 PROJECT BACKGROUND

As against the three modes of heat transfer, viz., conduction, convection and radiation, radiant heat transfer plays a dominant role in space applications. Whereas heat rejection in terrestrial applications is dominantly by convection, it is solely by radiant heat transfer in space systems. The basic difficulty associated with radiation heat transfer is its non-linear dependence on temperature. This non-linearity makes it difficult to analyze, except for simple configurations, by analytical methods.

Space radiators are used for waste heat rejection generated by electronic equipments, air-conditioning system etc. within the spacecraft. Irradiated energy from the sun which is absorbed the spacecraft is also to be rejected to outer space by radiation. The tubes from the heat source carry the hot coolant that dumps the internally generated heat to the radiator. The extended surfaces of the radiator reject the heat to space by thermal radiation.

The technological difficulties involved in the manufacture of heat exchanger for space applications include fabrication, surface finish, surface coatings, selective coatings, etc. Time and expense are involved with all such applications. To create an environment similar to that in space is very expensive but essential for testing purpose.

1.2 PROBLEM STATEMENT

The main problem that has to overcome is to remove heat generated inside spacecraft. Thus, space radiator is needed to remove this unwanted heat at high efficiency. But, to simulate conditions of outer space where convection is not present is difficult and costly. The heat exchanger thus designed has to be tested at the surface of earth for its performance before deployment at outer space.

1.3 OBJECTIVE OF THE PROJECT

The objective of this project is to evaluate fin performance of a space radiator to remove heat generated for outer space application.

1.4 PROJECT SCOPE

Cooling of the heat generated sources inside the space craft is based on conventional design methods involving conduction and convection with variable gravitational force term. Heat rejection to outer space is the area of present work where radiation is the dominant single mode. Extended surfaces are advantageous for current situation for heat dissipation to outer space. Hence convection cannot be excluded in the testing of space radiator. The present analysis includes convection and irradiation terms. This method of approach helps to estimate its performance under actual working conditions.

1.5 SUMMARY

In this chapter, the problem statement, objective and scope of the project is discussed to recognize the problem occur, purpose and range to evaluate fin performance at outer space.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Advance from chapter 1, this chapter present the field of heat transfer, space radiator, dimensionless number, numerical approach and FORTRAN Software. The result in the previous research is discussed to obtain the information about method and parameter used to verify present result.

2.2 SPACE RADIATOR

Space radiator is a thermal radiator system use on an outer space vehicle, which must survive a long period of nonuse and then radiate large amounts of heat for a limited period of time.

Figure 2.1: International Space Station (ISS)

Source: NASA (2001)

2.2.1 Different Car Radiator and Space Radiator

Radiators are used for cooling internal combustion engines, chiefly in automobiles. They operate by passing a liquid coolant through the engine block, where it is heated, then through the radiator itself where it loses this heat to the atmosphere by convection and radiation. This coolant is usually water-based, but may also be oil. It's usual for the coolant flow to be pumped, also for a fan to blow air through the radiator.

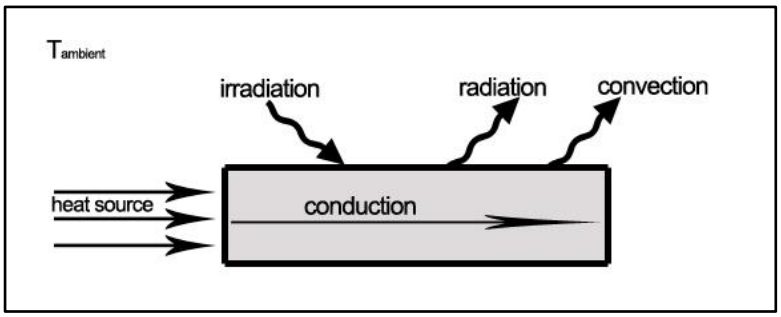

Figure 2.2: Block diagram for heat transfer component in car radiator (atmosphere condition)

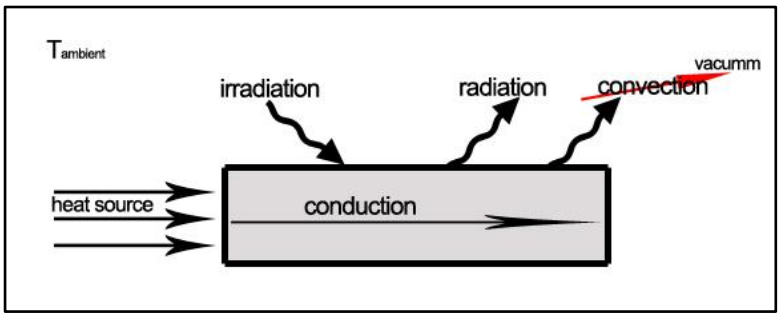

Figure 2.3: Block diagram for heat transfer component in space radiator (vacuum condition)

Source: Kumar et al. (1993)

A thermal radiator system is described for use on an outer space vehicle which must survive a long period of nonuse and then radiate large amounts of heat for a limited period of time. Space radiator has the same function with car radiator but in the outer space, heat can be transferred only by conduction, radiation and irradiation. Thus, radiation is the main parameter to know how efficient of the fin to remove heat.

2.2.2 Application of Space Radiator

There is a lot of heat generated inside the spacecraft such as air condition system, electronic and electrical equipment and human beings. Thus, that unwanted energy must be removed to maintain the equipment's temperature. Since no transferred medium in the outer space, convection heat transfer can't occur. It means, almost heat is transferred by radiation. The equipment can dump energy into space because any object at a reasonable temperature emits radiation. If its surroundings are colder than it is, it receives less radiation back than it radiates itself, and so it cools. This is just what a space radiator does. Nowadays, fin is famously used in radiator to remove unnecessary heat by evaluate surface area.

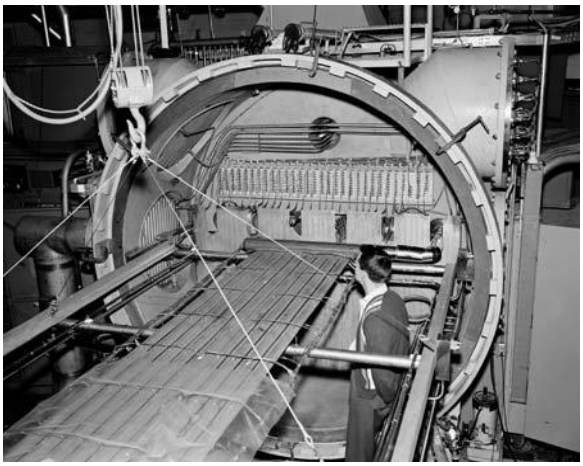

Figure 2.4: The International Space Station Radiator System

Source: NASA (2004)

2.2.3 Purpose of Thermal Control in Spacecraft

The space radiator has to control the operating temperature environment of spacecraft systems to maximize the efficiency of spacecraft equipment and to prevent from damage. There are some effect when the component inside spacecraft operate beyond their operating temperature where;

i. Most systems become less reliable when operated outside their design operating environment
ii. Propellant freezes
iii. Thermal cycling damage
iv. Instrument/antenna/camera alignment
v. Instrument requirements for very cold temperatures

Table 2.1: Typical spacecraft components temperatures

Component/ system	**Operating temperature (°C)**	**Survival temperature (°C)**
Digital electronics	0 to 50	-20 to 70
Analog electronics	0 to 40	-20 to 70
Batteries	10 to 20	0 to 35
IR detectors	-269 to -173	-269 to 35
Solid-state particle detectors	-35 to 0	-35 to 35
Momentum wheels	0 to 50	-20 to 70
Solar panels	-100 to 125	-100 to 125

Source: Pisacane, Vincent and Moore (1994)

2.3 DEFINITION OF FIN

Extended surface, in the forms of longitudinal or radial fins or spines are ubiquitous in applications where the need exits to enhance heat transfer between a surface and an adjacent fluid. Extended surface heat transfer is the study of high performance heat transfer components with respect to smaller weights, volumes, costs, accommodating shapes and of their behavior in a variety of thermal environments. In the design and construction of various types of heat transfer equipment, simple shapes such as cylinders, bars and plates are used to implement the flow of heat between sources and sink.

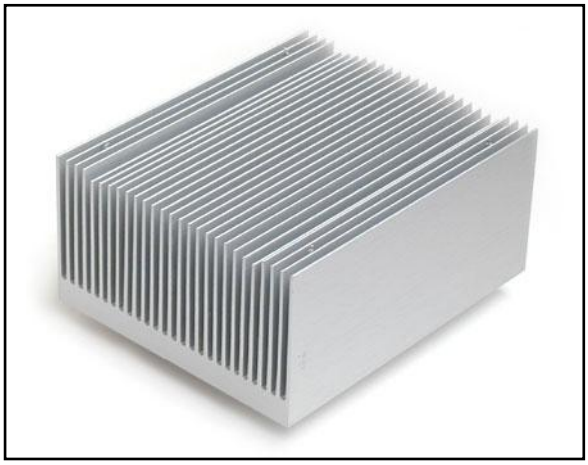

Figure 2.5: Arrangement of fin in forged fin

Source: Ixbtlabs (2010)

2.4 PREVIOUS RESEARCH

Extended surfaces (fins) are an effective and well established means of enhancing heat transfer between a primary surface and its environment. Practical applications of fins are numerous ranging from cooling of electronic equipment to heat rejection devices for the space vehicles.

The heat dissipation mechanism considered in most studies is either pure convection or pure radiation. In applications where fins operate in a free or natural convection environment, the contribution of radiation is equally significant, and therefore the design must allow for simultaneous convection and radiation. An example of such application is the stamped heat sink or extruded heat sink designed for cooling a transistor. Even if forced convection is employed for cooling, radiation can be significant if the operating temperatures are high as is the case with a finned regenerator. Cobble (1964) was perhaps the first to study the combined convection-radiation from a fin. He considered a horizontal circular pin fin and derived an approximate analytical solution. Experiment was conducted by Cobble to predict temperature distributions along the circular fin for aluminum and steel material. The temperature and fin efficiency was predicted by solving the nonlinear fin equation using a substitution developed by using the Gregory-Newton Forward Interpolation formula.

The temperature variation is expressed in Jacobian elliptic functions. The result is shown at Figure 2.6.

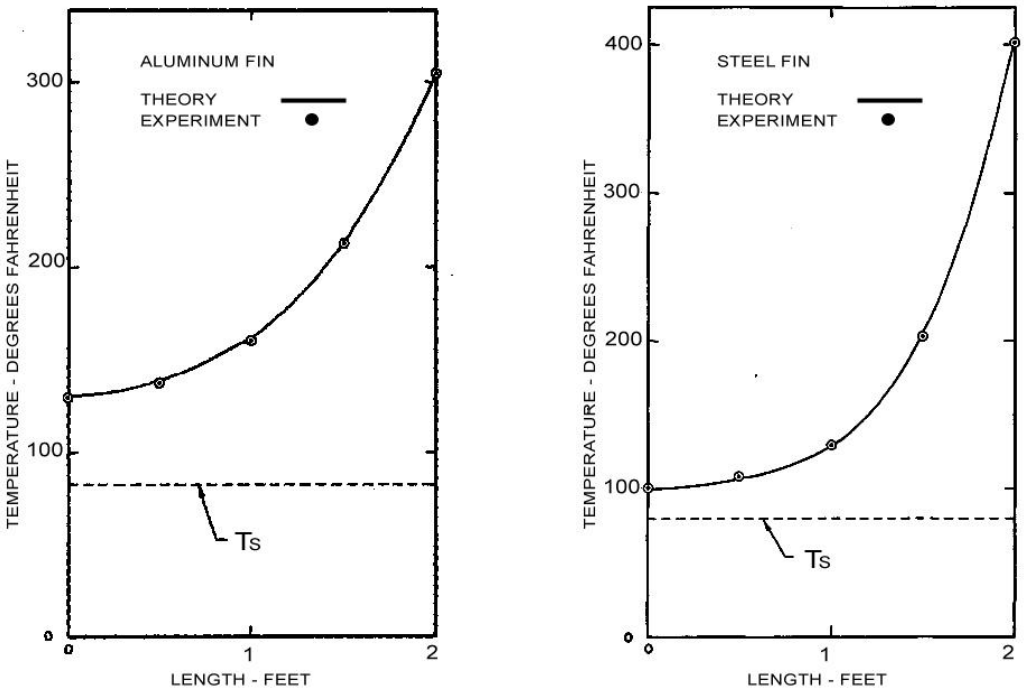

Figure 2.6: Temperature distributions for aluminum (left) and steel fin (right)

Source: Cobble (1964)

Aziz and Arlen (2009) show the effect of convection and radiation parameter to the heat loss from fin. Figure 2.7 shows how the temperature distribution in a fin of constant thermal conductivity with an insulated tip ($a = 0$, $N_{t1} = 0$, $N_{t2} = 0$) is affected by the variation of Bi characterizing the base convection process, the wall conduction resistance, and the contact resistance between the wall and the fin base.

To study the effect of varying the convection-conduction number N_C on the performance of the fin, the other variables were kept fixed at $a = 0$, $N_{t1} = 0$, $N_{t2} = 0$, $Bi = 1$, $N_R = 1$, $h_S = 0.2$. Figure 2.7 (left) shows the temperature distributions for $N_C = 1, 2, 3,$ and 4. As the surface convection gets stronger, the temperatures in the fin get increasingly depressed and the heat flow through the base of the fin increases but the efficiency decreases. The results are consistent with the well known performance characteristics of pure convective fins.

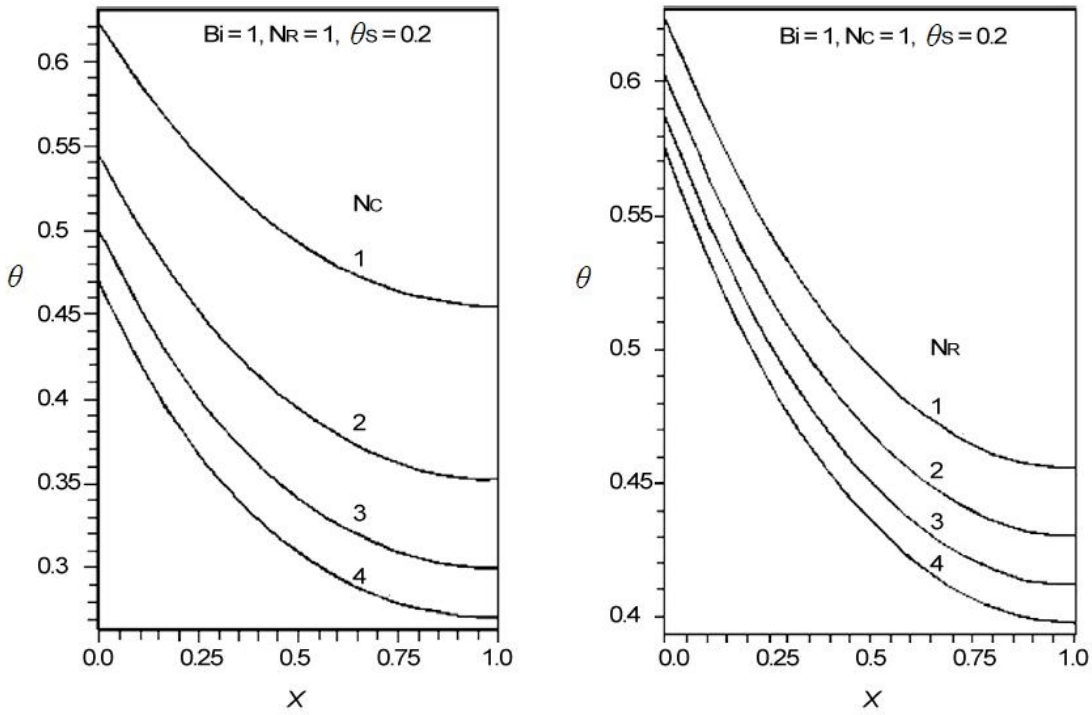

Figure 2.7: Effects of convection (left) and radiation parameter (right)

Source: Aziz and Arlen (2009)

Ngunyen and Aziz (1990) discuss about efficiency for the different geometries of fin and compare with various parameters such as dimensionless temperature, Biot Number, radiation parameter and heat transfer rate. They found that, as radiation parameter increase, the efficiency decrease significantly for all geometries. Their result is shown in Table 2.2.

Table 2.2: Efficiency of different profile fins $\theta_\infty = \theta_s = 0.8$, $Ncv = \alpha^2 Bi = 1$, $N_R = \alpha^2 Bi$

N_R	Rectangular	Trapezoidal (β = 0.25)	Triangular	Conc. parabolic
0	0.6968	0.6931	0.6845	0.6240
0.2	0.4679	0.4677	0.4631	0.4244
0.4	0.3631	0.3644	0.3616	0.3324
0.6	0.3030	0.3051	0.3033	0.2811
0.8	0.2638	0.2666	0.2655	0.2471
1	0.2365	0.2396	0.2390	0.2233

Source: Ngunyen and Aziz (1990)

2.5 HEAT TRANSFER

Heat transfer is due to a temperature difference. Heat transfer occurs whenever there is inhomogeneous temperature distribution within materials or between material and its surrounding environments. Heat is always conducted from warm to cold; never from cold to warm, and always moves via the shortest and easiest route.

2.5.1 Heat Transfer Mode

Heat transfer processes are classified into three types. The first is conduction, which is defined as transfer of heat occurring through intervening matter without bulk motion of the matter. It results from actual physical contact of one part of the same body with another part, or of one body with another. The greatest flow of heat possible between materials is where there is direct conduction between solids.

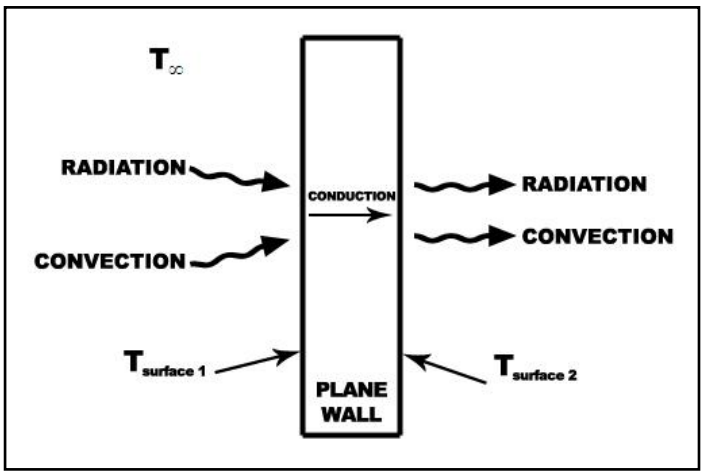

Figure 2.8: Diagram for heat transfer modes

Source: Incopera et al. (2007)

The second heat transfer process is convection, or heat transfer due to a flowing fluid. The fluid can be a gas or a liquid; both have applications in aerospace technology. In convection heat transfer, the heat is moved through bulk transfer of a non-uniform temperature fluid.

The third process is radiation or transmission of energy through space without the necessary presence of matter. Radiation is the only method for heat transfer in space. Radiation can be important even in situations in which there is an intervening medium; a familiar example is the heat transfer from a glowing piece of metal or from a fire.

2.5.2 Steady State Condition

A system in a steady state has numerous properties that are unchanging in time. This implies that for any property (such as heat flux, q'') of the system, the partial derivative with respect to time is zero:

$$\frac{dq''}{dt} = 0 \tag{2.1}$$

In heat transfer, steady state condition can give the approximate value if the variation of heat along distance is very small.

2.6 BASIC EQUATION IN HEAT TRANSFER

There are three basic equations which are conduction heat transfer, convection heat transfer and radiation heat transfer.

2.6.1 Heat Transfer by Conduction

For heat conduction, the rate equation is known as *Fourier's Law.* For the one-dimensional plane wall, having a temperature distribution *T(x)*, the rate equation is expressed as

$$q_{cond} = -kA\frac{dT}{dx} \tag{2.2}$$

The *heat flux* (W/m^2) is the heat transfer rate in the *x* direction per unit area perpendicular to the direction of heat transfer, and it is proportional to the temperature gradient, *dT/dx*, in this direction. The parameter *k* is a transport property known as the

thermal conductivity (W/m·K) and is a characteristic of the wall material. The minus sign is a consequence of the fact that heat is transferred in the direction of decreasing temperature. Under the steady state conditions, where the temperature distribution is linear, the temperature gradient may be expressed as

$$\frac{dT}{dx} = \frac{T_{out} - T_{in}}{L}$$

And the *heat flux* is then

$$q''_{cond} = -k\frac{T_2 - T_1}{L} \tag{2.3}$$

This equation provides a *heat flux*, that is, the rate of heat transfer per unit area. The heat rate by conduction, $q_x = q''_x \cdot A$

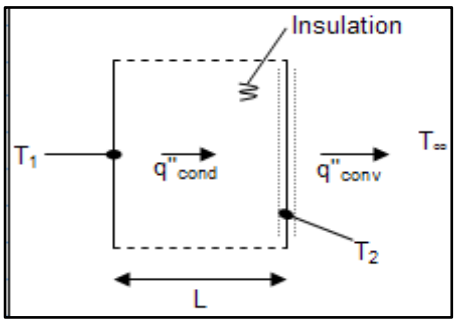

Figure 2.9: Block diagram for steady state heat conduction

Source: Tuwien (2003)

Table 2.3: Typical value of the conduction heat transfer coefficient

Materials	***k* (W/m·K)**
Diamond	3000
Copper	390
Stainless steel	15
Water	0.06
Air (1 atm)	0.026

Source: Kau-Fui Vincent Wong (2003)

a. 1 Dimensional Conduction

The one dimensional case gives rise to ordinary differential equations (ODEs), for which analytical solution may not be hard to obtain. Also, in many practical circumstances, it is possible to consider idealized cases in which one dimensional conduction may be assumed so that the numerical method may be compared with available analytical results for a check on accuracy, convergence, and stability.

b. Multidimensional Conduction

The analysis of multidimensional conduction is simplified by approximating the shapes as a combination of two or more semi-infinite or one dimensional geometries. For example, a short cylinder can be constructed by intersecting a one dimensional plate with a one dimensional cylinder. Similarly, a rectangular box can be constructed by intersecting three one dimensional plates, perpendicular to each other. In such cases, the temperature at any location and time within the solid is simply the product of the solutions corresponding to the geometries used to construct the shape.

2.6.2 Heat Transfer by Convection

Regardless of the particular nature of the convection heat transfer process, the appropriate rate equation is of the form

$$q''_{conv} = h(T_s - T_\infty) \tag{2.4}$$

Where *q''*, the convective *heat flux* (W/m^2), is the proportional to the difference between the surface and fluid temperatures, T_s and T_∞, respectively. This expression is known as *Newton's Law of cooling*, and the parameter h (W/m^2·K) is termed the convection heat transfer coefficient. It depends on conditions in the boundary layer, which are influenced by surface geometry, the nature of the fluid motion, and an assortment of fluid thermodynamics and transport properties.

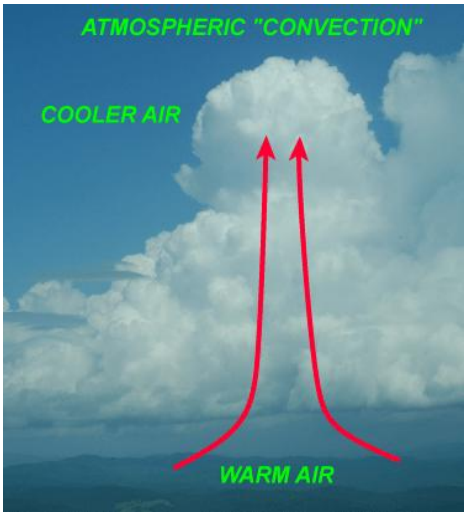

Figure 2.10: Free convection heat transfer occur at atmosphere

Source: Apollo (2000)

Table 2.4: Typical value of the convection heat transfer coefficient

Process	***h* (W/m^2·K)**
Free convection	
Gasses	2-25
Liquids	50-1000
Forced convection	
Gases	25-250
Liquids	100-20,000
Convection with phase change	
Boiling or condensation	2500-100,000

Source: Incopera (2007)

2.6.3 Heat Transfer by Radiation

Radiation that is emitted by the surface originates from the thermal energy of matter bounded by the surface and the rate at which energy is released per unit area is termed the *surface emissive power*, E. There is an upper limit to emissive power, which is prescribed by the *Stefan-Boltzmann law*

$$E_b = \sigma T_s^4 \tag{2.5}$$

Where T_s is the absolute temperature (K) of the surface and σ is the *Stefan-Boltzmann constant* (σ=5.67×10^{-8} W/m^2·K^4). Such a surface is called an ideal radiator or blackbody. The heat flux emitted by a real surface is less than that of a blackbody at the same temperature and is given by

$$E_{rad} = \varepsilon\sigma T_s^4 \tag{2.6}$$

Where ε is a radiative property of the surface termed the emissivity. With value in the range $0 \leq \varepsilon \leq 1$, these properties provide a measure of how efficiently a surface emits energy relative to a blackbody.

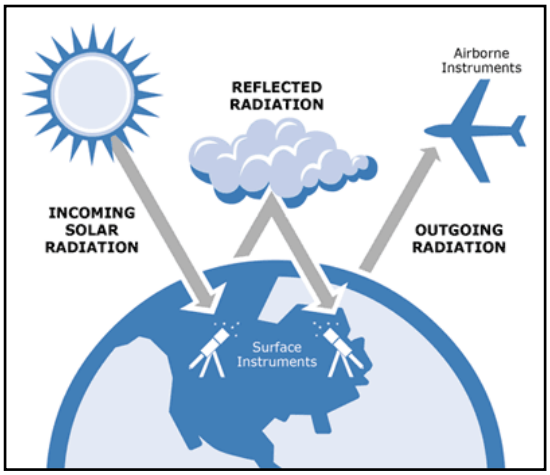

Figure 2.11: Solar radiation diagram

Source: Sieun (2009)

Radiation may also be incident on a surface from its surroundings. The radiation may originate from a special source such as the sun. The rate at which radiant energy is absorbed per unit surface area maybe evaluated from knowledge of a surface radiative property termed the *absorptivity* α. That is,

$$G_{abs} = \alpha G \tag{2.7}$$

Where $0 \leq \alpha \leq 1$

The net rate of radiation heat transfer from the surface, expressed per unit area of the surface, is

$$q''_{rad} = \varepsilon E_b(T_s) - \alpha G$$
$$= \varepsilon\alpha(T_s^4 - T_\infty^4) \quad (2.8)$$

This expression provides the difference between thermal energy that is released due to radiation emission and that which is gained due to radiation absorption.

2.7 ENERGY BALANCE OF FIN

To create a simplified equation for the heat transfer of a fin, many assumptions need to be made.

i. Steady state
ii. Constant material properties (independent of temperature)
iii. No internal heat generation
iv. One-dimensional conduction
v. Uniform cross-sectional area
vi. Uniform convection across the surface area

The conservation of energy can be used to create an energy balance. Under steady state,

$$Q_{entering} = Q_{leaving}$$
$$Q_{conduction\ at\ x} = Q_{conduction\ over\ dx} + Q_{convection}$$

Fourier's law states that

$$Q_{cond,x} = -kA_c\left(\frac{dT}{dx}\right) \quad (2.9)$$

Where A_c is the cross-sectional area of the differential element.

Heat leaving by conduction over d*x*,

$$Q_{x+dx} = Q_x + \left(\frac{dQ_x}{dx}\right) dx \tag{2.10}$$

Hence, it can also be expressed as

$$Q_{x+dx} = -kA_c\left(\frac{dT}{dx}\right) - k\frac{d}{dx}\left(A_c\frac{dT}{dx}\right) dx \tag{2.11}$$

Heat leaving by convection along d*x*,

$$\begin{aligned} Q_{conv} &= hA_s(T - T_\infty) \\ &= hP(T - T_\infty)dx \end{aligned} \tag{2.12}$$

Where *P* is the perimeter at the cross sectional area along d*x*. Then, the energy balance becomes

$$\begin{aligned} -kA_c\left(\frac{dT}{dx}\right) = -kA_c\left(\frac{dT}{dx}\right) - k\frac{d}{dx}\left(A_c\frac{dT}{dx}\right) dx + hP(T - T_\infty)dx \\ k\frac{d}{dx}\left(A_c\frac{dT}{dx}\right) dx - hP(T - T_\infty)dx = 0 \\ \frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{hP}{kA_c}\right)(T - T_\infty) = 0 \end{aligned} \tag{2.13}$$

2.7.1 Fin with Uniform Cross-sectional Area Cases

Since $\frac{dA_c}{dx} = 0$ for uniform cross sectional area, thus the energy balance reduced to,

$$\frac{d^2T}{dx^2} = \frac{hP}{kA_c}(T - T_\infty) \tag{2.14}$$

Since T_∞ is a constant, $\frac{d^2T}{dx^2} = \frac{d^2\theta}{dx^2}$. This equation is simplified as

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \tag{2.15}$$

Where,

$$m^2 = \frac{hP}{kA_c}$$

And,

$$\theta_x = T(x) - T_\infty$$

Equation above is a linear, homogeneous, second order-differential equation with constant coefficients. Its general solution is of the form,

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \tag{2.16}$$

2.7.2 Temperature Distribution and Heat Flow in Fins of Uniform Cross Section

Different boundary conditions will result in different temperature distributions in the fin. The temperature distributions are thus classified under the boundary conditions.

i. For the first case, the second boundary condition is that there is free convection at the tip. Therefore,

$$hA_c(T(L) - T_\infty) = -kA_c\left(\frac{dT}{dx}\right)\bigg|_{x=L} \quad at \quad x = L$$

Which simplifies to

$$h\theta(L) = -k\frac{d\theta}{dx}\bigg|_{x=L} \tag{2.17}$$

Knowing that

$$\theta_b = T_{base} - T_\infty \quad at \quad x = 0$$

The equations can be combined to produce

$$h(C_1 e^{mL} + C_2 e^{-mL}) = km(C_2 e^{-mL} - C_1 e^{mL})$$

The general solution for this case is

$$\theta(x) = c_1 \cosh m(L - x) + c_2 \sinh m(L - x) \qquad (2.18)$$

Where c_1 and c_2 are constant of integration to be determined by the boundary conditions.

$At\ x = L$

$$k[-c_1 m \sinh m(0) - c_2 m \cosh m(0)] + h[c_1 \cosh(0) + c_2 \sinh(0)] = 0$$

$$-kc_2 m + hc_1 = 0$$

$$c_1 = c_2 \frac{mk}{h} \qquad (2.18)$$

$At\ x = 0$

$$\theta_b = \frac{c_2 mk}{h} \cosh(mL) + c_2 \sinh mL$$

$$c_2 = \frac{\theta_b}{\sinh mL + \frac{mk}{h} \cosh(mL)}$$

$$c_1 = \frac{mk}{h}\left(\frac{\theta_0}{\sinh mL + \frac{mk}{h} \cosh(mL)}\right)$$

$$\frac{\theta(x)}{\theta_b} = \frac{\cosh m(L - x) + {}^{c_2}/_{c_1} \sinh m(L - x)}{{}^{\theta_0}/_{c_1}}$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L - x) + \left(\frac{h}{mk}\right) \sinh m(L - x)}{\cosh mL + \left(\frac{h}{mk}\right) \sinh mL} \qquad (2.19)$$

The rate of heat transfer from the extended surface to the surrounding fluid is

$$Q = \sqrt{hPkA_c}\theta_b \frac{\sinh mL + \left(\frac{h}{mk}\right)\cosh mL}{\cosh mL + \left(\frac{h}{mk}\right)\sinh mL} \tag{2.20}$$

ii. For the second case, the second boundary condition is that there is negligible heat flow at the tip. Therefore,

$$\frac{d\theta}{dx} = 0 \quad at \quad x = L$$

$$\theta_b = T_{base} - T_\infty \quad at \quad x = 0$$

The general solution for this case is

$$\theta(x) = c_1 \cosh m(L - x) + c_2 \sinh m(L - x) \tag{2.21}$$

From equation $\theta_b = c_1 \cosh mL$ and $c_1 = {}^{\theta_0}/_{\cosh mL}$

Thus,

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L - x)}{\cosh mL} \tag{2.22}$$

Now,

$$\begin{aligned} Q &= \int_0^L h\theta P dx \\ &= \int_0^L \frac{hP}{m^2}\frac{d^2\theta}{dx^2} dx \\ &= Ak\left(\left.\frac{d\theta}{dx}\right|_L - \left.\frac{d\theta}{dx}\right|_0\right) \end{aligned}$$

Since $\left.\frac{d\theta}{dx}\right|_L = 0$, the equation become

$$= -Ak\left(\left.\frac{d\theta}{dx}\right|_0\right)$$

$$= -Ak\left(\frac{\theta_b \sinh mL}{\cosh mL}\right)$$

$$= \theta_b Akm \tanh mL$$

$$Q = \sqrt{hPkA_c}\theta_b \tanh mL \quad where \quad m^2 = \frac{Ph}{Ak} \tag{2.23}$$

iii. For the final case, the second boundary condition is that the heat flow at the tip almost zero. Therefore,

$$\theta(x) \to 0 \quad as \quad x \to \infty$$

$$\theta_b = T_{base} - T_\infty \quad at \quad x = 0$$

The general solution for this case is $\theta(x) = c_1 e^{-mx} + c_2 e^{mx}$

As $x \to \infty$, $\quad \theta \to \lim_{x\to\infty}(c_1 e^{-mx} + c_2 e^{mx})$

Hence, $c_2 = 0$ to satisfy the boundary condition. At $x = 0$,

$$\theta(0) = \theta_b = T_b - T_\infty = c_1 e^{-m.0}$$

Thus, $c_1 = T_b - T_\infty$. So,

$$\frac{\theta}{\theta_b} = e^{-mx} \;\; or \; \theta = \theta_b e^{-mx} \tag{2.24}$$

The heat flux, Q, can be found from

$$Q = \int_{x=0}^{\infty} h\theta(x)Pdx$$

$$Since\ \theta = \frac{1}{m^2}\frac{d^2\theta}{dx^2}\ from\ the\ governing\ equation,$$

$$Q = \frac{hP}{m^2}\int_0^\infty \frac{d^2\theta}{dx^2}dx$$

$$= \frac{hP}{hP/_{Ak}}\int_0^\infty \frac{d^2\theta}{dx^2}dx$$

$$= Ak\left[\left.\frac{d\theta}{dx}\right|_\infty - \left.\frac{d\theta}{dx}\right|_0\right]$$

As $x \to \infty, \theta \to 0\ and\ \left.\frac{d\theta}{dx}\right|_\infty \to 0$

Recall that since $\theta = \theta_b e^{-mx}$,

$$\frac{d\theta}{dx} = -m\theta_b e^{-mx}\quad and\quad \left.\frac{d\theta}{dx}\right|_0 = -m\theta_b e^{-mx} = -m\theta_b$$

$$Q = -Ak(-m\theta_b) = \sqrt{hPkA_c}\theta_b\ \ since\ m^2 = \frac{Ph}{Ak} \qquad (2.25)$$

Table 2.5: Temperature distributions for fin of uniform cross sectional area.

Tip Condition (x=L)	**Temperature Distribution**
Convection heat transfer	$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + \left(\frac{h}{mk}\right)\sinh m(L-x)}{\cosh mL + \left(\frac{h}{mk}\right)\sinh mL}$
Adiabatic	$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$
Constant Temperature	$\frac{\theta}{\theta_b} = \frac{\frac{\theta_L}{\theta_b}\sinh mx + \sinh m(L-x)}{\sinh mL}$
Infinite Fin Length	$\frac{\theta}{\theta_b} = e^{-mx}$

Source: Cengel (2006)

Table 2.6: Heat transfer rate for fins of uniform cross sectional area

Tip Condition (x=L)	Fin Heat Transfer Rate
Convection heat transfer	$\sqrt{hPkA_c}\theta_b \dfrac{\sinh mL + \left(\frac{h}{mk}\right)\cosh mL}{\cosh mL + \left(\frac{h}{mk}\right)\sinh mL}$
Adiabatic	$\sqrt{hPkA_c}\theta_b \tanh mL$
Constant Temperature	$\sqrt{hPkA_c}\theta_b \dfrac{\cosh mL - \frac{\theta_L}{\theta_b}}{\sinh mL}$
Infinite Fin Length	$\sqrt{hPkA_c}\theta_b$

Source: Cengel (2006)

2.8 NUMERICAL APPROACH STUDY

There are several numerical approaches to determine temperature distribution, two of the most commonly used ones being the finite difference and the finite elements methods.

The governing differential equation is written in terms of the temperatures at a finite number of points in the conduction region. The spatial derivatives at a point are replaced by expressions written in terms of the temperatures at the neighboring locations and the distances between them. This procedure gives rise to algebraic equations, one for each grid location shown, and the set of simultaneous equations is solved to obtain the temperatures at various points in the conduction region. Each grid point is assumed to represent a finite region in its neighborhood, and the particular arrangement shown is one without overlap of regions. Various other configurations of the grid have been employed, depending on the nature of the problem. The points at the surfaces considered in terms of the given boundary conditions, and special equations are often obtained. All these equations are then solved numerically to obtain the temperature distribution, starting with the given initial temperature distribution.

2.8.1 Finite Difference Method

Finite differences are frequently used to approximate differential equations. The result is a set of algebraic equations for the nodal unknowns with information from a continuum replaced by the information at discrete nodal points. The algebraic equations must be solved simultaneously (for steady state problem) or simultaneously at discrete time levels (for transient problem). The way to solve ordinary differential equations, particularly boundary value problems, is based on obtaining the finite difference approximation of the differential equation. This lead to a system which are solved to obtain the dependent variable at discrete value of the independent variable. As an example of the application of finite difference methods to ordinary differential equations, let consider the simple second-equation for an extended surface, given by equation (2.15) where,

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

The boundary conditions may be specified values of θ at the two ends. If the length L is divided into n subdivisions of length Δx, then $x=i\Delta x$, where i=0, 1, 2,…., n. There are (n +1) nodal points and the temperature θ at x=0, θ_0, is given as 1.0. For simplicity, let take θ=0.5 at x=L as the second boundary condition. The finite difference approximation of the given ordinary differential equation is obtained by replacing the second-order derivative by the second central difference as:

$$\frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{(\Delta x)^2} - m^2\theta_i = 0 \quad for\ i = 1,2,3,\dots,n-1 \qquad (2.26)$$

Then the system of equations to be solved is:

$$\theta_{i+1} = [2 + m^2(\Delta x)^2]\theta_i - \theta_{i-1} \quad for\ i = 1,2,3,\dots,n-1 \qquad (2.27)$$

Along with the conditions:

$$\theta_0 = 1.0\ and\ \theta_n = 0.5$$

2.9 DIMENSIONLESS NUMBER

A dimensionless number is a quantity which describes a certain physical system and which is a pure number without any physical units. Such a number is typically defined as a product or ratio of quantities which do have units, in such a way that all units cancel. Dimensionless numbers are widely applied in the field of mechanical and chemical engineering.

2.9.1 Properties

A dimensionless quantity has no physical unit associated with it. However, it is sometimes helpful to use the same units in both the numerator and denominator, such as kg/kg, to show the quantity being measured (for example to distinguish a mass ratio from a volume ratio). A dimensionless proportion has the same value regardless of the measurement units used to calculate it. It has the same value whether it was calculated using the SI system of units or the imperial system of units. This doesn't hold for all dimensionless quantities; it is guaranteed to hold only for proportions.

2.9.2 Buckingham π theorem

According to the Buckingham π theorem of dimensional analysis, the functional dependence between a certain number (e.g., n) of variables can be reduced by the number (e.g., k) of independent dimensions occurring in those variables to give a set of $p = n - k$ independent, dimensionless quantities. For the purposes of the experimenter, different systems which share the same description by dimensionless quantity are equivalent.

2.10 FORTRAN SOFTWARE

Formula Translation, known as FORTRAN, is a language that is use throughout the world to write programs for solving problems in science and engineering. This software is a general-purpose, procedural, imperative programming language that is especially suited to numeric computation and scientific computing. A FORTRAN

program generally consists of a main program (or driver) and possibly several subprograms (or procedures or subroutines). A FORTRAN program consists of a main program, possibly followed by one or more subprograms. The structure of a program is given in the following scheme,

Main program:
  program statement
  declarations
  statements
  end

Subprogram:
  subroutine or function
  declarations
  statements
  end

2.10.1 Content of Program

Organizing a problem's data is an important part of developing a program to the mathematical problem. This may be numeric data representing times or temperatures, or character data representing names, or logical data used in designing a circuit. Consequently, a program for solving a problem must be written in a language that can store and process various types of data. FORTRAN is design to handle six types of data:

i. Integer
ii. Real or single precision
iii. Double precision
iv. Complex
v. Character
vi. Logical

The first four are numeric types used to store and process various kinds of numbers, the character type is used to store and process strings of characters, and the logical type is used to store and process logical data values (.FALSE. and .TRUE.)

2.10.2 Character Strings

Character constant, also called strings, are sequences of symbols from the FORTRAN character set. The ANSI standard character set for FORTRAN is given in Table 2.5. Many versions of FORTRAN also include lowercase letters and other special symbols in their characters sets.

Table 2.7: Example of character strings

Character	Meaning
blank	Blank or space
0,…,9	Digits
A,…,Z	Uppercase letters
‘	Apostrophe (single quote)
*	Asterisk
+	Plus sign
-	Minus sign
/	Slash
,	Comma
.	Period
:	Colon
=	Equal sign

Source: Nyhoff and Leestma (1996)

2.10.3 Arithmetic Operations

Variables and constant can be processed by using operations and functions appropriate to their types. Table 2.7 summarizes these arithmetic operations

Table 2.8: Example of arithmetic operations

Operator	Operation
+	Addition, unary plus
-	Subtraction, unary minus
*	Multiplication
/	Division
**	Exponentiation

Source: Nyhoff and Leestma (1996)

2.10.4 Functions

FOTRAN provides functions for many of the common mathematical operations and functions. To use of any these functions, we simply give the function name followed by the arguments enclosed in parentheses. In each case, the arguments must be of the type specified for that function in table below;

Table 2.9: Example of functions

Function	Description
ABS (x)	Absolute value of x
COS (x)	Cosine of x radians
EXP (x)	Exponential function e^x
INT (x)	Integer part of x
LOG (x)	Natural logarithm of x
MAX (x_1, ... , x_n)	Maximum of x_1, ... , x_n
MIN (x_1, ... , x_n)	Minimum of x_1, ... , x_n
MOD (x, y)	x (mod y); x – INT (x/y) * y
NINT (x)	x rounded to nearest integer
REAL (x)	Conversion of x to real type
SQRT (x)	Square root of x

Source: Nyhoff and Leestma (1996)

2.11 CONCLUSION

The related information to analysis the fin performance has review to make the research methodology. The information and method used has chosen and is clarify in the next chapter.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

This chapter is really focused about method to predict the temperature gradient, efficiency and fluid temperature. Before model the calculation, there are some method needs to be determined. Taylor series central difference scheme and Jacobi method is chosen to predict temperature gradient. FORTRAN Software is used to compute a present mathematical model with several repetitions.

3.2 RESEARCH FLOW CHART

From the flow chart in figure 3.1, the project starts with gain related information about basic heat transfer, heat transfer mode, basic equation, space radiator, fin, numerical approach and FORTRAN Software. Article from previous research is studied to know parameter and method used to adapt in this research. FORTRAN Software is important tool to predict temperature distribution at varies radiation parameter, N_R.

After successfully understand basic concept of heat transfer in radiating fin, energy balance equation is developed to gather the related parameter into simplest equation. There are three main equations involve in this research which are temperature gradient, efficiency and fluid temperature. Nondimensionalization the equation is important approach to compare with previous research because this approach has value to represent the executed data.

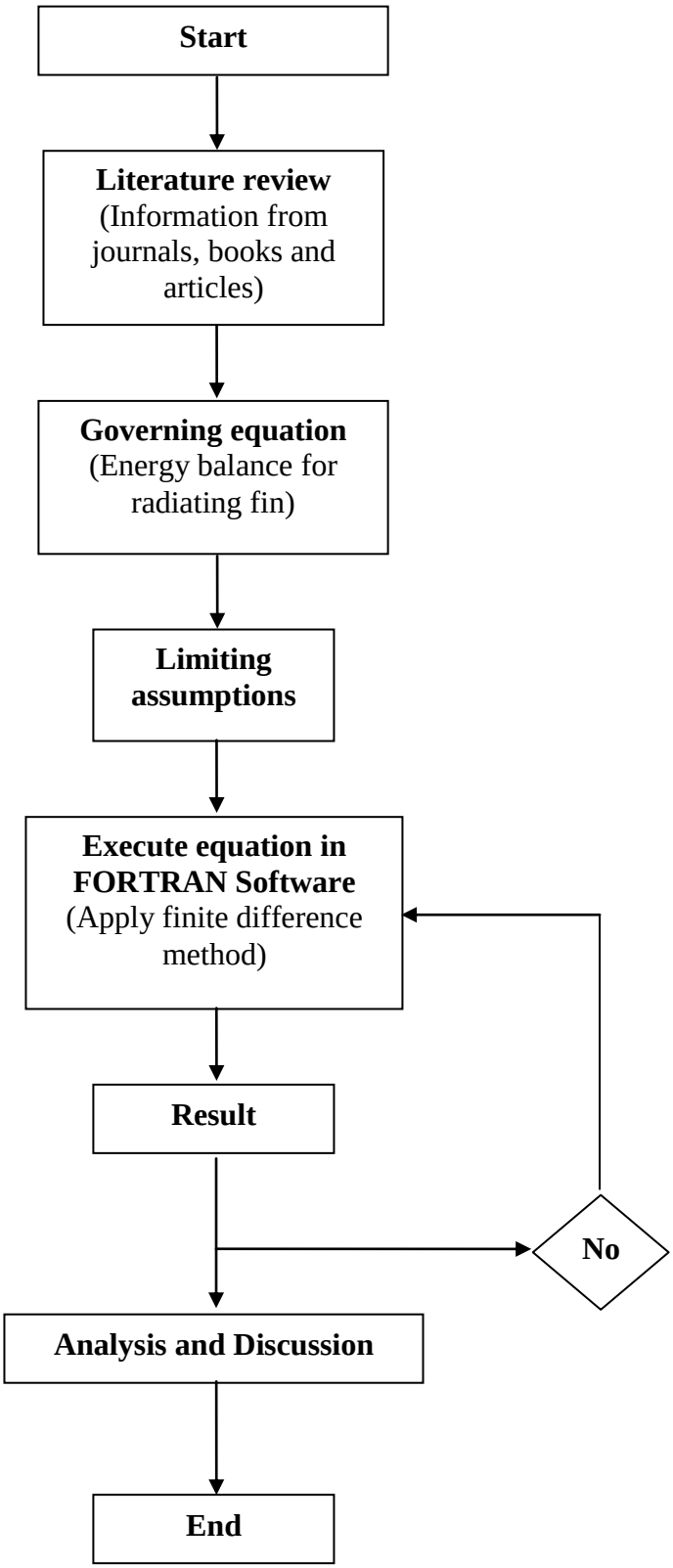

Figure 3.1: Research flowchart.

A lot of assumptions are made to limit and simplify the solution. There are 10 limiting assumptions which are referred to Murray-Gardner assumptions. Example for this assumption is the fin material is homogeneous, its thermal conductivity is the same in all directions, and it remains constant.

After that, the temperature gradient is solve numerically using finite difference method and is execute in FORTRAN software. The program needs five parameters to generate the equations which are dimensionless temperature at point 2, θ (2), dimensionless temperature ratio, ψ, convection parameter, $N_{C,}$ radiation parameter, N_R and irradiation parameter, N_G. Efficiency and fluid temperature is calculate base on temperature gradient (after some repetition with initial guess) at some parameter value.

After that, the result is obtain and is save in Microsoft Excel before graph is draw. If the result is not present the actual or logical value, the equation is executed again after some method or parameter is altered.

The result after the equation has solved, is present in graph. The first graph is relation between dimensionless temperature, θ and distance from base to tip. Second, the graph about effect of efficiency with the change of radiation parameter, N_R. The third graph is relation between fluid temperature and radiation parameter, N_R. The variety of dimensionless temperature ratio, ψ, convection parameter, $N_{C,}$ radiation parameter, N_R and irradiation parameter, N_G means there are many relations at various conditions.

All the relationship is discuss to predict heat transfer at outer space before and after convection parameter is taken off.

3.3 LIMITING ASSUMPTIONS

Fins of various geometries and thermal conductivities respond differently to identical and uniform heat source and sinks. Similarly, there are numerous ways in which the temperatures and heat transfer coefficients of source and sinks may vary. Important to the analysis of fin geometries are the constraints or assumptions that are

employed to define and limit the problem and often to simplify its solution. These limiting assumptions, which are almost always referred to as the Murray-Gardner assumptions are,

i. The heat flow in the fin and its temperatures remain constant with time.
ii. The fin material is homogeneous, its thermal conductivity is the same in all directions, and it remains constant.
iii. The convective heat transfer coefficient on the faces of the fin is constant and uniform over the entire surface of the fin.
iv. The temperature of the medium surrounding the fin is uniform.
v. The fin thickness is small, compared with its height and length, so that temperature gradients across the fin thickness and heat transfer from the edge of the fin may be neglected.
vi. The temperature at the base of the fin is uniform.
vii. There is no contact resistance where the base of the fin joins the prime surface.
viii. There are no heat sources within the fin itself.
ix. The heat transferred through the tip of the fin is negligible compared with the heat leaving its lateral surface.
x. Heat transfer to or from the fin is proportional to the temperature excess between the fin and the surrounding medium.
xi. The heat transferred at the outer surface of tube is negligible.

3.4 DEVELOPMENT OF ENERGY BALANCE EQUATION AND NONDIMENSIONALIZATION

The governing equation for conductive, convective, radiative and irradiative heat transfer processes is obtained by applying the principle of conservation of energy under steady state condition to a differential element of width, dx, and cross section $A = \delta\, dz$ and assume one-dimensional conduction, constant thermal parameters, and ignoring fin-to-base and fin-to-fin radiation interaction.

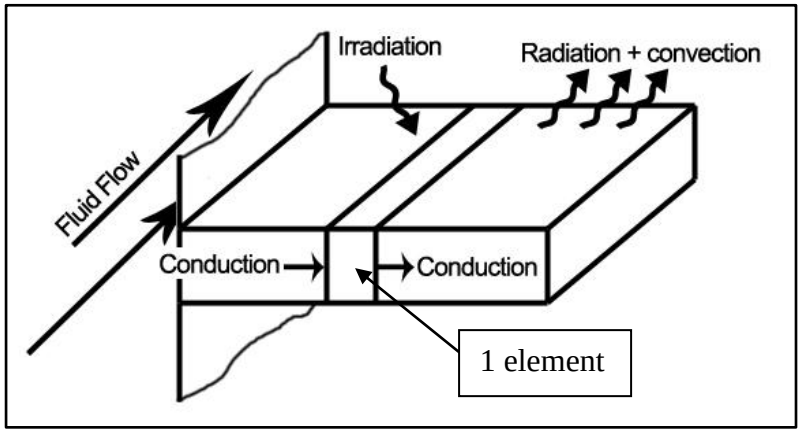

Figure 3.2 Block diagram for heat transfer components

Source: Kraus (2001)

The energy balance can be written as

$$\begin{bmatrix}\text{Rate of heat conducted in,}\\ \text{at } x = x\end{bmatrix} + \begin{bmatrix}\text{Rate of heat entering by}\\ \text{irradiation over width, d}x\end{bmatrix} = \begin{bmatrix}\text{Rate of heat conducted out,}\\ \text{at } x = x+\text{d}x\end{bmatrix}$$
$$+ \begin{bmatrix}\text{Rate of heat loss by convection}\\ \text{over the width, d}x\end{bmatrix} + \begin{bmatrix}\text{Rate of heat loss by radiation}\\ \text{over the width, d}x\end{bmatrix}$$

A simple analysis of an extended surface assumes a constant temperature over the edge that is in contact with the coolant tube. Generally flat profile of constant thickness is used for the fin, as it is easy to fabricate.

Consider an elemental length dx of the fin. As the surface is meant for space application, certain amount of energy can enter the system from outer space by irradiation. Let the absorptivity of the surface for irradiation G, to be α. Let the elemental surface emissivity be ε (not necessarily the same as absorptivity α). The total radiation exchange from the both surface is 2(heat loss by convection + heat loss by radiation – heat entering by irradiation), considering the heat rejection from the top and bottom surfaces and convection losses. This must equal the net energy conducted into the element. Thus, the governing equation becomes

$$-kA_c\left(\frac{dT}{dx}\right)dx = -kA_c\left(\frac{dT}{dx}\right)dx - kA_c\left(\frac{d^2T}{dx^2}\right)dx$$
$$+2\{h(T-T_\infty)+\varepsilon\sigma(T^4-T_\infty^4)-\alpha G\}Hdx$$
$$kA_c\left(\frac{d^2T}{dx^2}\right)dx = 2\{h(T-T_\infty)+\varepsilon\sigma(T^4-T_\infty^4)-\alpha G\}Hdx$$
$$H\delta\left(\frac{d^2T}{dx^2}\right)dx = \frac{2}{k}\{h(T-T_\infty)+\varepsilon\sigma(T^4-T_\infty^4)-\alpha G\}Hdx$$
$$\delta\left(\frac{d^2T}{dx^2}\right)dx = \frac{2}{k}\{h(T-T_\infty)+\varepsilon\sigma(T^4-T_\infty^4)-\alpha G\}dx \tag{3.1}$$

In the energy balance equation above, convection has been considered. The convection component has been taken into account keeping in view the possibility for testing the exchanger in a controlled atmosphere. Figure 3.2 shows the schematic diagram of a flat fin under consideration. The inclusion of convection in the problem has resulted in the addition of two dimensionless terms, N_C and ψ.

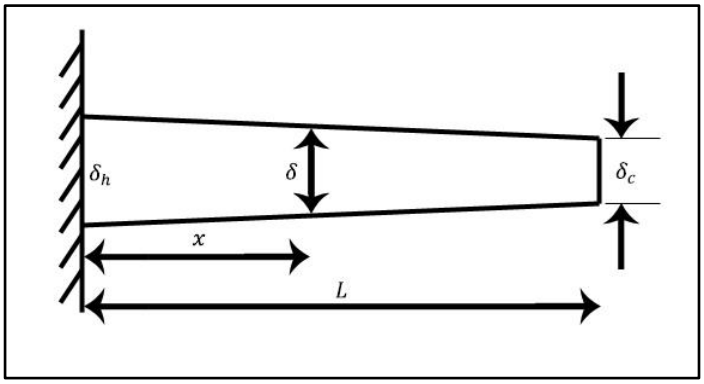

Figure 3.3 Geometry relations in fin

Source: Sasikumar et al. (2002)

From the geometry we have, $\frac{\delta-\delta_c}{L-x} = \frac{\delta_h-\delta_c}{L}$ or on rearrangement

$$\delta = \delta_h\left[1-\frac{x}{L}\left(1-\frac{\delta_c}{\delta_h}\right)\right] \tag{3.2}$$

From equation (3.1) above,

$$\delta\left(\frac{d^2T}{dx^2}\right)dx = \frac{2}{k}\{h(T-T_\infty)+\varepsilon\sigma(T^4-T_\infty^4)-\alpha G\}dx$$

$$\delta_h\left[1-\frac{x}{L}\left(1-\frac{\delta_c}{\delta_h}\right)\right]\left(\frac{d^2T}{dx^2}\right)dx = \frac{2}{k}\{h(T-T_\infty)+\varepsilon\sigma(T^4-T_\infty^4)-\alpha G\}dx$$

$$\delta_h\frac{T_b-T_\infty}{L^2}\left\{\left[1-\frac{x}{L}\left(1-\frac{\delta_c}{\delta_h}\right)\right]\left(\frac{d^2T}{dx^2}\right)-\left(1-\frac{\delta_c}{\delta_h}\right)\frac{dT}{dx}\right\}$$
$$= \frac{2}{k}\{h(T-T_\infty)+\varepsilon\sigma(T^4-T_\infty^4)-\alpha G\}$$

$$\left[1-\frac{x}{L}\left(1-\frac{\delta_c}{\delta_h}\right)\right]\left(\frac{d^2T}{dx^2}\right)-\left(1-\frac{\delta_c}{\delta_h}\right)\frac{dT}{dx}$$
$$= \frac{2L^2}{k\delta_h T_b-T_\infty}\{h(T-T_\infty)+\varepsilon\sigma(T^4-T_\infty^4)-\alpha G\}$$

$$\left[1-\frac{x}{L}\left(1-\frac{\delta_c}{\delta_h}\right)\right]\left(\frac{d^2T}{dx^2}\right)-\left(1-\frac{\delta_c}{\delta_h}\right)\frac{dT}{dx}$$
$$= \frac{2L^2(T_b-T_\infty)^3}{k\delta_h}\left\{\frac{h(T-T_\infty)}{(T_b-T_\infty)(T_b-T_\infty)^3}+\frac{\varepsilon\sigma(T^4-T_\infty^4)}{(T_b-T_\infty)(T_b-T_\infty)^3}\right.$$
$$\left.-\frac{\alpha G}{(T_b-T_\infty)(T_b-T_\infty)^3}\right\}$$

$$\left[1-\frac{x}{L}\left(1-\frac{\delta_c}{\delta_h}\right)\right]\left(\frac{d^2T}{dx^2}\right)-\left(1-\frac{\delta_c}{\delta_h}\right)\frac{dT}{dx}$$
$$= \frac{2L^2(T_b-T_\infty)^3}{k\delta_h}\left\{\frac{h}{(T_b-T_\infty)^3}\frac{(T-T_\infty)}{(T_b-T_\infty)}+\frac{\varepsilon\sigma(T^4-T_\infty^4)}{(T_b-T_\infty)^4}-\frac{\alpha G}{(T_b-T_\infty)^4}\right\}$$

$$\left[1-\frac{x}{L}\left(1-\frac{\delta_c}{\delta_h}\right)\right]\left(\frac{d^2T}{dx^2}\right)-\left(1-\frac{\delta_c}{\delta_h}\right)\frac{dT}{dx}$$
$$=\frac{2\varepsilon\sigma L^2(T_b-T_\infty)^3}{k\delta_h}\left\{\frac{h}{\varepsilon\sigma(T_b-T_\infty)^3}\frac{(T-T_\infty)}{(T_b-T_\infty)}+\frac{(T^4-T_\infty^4)}{(T_b-T_\infty)^4}\right.$$
$$\left.-\frac{\alpha G}{\varepsilon\sigma(T_b-T_\infty)^4}\right\}$$

$$\left[1-\frac{x}{L}\left(1-\frac{\delta_c}{\delta_h}\right)\right]\left(\frac{d^2T}{dx^2}\right)-\left(1-\frac{\delta_c}{\delta_h}\right)\frac{dT}{dx}$$
$$-\frac{2\varepsilon\sigma L^2(T_b-T_\infty)^3}{k\delta_h}\left\{\frac{h}{\varepsilon\sigma(T_b-T_\infty)^3}\frac{(T-T_\infty)}{(T_b-T_\infty)}+\frac{(T^4-T_\infty^4)}{(T_b-T_\infty)^4}\right.$$
$$\left.-\frac{\alpha G}{\varepsilon\sigma(T_b-T_\infty)^4}\right\}=0 \qquad (3.3)$$

$Rearrange\ \frac{(T^4-T_\infty^4)}{(T_b-T_\infty)^4}$

$$\frac{(T^4-T_\infty^4)}{(T_b-T_\infty)^4}=\frac{T^4}{(T_b-T_\infty)^4}-\frac{T_\infty^4}{(T_b-T_\infty)^4}$$
$$=\left(\frac{T}{T_b-T_\infty}\right)^4-\left(\frac{T_\infty}{T_b-T_\infty}\right)^4$$
$$=\left(\frac{T-T_\infty+T_\infty}{T_b-T_\infty}\right)^4-\left(\frac{T_\infty}{T_b-T_\infty}\right)^4$$
$$=\left(\frac{T-T_\infty}{T_b-T_\infty}+\frac{T_\infty}{T_b-T_\infty}\right)^4-\left(\frac{T_\infty}{T_b-T_\infty}\right)^4$$
$$=(\theta+\psi)^4-\psi^4 \qquad (3.4)$$

Substitute equation (3.4) in to equation (3.3), then the non-dimensional form gives,

$$\left[1-\xi(1-N_p)\right]\frac{d^2\theta}{d\xi^2}-\left(1-N_p\right)\frac{d\theta}{d\xi}-N_R[N_c\theta+\{(\theta+\psi)^4-\psi^4\}-N_G]=0 \qquad (3.5)$$

Where,

$$\xi = \frac{x}{L}$$

$$N_p = \frac{\delta_c}{\delta_h}$$

$$\theta = \frac{T - T_\infty}{T_b - T_\infty}$$

$$\psi = \frac{T_\infty}{T_b - T_\infty}$$

$$N_R = \frac{2\varepsilon\sigma L^2 (T_b - T_\infty)^3}{k\delta_h}$$

$$N_c = \frac{h}{\varepsilon\sigma (T_b - T_\infty)^3}$$

$$N_c = \frac{h}{\varepsilon\sigma (T_b - T_\infty)^3}$$

$$N_G = \frac{\alpha G}{\varepsilon\sigma (T_b - T_\infty)^4}$$

3.4.1 Boundary Conditions

The boundary conditions at the base and at the tip with no heat flow can be written as

$$x = 0, \;\; T = T_b; \qquad\qquad x = L, \;\; \frac{dT}{dx} = 0 \tag{3.6}$$

or in non-dimensional form as

$$\xi = 0, \;\; \theta = 1; \qquad\qquad \xi = 1, \;\; \frac{d\theta}{d\xi} = 0 \tag{3.7}$$

There are four parameters N_R, N_c, N_G *and* N_P that influence the temperature distribution and are governed by the non-linear differential equation (3.5) which have to be solved numerically.

3.5 DETERMINE THE VARIATION OF TEMPERATURE

From energy balance (in non-dimensional form)

$$\left[1-\xi\left(1-N_p\right)\right]\frac{d^2\theta}{d\xi^2}-\left(1-N_p\right)\frac{d\theta}{d\xi}-N_R[N_c\theta+\{(\theta+\psi)^4-\psi^4\}-N_G]=0$$

$$\left[1-\xi\left(1-N_p\right)\right]\frac{d^2\theta}{d\xi^2}=\left(1-N_p\right)\frac{d\theta}{d\xi}+N_R[N_c\theta+\{(\theta+\psi)^4-\psi^4\}-N_G] \qquad (3.8)$$

The definition of the derivative of $\theta(\xi)$ at ξ_n is given by,

$$\left.\frac{d\theta}{d\xi}\right|_{\xi_n}\approx\frac{\theta_n-\theta_{n-1}}{\xi}$$

The second derivative of $\theta(\xi)$ can be written in central difference approximation as,

$$\left.\frac{d^2\theta}{d\xi^2}\right|_{\xi_n}\approx\frac{\left.\frac{d\theta}{d\xi}\right|_{\xi_n+\Delta\xi/2}-\left.\frac{d\theta}{d\xi}\right|_{\xi_n-\Delta\xi/2}}{\xi}$$

Substitute the central difference forms,

$$\left.\frac{d^2\theta}{d\xi^2}\right|_{\xi_n}\approx\frac{\theta_{n+1}-2\theta_n+\theta_{n-1}}{\xi^2} \qquad (3.9)$$

Thus, the energy balance can be write as,

$$[\theta_{n+1}-2\theta_n+\theta_{n-1}][1-\xi(n-1)(1-N_P)]\left(\frac{1}{\xi^2}\right)$$
$$=\left(1-N_p\right)[\theta_n-\theta_{n-1}]\left(\frac{1}{\xi}\right)+N_R[N_c\theta+\{(\theta+\psi)^4-\psi^4\}-N_G]$$

Rearrange the equation,

$$\theta_{n+1} = 2\theta_n - \theta_{n-1} + \xi^2(TERM3 + TERM2 \times TERM4)\left(\frac{1}{TERM1}\right) \quad (3.10)$$

Where,

$$TERM1 = [1 - \xi(n-1)(1 - N_P)]$$
$$TERM2 = (1 - N_p)$$
$$TERM3 = N_R[N_c\theta + \{(\theta + \psi)^4 - \psi^4\} - N_G]$$
$$TERM4 = [\theta_n - \theta_{n-1}]\left(\frac{1}{\xi}\right)$$

3.5.1 Numerical Approach

The numerical solution is obtained for the present problem by developing an algorithm where the domain is discretized by Taylor series central difference scheme and finally difference equations have been solved by Jacobi method. Equation (3.10) is compute using FORTRAN Software at certain parameter value. The length of fin is divided to 100 elements and thus the dimensionless distance, ξ remain constant at 0.01. The initial dimensionless temperature ratio, θ at $\xi = 0$ is 1. The iterations stop when the different of $\theta(99)$ and $\theta(100)$ less than 0.0001 which mean by constant temperature at the tip.

3.5.2 Development of Program to Determine the Variation of Temperature

The FORTRAN coding is developed from equation (3.10) to predict temperature gradient which is depending on several parameters. This computational is for a straight fin of rectangular profile with a constant cross-sectional area, A_c of an arbitrary form having the perimeter, P.

```
      PROGRAM TEMPERATURE GRADIENT FOR SPACE RADIATOR
C
*****************************************************************
C              S P A C E   R A D I A T O R
C
*****************************************************************
C     SOLUTION OF TEMPERATURE GRADIENT USING RUNGE KUTTA METHOD
C
C-----------------------------------------------------------------


      REAL NR,NC,NG,NP
      DOUBLE PRECISION T(100),TE(2000),TG(2000)
      WRITE(*,*)'INPUT VALUES T(2),SI,NP,NR,NC,NG'
      READ(*,*)T(2),SI,NP,NR,NC,NG

      T(1)=1.0
      KK=1
      TG(KK)=T(2)
      J=100
      H=1./J

 40   DO 100 N=2,J-1
      TERM1=(1.-H*(N-1)*(1-NP))
      TERM2=(1-NP)
      TERM3=NR*(NC*T(N)+((T(N)+SI)**4-SI**4)-NG)
      TERM4=(T(N)-T(N-1))/H
      T(N+1)=2*T(N)-T(N-1)+H*H*(TERM3+TERM2*TERM4)/TERM1

 100  CONTINUE


      TE(KK)=(T(J)-T(J-1))/H
      IF(ABS(TE(KK)).LE.1.E-4)GO TO 30
      KK=KK+1
      IF(KK.LE.2)THEN
      TG(2)=TG(1)*0.99
      T(2)=TG(2)
      ELSE
      TG(KK)=(TG(KK-2)*TE(KK-1)-TG(KK-1)*TE(KK-2))/(TE(KK-1)-TE(KK-2))
      T(2)=TG(KK)
      ENDIF

      GO TO 40
 30   CONTINUE

      WRITE(*,*)'N,T(N)'
      DO 101 N=1,J
      WRITE(*,500)N,T(N)

 101  CONTINUE
 300  CONTINUE
 500  FORMAT((I4,F8.4))
      pause
      CLOSE(2)
      STOP
      END
```

3.6 DETERMINE THE HEAT TRANSFER AND FIN EFFICIENCY

The heat loss from the fin is equal to the conductive heat entering at the base. This is given by

$$Q_{loss} = heat\ transfer\ by\ conduction\ at\ \xi = 0$$

$$Q_{loss} = -\frac{k\delta_h H(T_b - T_\infty)}{L}\left(\frac{d\theta}{d\xi}\right)_{\xi=0} \tag{3.11}$$

The maximum heat that could be lost from the fin is given by the amount of heat lost if the fin is maintained throughout at the base temperature T_b and no radiation is incident on it.

$$Q_{max} = heat\ transfer\ by\ convection\ and\ radiation$$

$$Q_{max} = 2LH\{h(T_b - T_\infty) + \varepsilon\sigma(T_b^4 - T_\infty^4)\} \tag{3.12}$$

The efficiency, η is defined as the ratio of Q_{loss} to $Q_{max.}$ Thus,

$$\eta = \frac{-\frac{k\delta_h H(T_b - T_\infty)}{L}\left(\frac{d\theta}{d\xi}\right)_{\xi=0}}{2LH\{h(T_b - T_\infty) + \varepsilon\sigma(T_b^4 - T_\infty^4)\}}$$

$$= \frac{-k\delta_h(T_b - T_\infty)\left(\frac{d\theta}{d\xi}\right)_{\xi=0}}{2L^2\{h(T_b - T_\infty) + \varepsilon\sigma(T_b^4 - T_\infty^4)\}}$$

$$= \frac{-\left(\frac{d\theta}{d\xi}\right)_{\xi=0}}{\frac{2L^2}{k\delta_h(T_b - T_\infty)}\{h(T_b - T_\infty) + \varepsilon\sigma(T_b^4 - T_\infty^4)\}}$$

$$= \frac{-\left(\frac{d\theta}{d\xi}\right)_{\xi=0}}{\frac{2L^2}{k\delta_h}\left[h + \varepsilon\sigma\left\{\left(\frac{T_b^4}{(T_b - T_\infty)}\right) - \left(\frac{T_\infty^4}{(T_b - T_\infty)}\right)\right\}\right]}$$

$$= \frac{-\left(\frac{d\theta}{d\xi}\right)_{\xi=0}}{\frac{2\varepsilon\sigma L^2(T_b-T_\infty)^3}{k\delta_h}\left[\frac{h}{\varepsilon\sigma(T_b-T_\infty)^3}+\left\{\left(\frac{T_b^4}{(T_b-T_\infty)(T_b-T_\infty)^3}\right)-\left(\frac{T_\infty^4}{(T_b-T_\infty)(T_b-T_\infty)^3}\right)\right\}\right]}$$

$$= \frac{-\left(\frac{d\theta}{d\xi}\right)_{\xi=0}}{\frac{2\varepsilon\sigma L^2(T_b-T_\infty)^3}{k\delta_h}\left[\frac{h}{\varepsilon\sigma(T_b-T_\infty)^3}+\left\{\left(\frac{T_b-T_\infty+T_\infty}{T_b-T_\infty}\right)^4-\left(\frac{T_\infty}{T_b-T_\infty}\right)^4\right\}\right]}$$

$$= \frac{-\left(\frac{d\theta}{d\xi}\right)_{\xi=0}}{\frac{2\varepsilon\sigma L^2(T_b-T_\infty)^3}{k\delta_h}\left[\frac{h}{\varepsilon\sigma(T_b-T_\infty)^3}+\left\{\left(1+\frac{T_\infty}{T_b-T_\infty}\right)^4-\left(\frac{T_\infty}{T_b-T_\infty}\right)^4\right\}\right]}$$

$$= \frac{-\left(\frac{d\theta}{d\xi}\right)_{\xi=0}}{N_R[N_C+\{(1+\psi)^4-\psi^4\}]} \tag{3.13}$$

3.6.1 Development of Program to Determine the Heat Transfer and Fin Efficiency

Fin efficiency is predicted after temperature gradient has been obtained. The efficiency will vary with varies radiation parameter.

```
      PROGRAM FIN EFFICIENCY OF SPACE RADIATOR

C*****************************************************************
C              S P A C E   R A D I A T O R
C
*****************************************************************
C     SOLUTION OF TEMPERATURE GRADIENT USING RUNGE KUTTA METHOD
C
C-----------------------------------------------------------------

      REAL NR,NC,NG,NP
      DOUBLE PRECISION T(100),TE(2000),TG(2000)
      WRITE(*,*)'INPUT VALUES T(2),SI,NC,NG'
      READ(*,*)T(2),SI,NC,NG

      T(1)=1.0
      NP=1.
      KK=1
      TG(KK)=T(2)
      J=100
      H=1./J

      DO 200 IXX=1,3400,100
      NR=IXX/1000.
 40   DO 100 N=2,J-1
```

```
      TERM1=(1.-H*(N-1)*(1-NP))
      TERM2=(1-NP)
      TERM3=NR*(NC*T(N)+((T(N)+SI)**4-SI**4)-NG)
      TERM4=(T(N)-T(N-1))/H
      T(N+1)=2*T(N)-T(N-1)+H*H*(TERM3+TERM2*TERM4)/TERM1

100   CONTINUE
      TE(KK)=(T(J)-T(J-1))/H
      IF(ABS(TE(KK)).LE.1.E-4)GO TO 30
      KK=KK+1
      IF(KK.LE.2)THEN
      TG(2)=TG(1)*0.99
      T(2)=TG(2)
      ELSE
      TG(KK)=(TG(KK-2)*TE(KK-1)-TG(KK-1)*TE(KK-2))/(TE(KK-1)-TE(KK-2))
      T(2)=TG(KK)
      ENDIF

      GO TO 40
30    CONTINUE
      TERM5=(NC+(1.0+SI)**4-SI**4)
      EFFI=-(T(2)-T(1))/(H*NR*TERM5)

20    CONTINUE
      WRITE(*,400)NR,EFFI

200   CONTINUE
400   FORMAT(F8.2,F8.4)
      pause
      CLOSE(2)
      STOP
      END
```

3.7 DETERMINE THE FLUID TEMPERATURE

In many situations it is required that the length of the tube, H for a specific temperature drop is known for the evaluation of the size of the radiator.

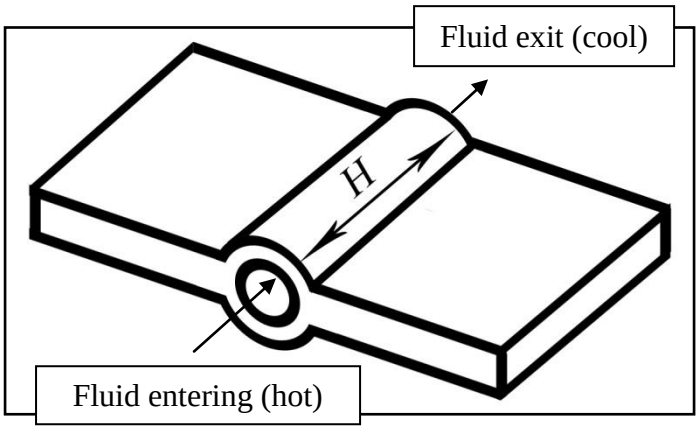

Figure 3.4 Movement of fluids along fin's tube

The rate at which energy is conducted through the plate to one fluid tube from both sides is

$$dQ_{fin} = -2k_p\delta_h\left[\frac{dT}{dx}\right]_{x=0} dz \tag{3.14}$$

$$dQ_{fluid} = mc_p dT_f \tag{3.15}$$

Equating Equations (3.14) and (3.15) then,

$$dQ_{fin} = dQ_{fluid}$$

$$-2k_p\delta_h\left[\frac{dT}{dx}\right]_{x=0} dz = mc_p dT_f$$

Rearranging

$$dT_f = -\frac{2k_p\delta_h}{mc_p}\left[\frac{dT}{dx}\right]_{x=0} dz \tag{3.16}$$

The Eq. (3.16) in non-dimensional form gives

$$T_{fi} - T_{fo} = -\frac{2k_p\delta_h}{mc_p}\left[\frac{(T_{fi} - T_\infty)d\theta}{Ld\xi}\right]_{\xi=0} dz$$

$$\frac{T_{fi} - T_{fo}}{T_{fi} - T_\infty} = -\frac{2k_p\delta_h}{mc_pL}\frac{d\theta}{d\xi}\bigg|_{\xi=0} dz \tag{3.17}$$

Integrating Eq. (3.17) and using the fluid inlet and outlet conditions at $z = 0$ and at $z = H$ and with the assumption that the temperature at the base is the same as that of the fluid or $T_b = T_f$

$$\frac{T_{fi} - T_{fo}}{T_{fi} - T_\infty} = 1 - \exp\left[-\frac{2k_p\delta_h H}{mc_pL}\frac{d\theta}{d\xi}\bigg|_{\xi=0}\right]$$

or

$$\frac{T_{fi} - T_{fo}}{T_{fi} - T_\infty} = 1 - \exp\left[-N_f\frac{d\theta}{d\xi}\bigg|_{\xi=0}\right] \tag{3.18}$$

$$\text{Where } N_f = \frac{2k_p\delta_h H}{mc_p L}$$

3.7.1 Development of Program to Determine the Fluid Temperature

Longitudinal heat conduction parameter, N_F, is introduced to make comparison about fluid and fin properties. The fluid temperature will vary along tube length. The total heat dissipated to fin is differential of heat enter and exit from the tube.

```
      PROGRAM FLUID TEMPERATURE

C    ****************************************************************
C                S P A C E   R A D I A T O R
C    ****************************************************************
C      SOLUTION OF TEMPERATURE GRADIENT USING RUNGE KUTTA METHOD
C
C    ----------------------------------------------------------------

      REAL NR,NC,NG,NP
      DOUBLE PRECISION T(100),TE(2000),TG(2000)
      WRITE(*,*)'INPUT VALUES T(2),NF,SI,NC,NG'
      READ(*,*)T(2),NF,SI,NC,NG

      T(1)=1.0
      NP=1.
      KK=1
      TG(KK)=T(2)
      J=100
      H=1./J

      DO 200 IXX=1,3400,100
      NR=IXX/1000.
 40   DO 100 N=2,J-1
      TERM1=(1.-H*(N-1)*(1-NP))
      TERM2=(1-NP)
      TERM3=NR*(NC*T(N)+((T(N)+SI)**4-SI**4)-NG)
      TERM4=(T(N)-T(N-1))/H
      T(N+1)=2*T(N)-T(N-1)+H*H*(TERM3+TERM2*TERM4)/TERM1

 100  CONTINUE

      TE(KK)=(T(J)-T(J-1))/H
      IF(ABS(TE(KK)).LE.1.E-4)GO TO 30
      KK=KK+1
      IF(KK.LE.2)THEN
      TG(2)=TG(1)*0.99
      T(2)=TG(2)
      ELSE
      TG(KK)=(TG(KK-2)*TE(KK-1)-TG(KK-1)*TE(KK-2))/(TE(KK-1)-TE(KK-2))
      T(2)=TG(KK)
      ENDIF
      GO TO 40
 30   CONTINUE
```

```
      TGRAD=(T(1)-T(2))/H
      FT=1.-EXP(-NF*TGRAD)

20    CONTINUE

      WRITE(*,400)NR,FT

200   CONTINUE
400   FORMAT(F8.2,F8.4)
500   FORMAT((I4,F8.4))
      pause
      CLOSE(2)
      STOP
      END
```

3.8 CONCLUSION

The methodology to predict the temperature gradient, fin efficiency and fluid temperature has been done by developing an algorithm where the domain is discretized by Taylor series central difference scheme and solved by Jacobi method. Finally, FORTRAN Software will compute the final equation with present methodology with several assumptions or limitations.

CHAPTER 4

RESULT AND DISCUSSION

4.1 INTRODUCTION

In the following sections, there are three types of result presented temperature distribution, fin efficiency and fluid temperature. The temperature distributions obtained by the finite difference method are used for efficiency comparison and effect of fluid temperatures. The temperature distributions and efficiency is matched and compared with previous research to verify present methodology.

4.2 VERIFICATION OF PRESENT MODEL

Before furnishing the result obtained from the present analysis, the accuracy of the present formulation is ascertained by comparing its results with published solutions.

4.2.1 Temperature Gradient

Based on the above analysis, temperature distribution over the fin surface and the overall fin performances are estimated for a design condition of constant thermal conductivity, k, convection heat transfer coefficient, h, profile parameter, N_P and base temperature, T_b.

In order to make a comparison between the present and the published results as well as to provide a useful assessment of the accuracy of the present method, the results obtained from the present analysis are compared with that of the Cobble (1964) models. Numerical results were generated for a wide range of variables. The specific values

chosen were ψ = 0.2323, N_C = 16.78, N_R = 0.1738 and N_G = 0. According to the Mueller and Mulaweh (2005), the temperature from both the experimental data and the numerical solution is a maximum at the base and decays to nearly the ambient temperature at the tip. Agreement between the present and previous solution is good. As is shown in the Figure 4.1, the dimensionless temperature ratio term, θ, decrease rapidly in the initial length of fin before slightly decrease at the end of the fin (tip). Since the present analysis applied constant convection heat transfer coefficient, h the present θ value at fin tip (ξ =1) is 0.3507 is lower than value 0.3611 given by Cobble (1964) for varies h. It is because the h value is affected by temperature along the fin.

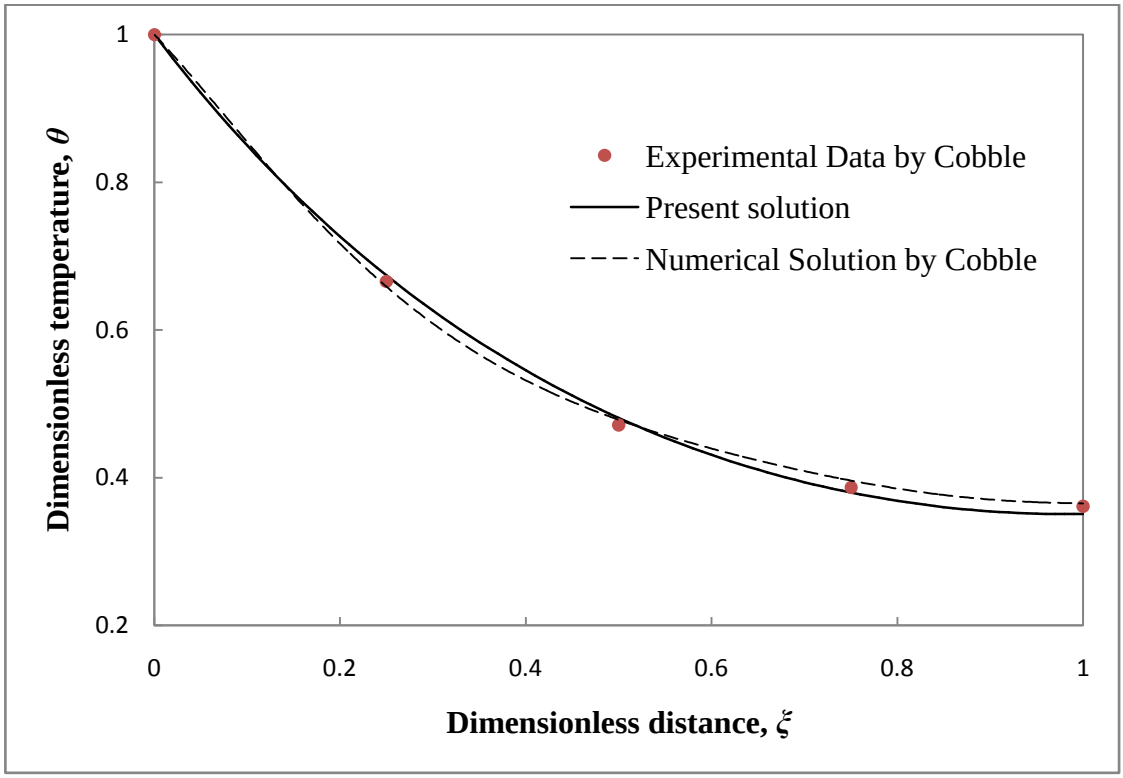

Figure 4.1: Temperature distributions at ψ = 0.2323, N_C = 16.78, N_R = 0.1738 and N_G = 0

4.2.2 Fin Efficiency

The efficiencies calculated using Equation 3.13 are presented in Figure 4.2 for $N_C = 1$ and $\psi = 0.8$. The closed agreement between the corresponding curves for $N_G = 0$ and the range of Ngunyen and Aziz (1990) for a rectangular fin confirms the accuracy of the present model. However, the Ngunyen and Aziz, (1990) value is lower because of the difference in tip conditions. While their model allow for simultaneous convection and radiation from the tip, present analysis use an insulated tip as assumption. As N_R increases, the efficiency decreases significantly. The same conclusion was reached by Sparrow and Niewerth (1968) and Campo (1976) who studied the rectangular fin.

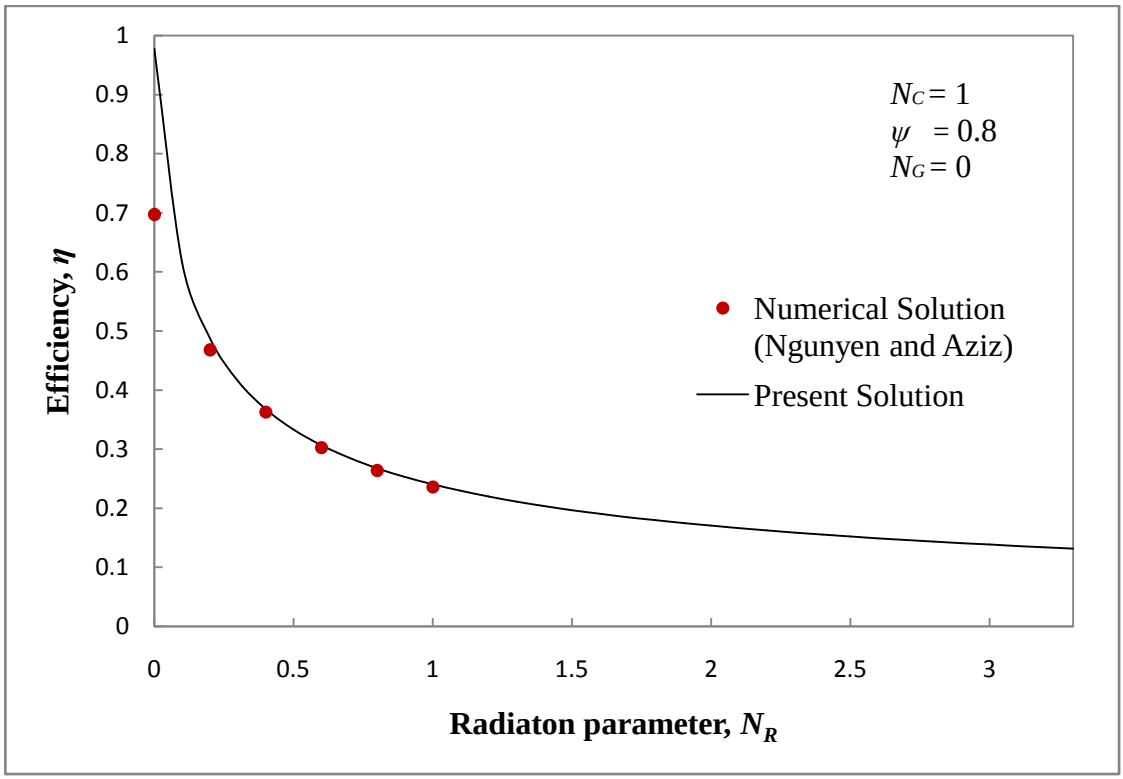

Figure 4.2: Fin efficiency at $N_C = 1$, $N_G = 0$ and $\psi = 0.8$

4.3 PERFORMANCE OF FIN AT SURFACE OF EARTH

At the surface of earth, the analysis is concerned with the heat transfer performance of a fin controlled by the interaction of conduction, convection and radiation processes.

4.3.1 Temperature Gradient

To illustrate the effect of varying convection parameter, N_C, parameters $N_R = 1$ and $N_G = 0.4$ is selected and shown at Figure 4.3. Several values for dimensionless temperature ratio term, ψ, have been tested and it was concluded that $\psi = 0.5$ produce reasonably good result to compute the convection comparison. Note that at $N_C = 0$, the present of convection is negligible and the heat is discharge only by radiation. This is what actually happens at outer space. As expected, the increasing of convection parameter will increase the heat removing to ambient. For reference, $\theta = 1$ indicate that fin temperature is equal to base temperature (normally occur at fin base) while $\theta = 0$ shows that fin temperature nearly same as ambient temperature.

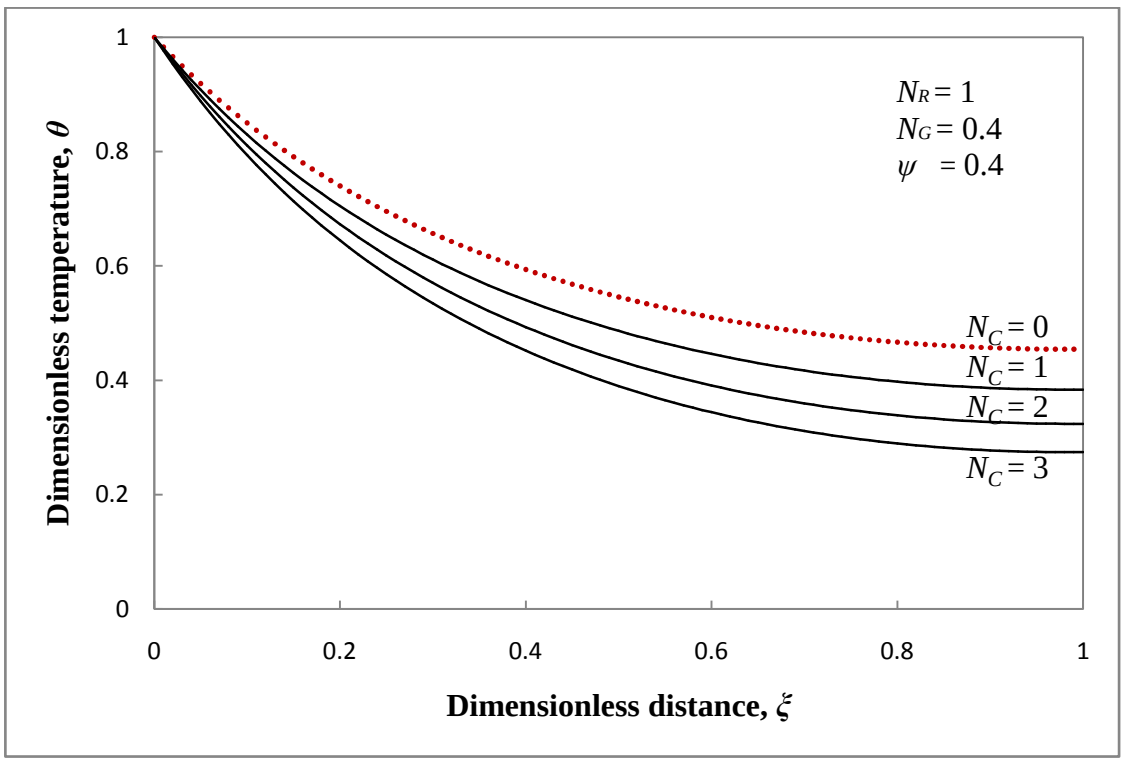

Figure 4.3: Temperature distributions at $\psi = 0.5$, $N_R = 1$ and $N_G = 0.4$

4.3.2 Fin Efficiency

The results of fin efficiency in radiative-convective fins are presented. The variation of fin efficiencies for different values of radiation parameter N_R is shown in Figure 4.4 to Figure 4.7. The efficiencies continuously decrease with the increasing of N_R. From Equation 3.13, it can see that efficiency is inversely proportional to N_R. However, when $N_R \rightarrow 0$ and the irradiation parameter $N_G = 0$, the equation is reduced to the standard case which has exact solution. Later, the effect of increase of N_G is to increase thermal load on the exchanger and hence the efficiencies decreases.

Comparison between Figure 4.4 and Figure 4.5 showed that efficiency is slightly increase when convection parameter N_C increase. The dimensionless temperature ratio term ψ is less significant when N_C is increase. This comparison is shown between Figure 4.5 and Figure 4.6. From Figure 4.7, the increasing of N_C and ψ means, N_G is less significant.

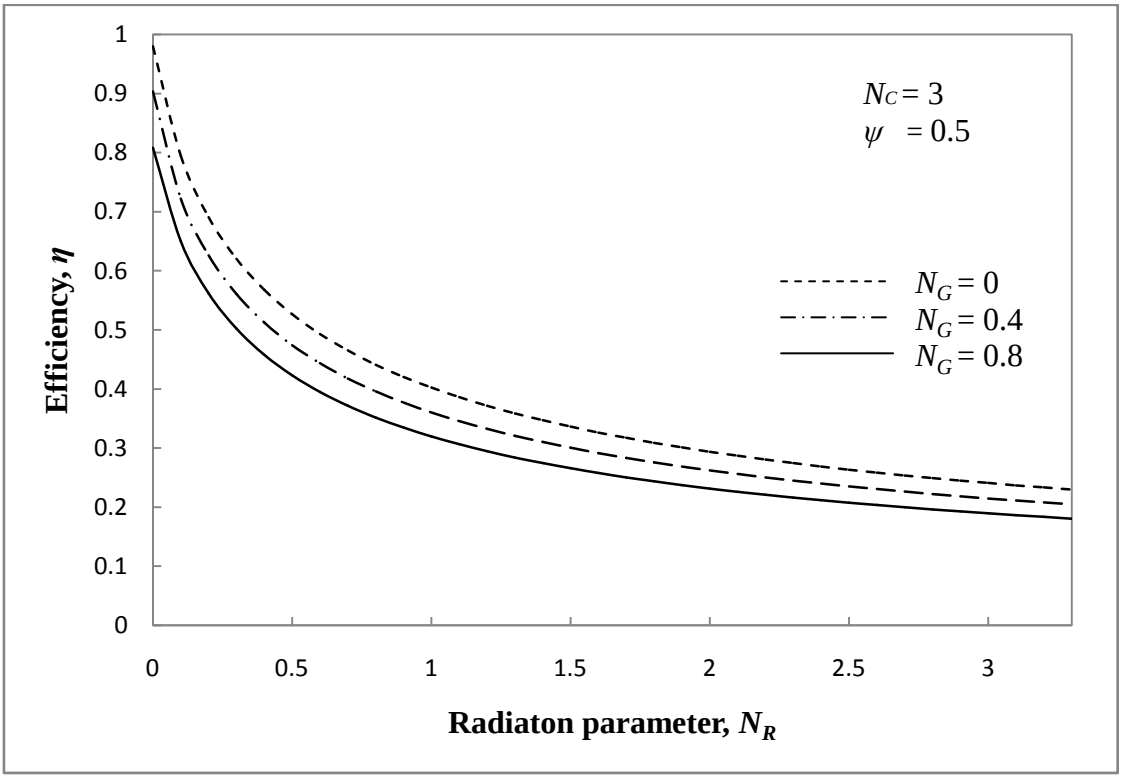

Figure 4.4: Fin efficiency at $N_C = 3$ and $\psi = 0.1$

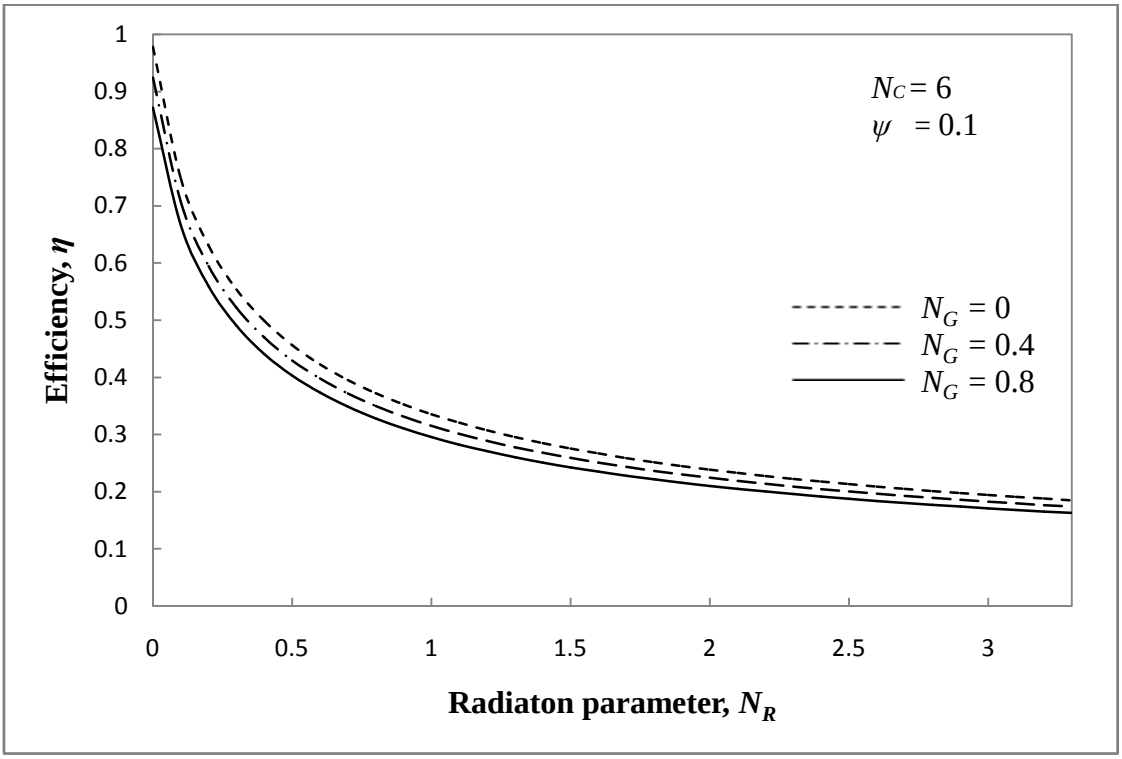

Figure 4.5: Fin efficiency at $N_C = 6$ and $\psi = 0.1$

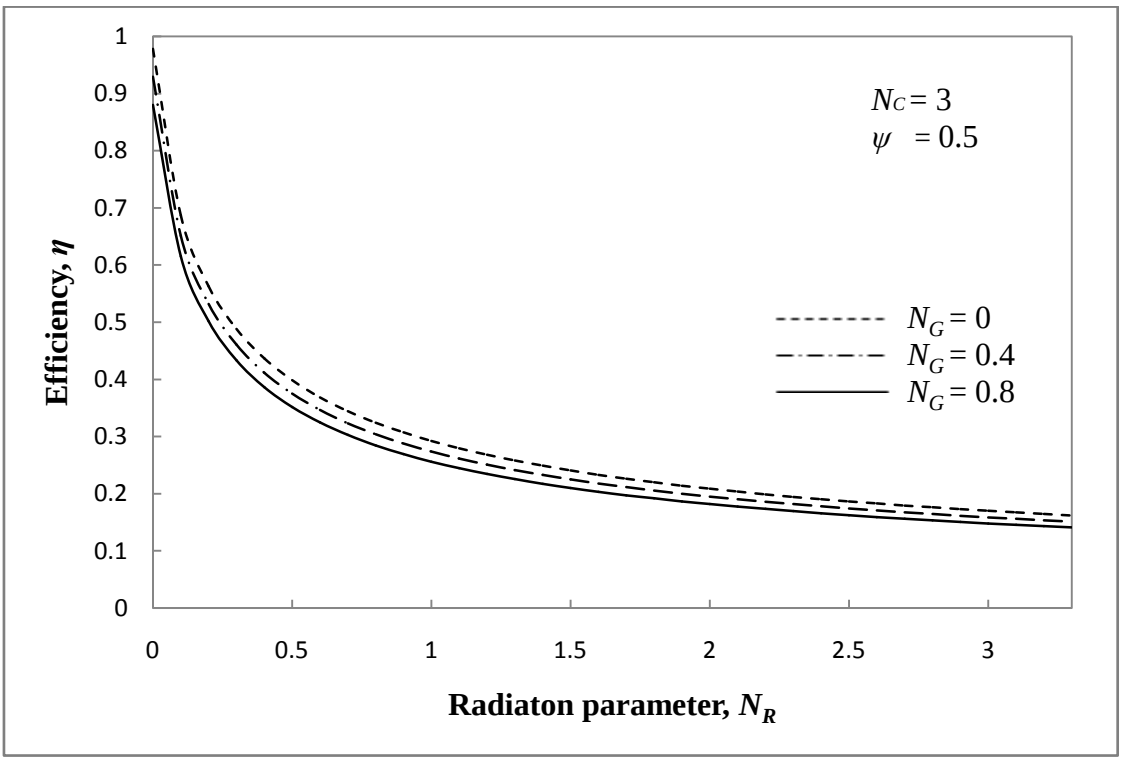

Figure 4.6: Fin efficiency at $N_C = 3$ and $\psi = 0.5$

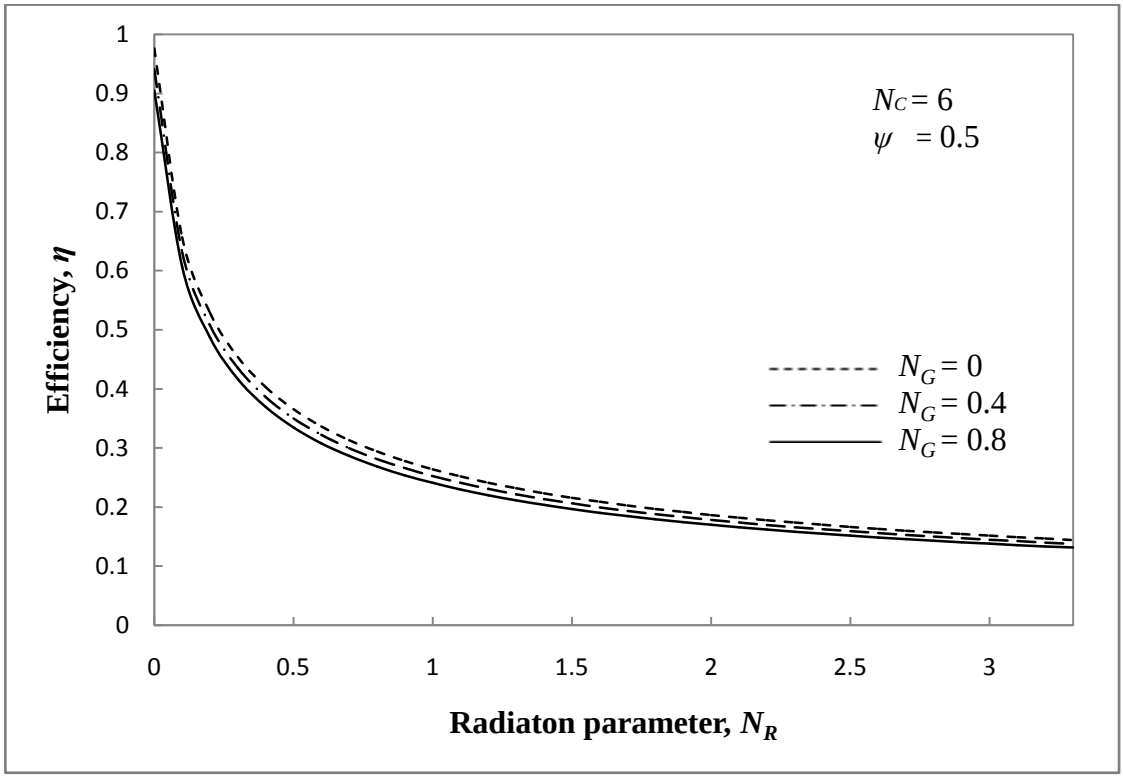

Figure 4.7: Fin efficiency at $N_C = 6$ and $\psi = 0.5$

4.3.3 Fluid Temperature

The variation of dimensionless fluid temperature for different values of longitudinal heat conduction parameter, N_F, irradiation parameter N_G, for different values of N_R when $N_C = 3$ and $\psi = 0.1$ is shown in Figure 4.8. The dimensionless fluid temperature θ_b continuously increases with increase of N_R. It is expected because the higher fluid temperature need more heat dissipated amount to ambient by radiation. In addition, irradiation parameter N_G will increase thermal load to fin and hence reduce the total heat dissipated from fluid to ambient. The fin is fully efficient when outer fluid temperature, T_{fo} is equal to ambient temperature, T_∞. Figure 4.8 also show that the effect of irradiation parameter N_G at lower value of convection parameter N_C significantly reduce the total heat dissipated.

It has also been demonstrated that the longitudinal heat conduction parameter, (fin to fluid properties parameter) N_F, will increase the amount of heat dissipated to ambient and fin efficiency itself. This is more prominence at higher convection

parameter N_C as shown in Figure 4.9. Comparison between Figure 4.8 and Figure 4.9 shows the effect of convection parameter N_C to dissipate heat. At N_C = 3, the fluid temperature is increase slowly compared to N_C = 6. The increasing of dimensionless temperature ratio term ψ will slightly increase the efficiency of the fin as compared between Figure 4.8 and Figure 4.10. From Figure 4.11, the increasing of N_C and ψ means, N_G is less significant.

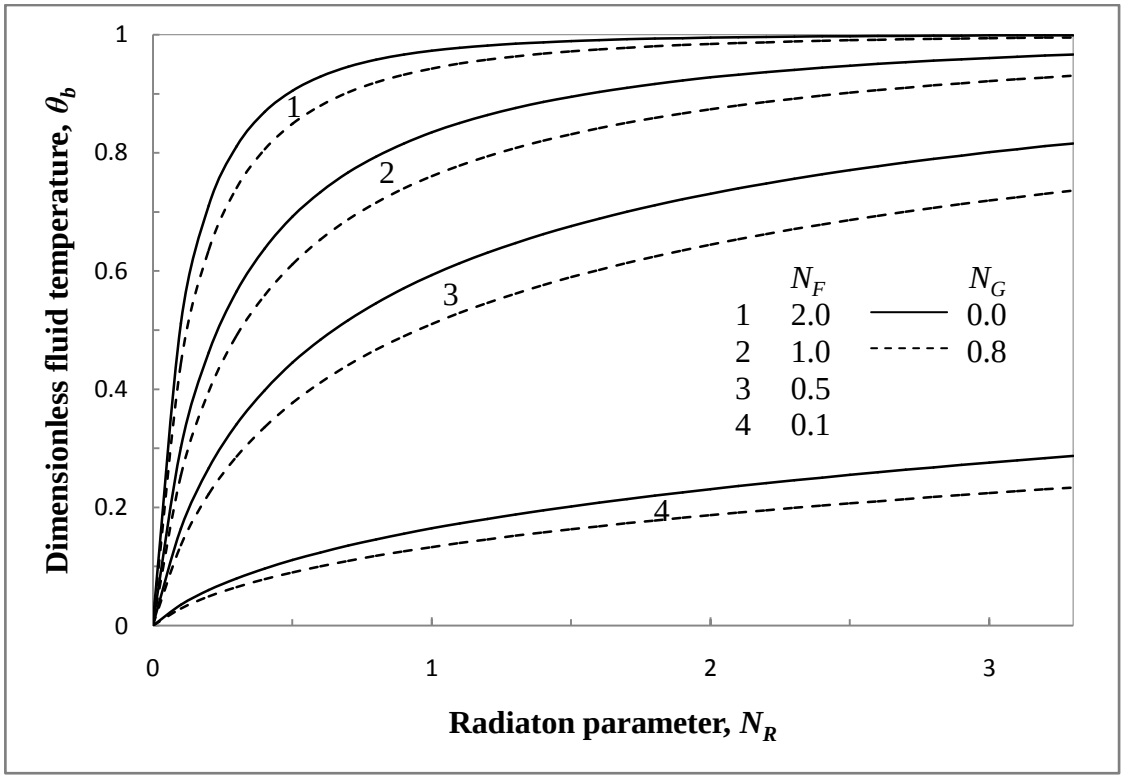

Figure 4.8: Fluid temperature at N_C = 3 and ψ = 0.1

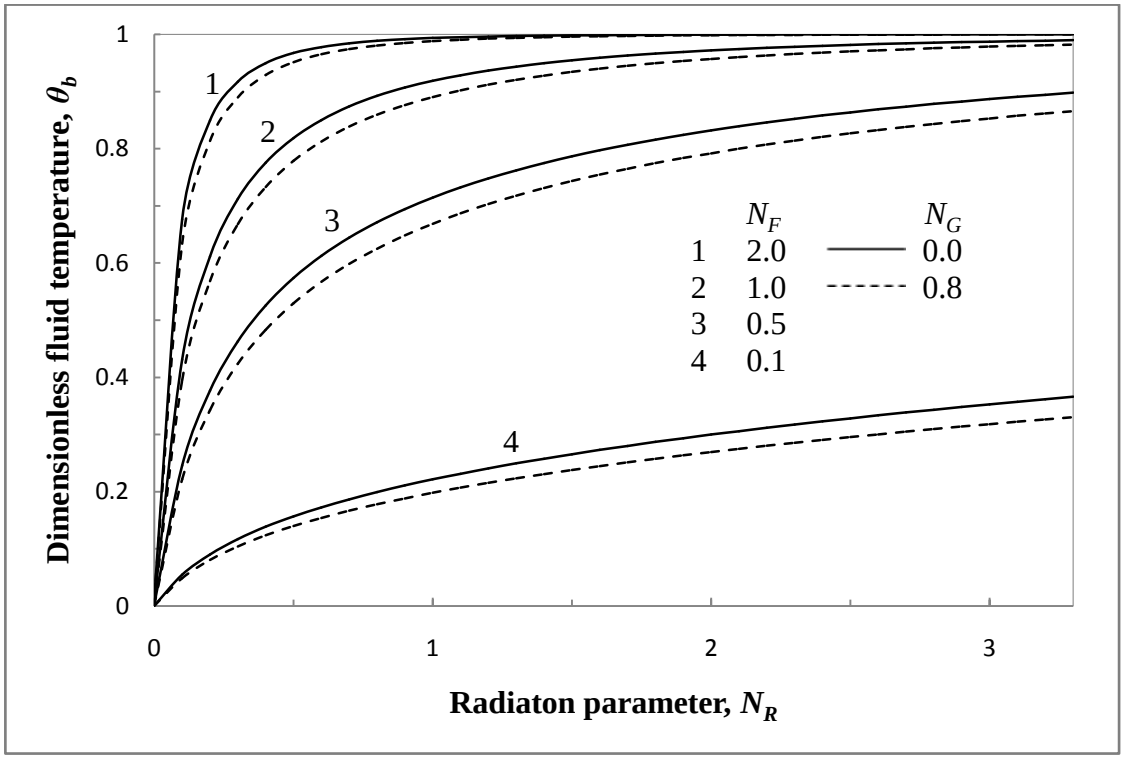

Figure 4.9: Fluid temperature at $N_C = 6$ and $\psi = 0.1$

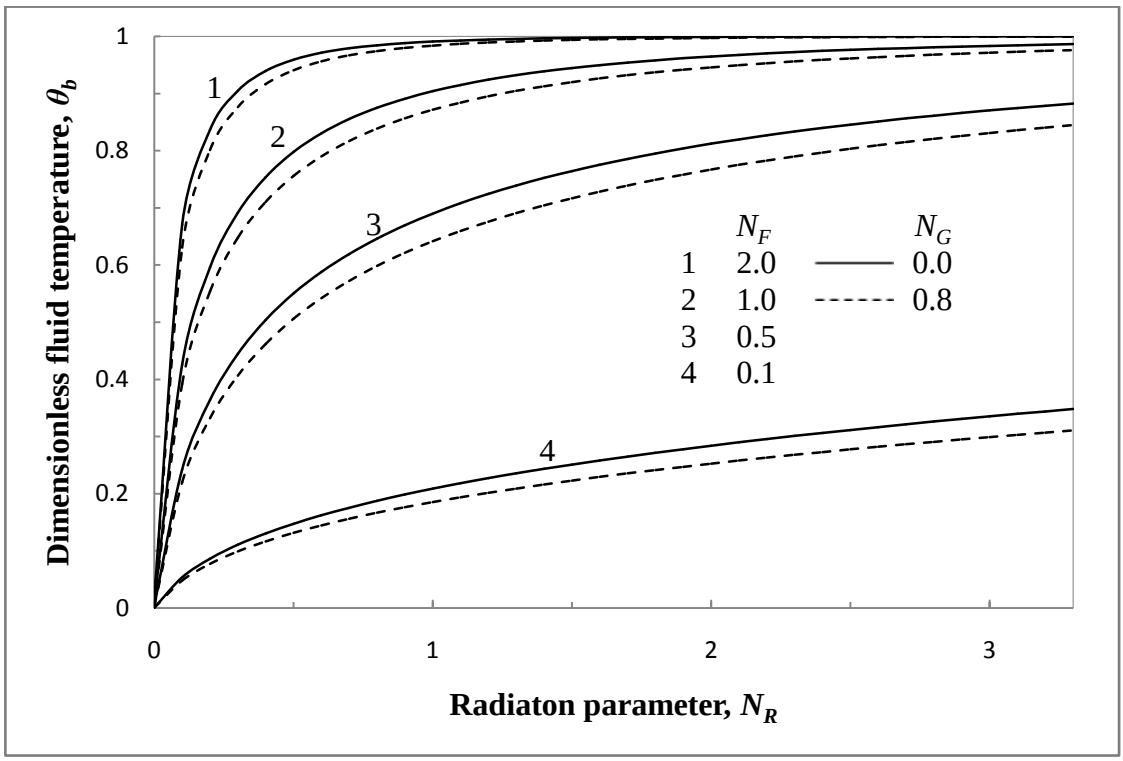

Figure 4.10: Fluid temperature at $N_C = 3$ and $\psi = 0.5$

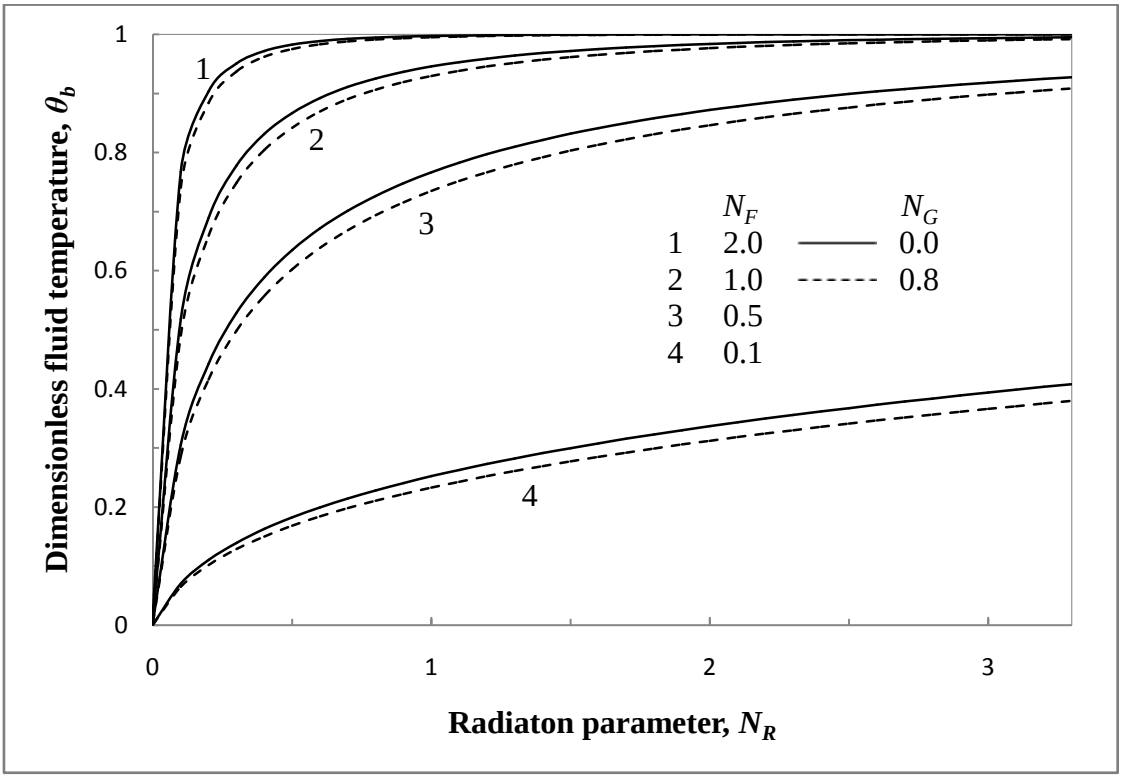

Figure 4.11: Fluid temperature at $N_C = 6$ and $\psi = 0.5$

4.4 PERFORMANCE OF FIN AT OUTER SPACE

The need to dissipate energy from vehicles operating in atmosphere-free space has stimulated considerable interest in the characteristics of finned surfaces that transfer heat by thermal radiation. The initial analyses of radiating fins were concerned with a single isolated fin that radiates freely to space and which in turn may be irradiated from space. Later, temperature gradient, fin efficiency and fluid temperature are discussed within outer space condition where the convection not presents.

4.4.1 Temperature Gradient

Figure 4.12 depicts the variation of temperature along the fin for $N_C = 0$, $N_G = 0.4$ and $\psi = 0.5$. As expected, the temperature is a maximum at the base and decays to nearly the ambient temperature at the tip. Also, the increasing of N_R will increase the heat removing to ambient.

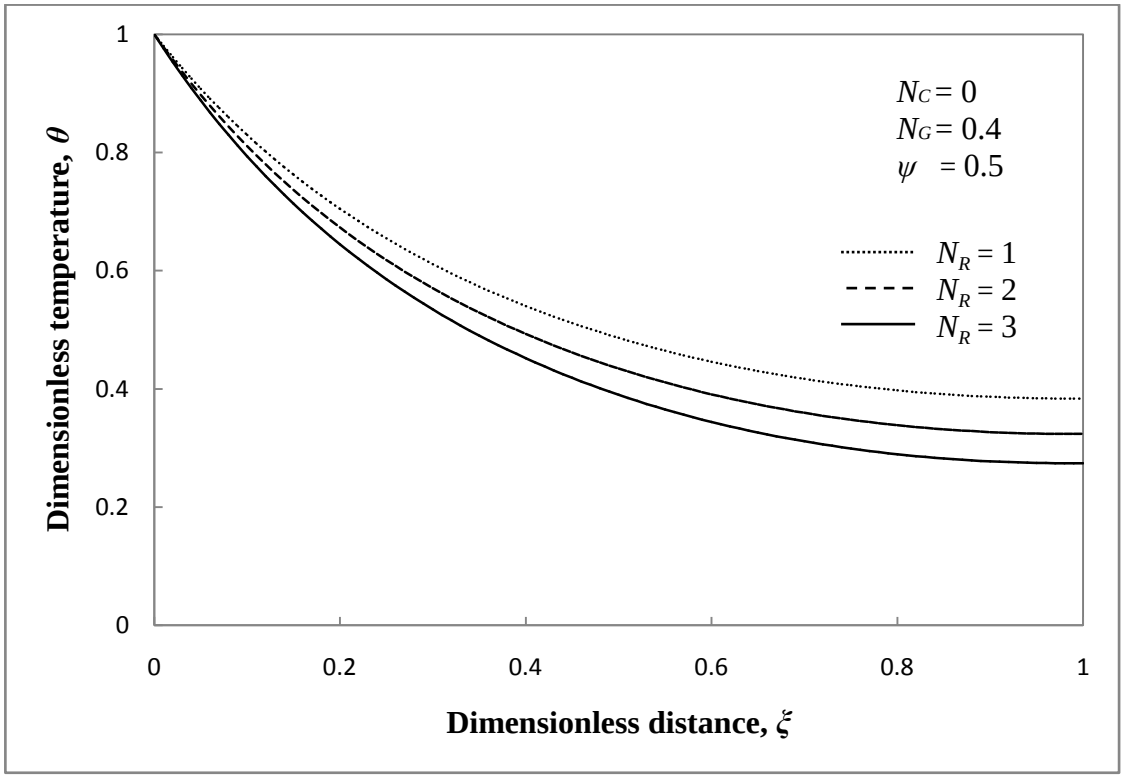

Figure 4.12: Temperature gradient at $N_C = 0$, $N_G = 0.4$ and $\psi = 0.5$

4.4.2 Fin Efficiency

To demonstrate the efficiency of fin with the absence of convection, four graphs are analyze and presented. Figure 4.13 shows the fin efficiency profile development with N_R when $N_C = 0$ and $\psi = 0.1$. At $N_C = 0$, the total heat dissipation to ambient is totally by radiation. Such as condition at surface of earth, the efficiency is decrease with the increasing of N_R. The effect of N_C is to dissipate more heat and hence the efficiencies is more when compared with $N_C = 0$. This is more pronounced at lower values of N_R and larger values of N_G which is evident from a comparison of Figure 4.5 and Figure 4.13.

Note that, $\psi \rightarrow 0$ indicate that the ambient temperature, T_∞ is extremely small compared to base temperature, T_b ($T_\infty << T_b$).. A comparison of Figure 4.4 and 4.6 shows the effect of increase in convection parameter N_C.

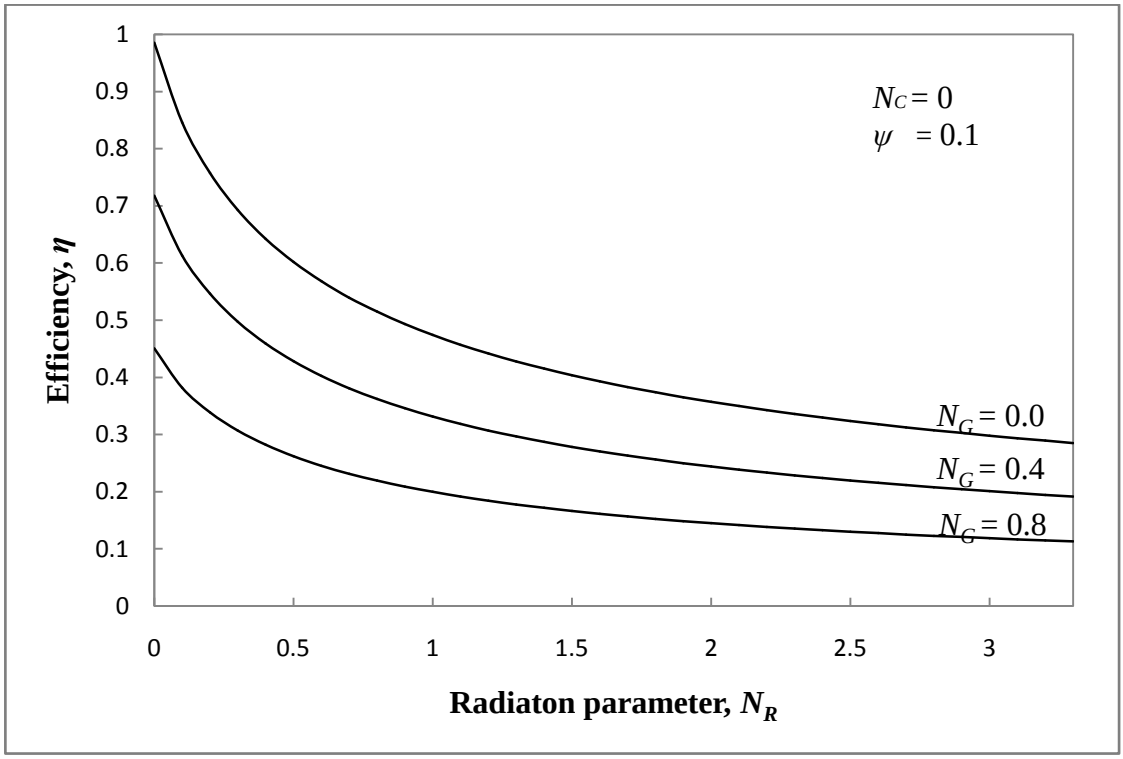

Figure 4.13: Fin efficiency at $N_C = 0$ and $\psi = 0.1$

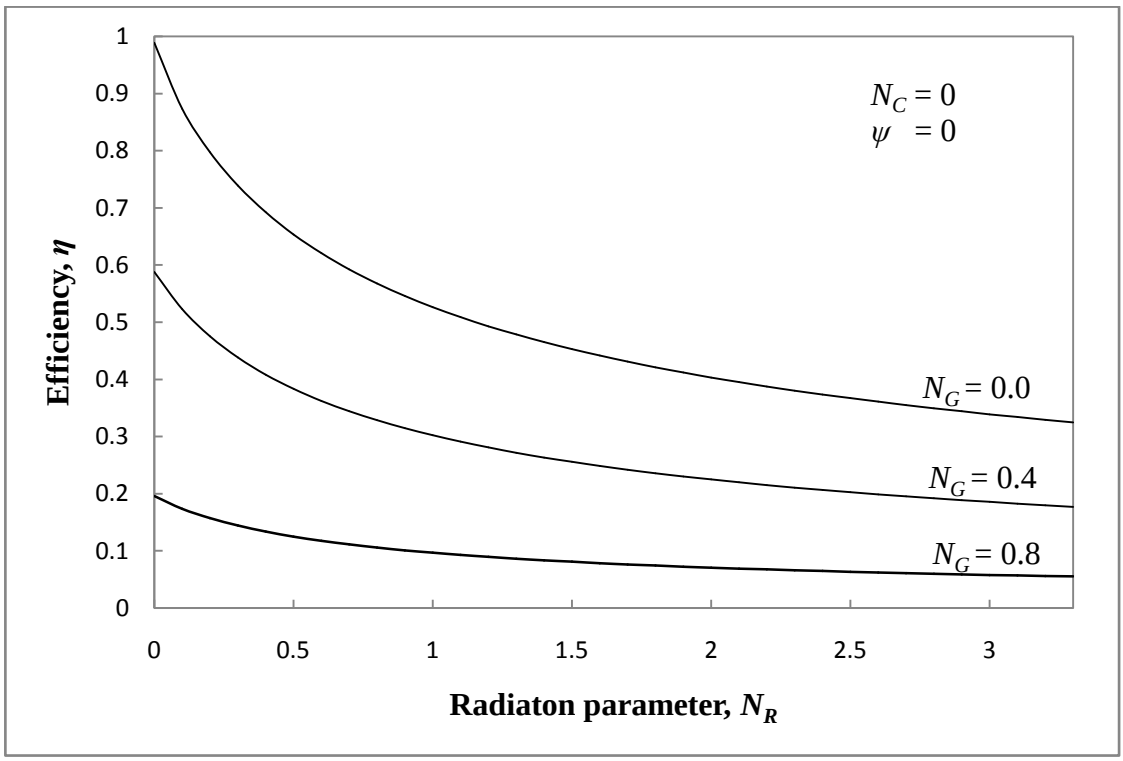

Figure 4.14: Fin efficiency at $N_C = 0$ and $\psi = 0$

4.4.3 Fluid Temperature

From among the totality of θ values determined from these solutions, the θ_b values are perhaps of greatest interest because they correspond most closely to the distribution of the fluid temperature. In particular, θ_b corresponds to the ratio of the fluid temperature at the outlet to the fluid temperature at the inlet. Therefore, T_{fo} is an indication of the heat loss from the fin. The profile of θ_b from the solutions of equation (3.18) are shown in Figure 4.15 at $N_C = 0$ and $\psi = 0.1$. To illustrate the fluid temperature changes with various N_R, it is seen from Figure 4.15 that the increasing of N_R will increase the total heat transfer to ambient. It is expected because the higher fluid temperature need more heat dissipated amount to ambient by radiation. In addition, irradiation parameter N_G will increase thermal load to fin and hence reduce the total heat dissipated from fluid to ambient. In the absent of convection, irradiation become more significant as shown at Figure 4.9 and Figure 4.15.

It has also been demonstrated that N_F will increase the amount of heat dissipated to ambient. The increasing of ψ will increase the efficiency of the fin as compared between Figure 4.15 and Figure 4.16. Note that at higher value of ψ, the fluid temperature is nearly to ambient temperature. It means, at $\psi \rightarrow \infty$, the total amount of heat transferred to ambient is at minimum level. Even if forced convection is employed for cooling, radiation can be significant if the operating temperatures are high ($\psi \rightarrow 0$) as is the case with a finned regenerator (Ngunyen and Aziz 1990).

It can be seen that the inclusion of convection is quite significant and helps to predict the efficiency and total heat transfer of the radiator under actual working condition.

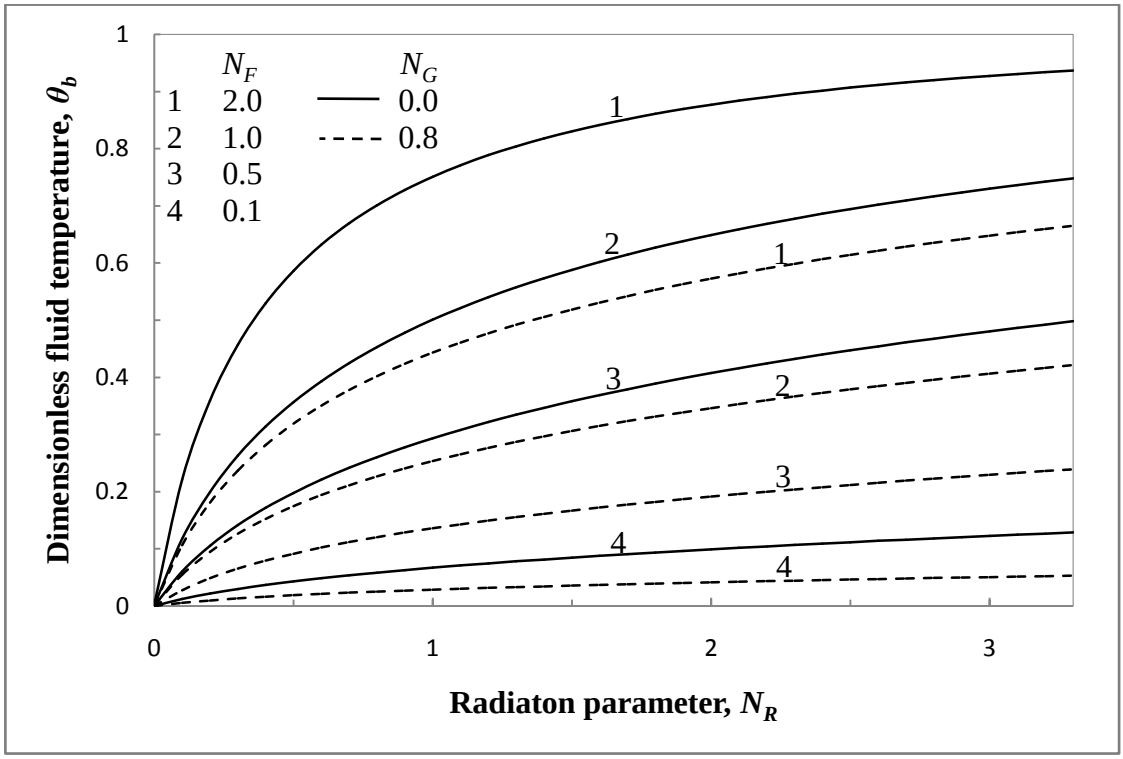

Figure 4.15: Fluid temperature at $N_C = 0$ and $\psi = 0.1$

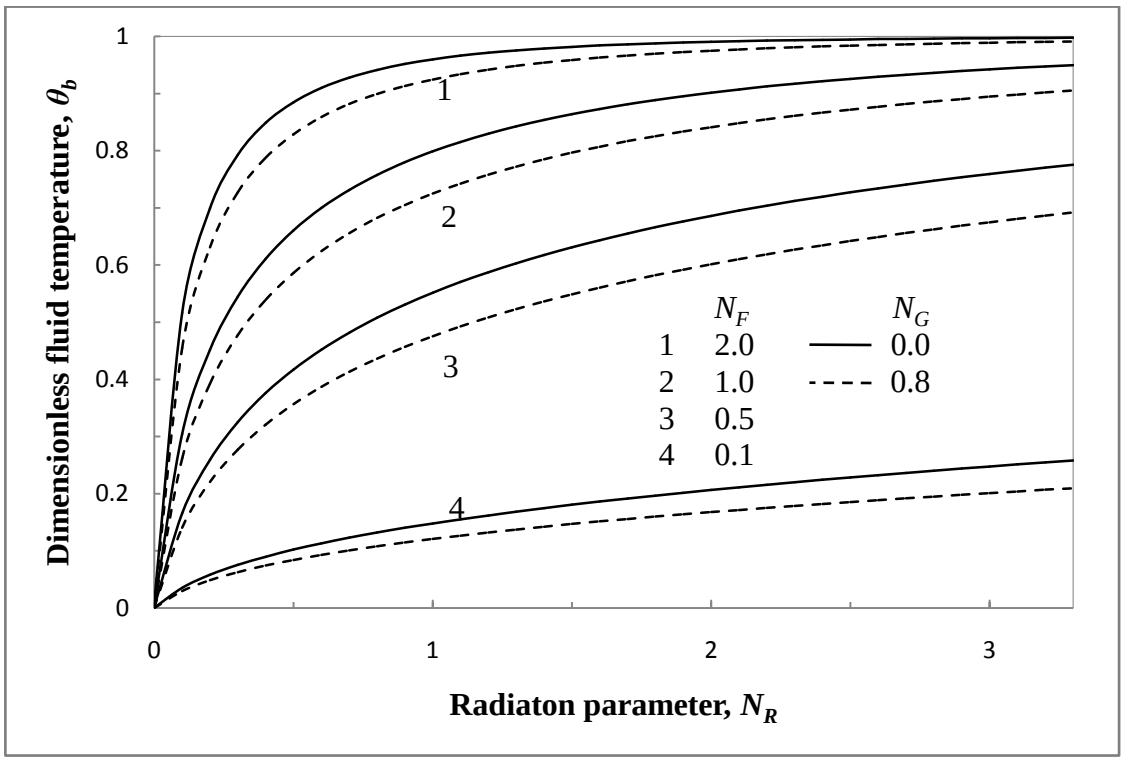

Figure 4.16: Fluid temperature at $N_C = 0$ and $\psi = 0.5$

4.5 CONCLUSION

The result for temperature gradient, fin efficiency and fluid temperature is obtained to know the correlation of convection parameter N_C, radiation parameter N_R, irradiation parameter N_G and longitudinal heat conduction parameter N_F. The result is shown in graph at various parameters.

CHAPTER 5

CONCLUSION

5.1 INTRODUCTION

This final chapter consist of overall project conclusion include introduction, literature review, project methodology, result and discussion. Recommendation for further research is included to give some view of present research upgrading.

5.2 RESEARCH CONCLUSION

Heat transfer from extended surfaces subject to heat transfer by conduction, convection, radiation and radiation has been studied. The temperature distribution in a steady state fin, exchanging heat with the surroundings by convection, radiation and irradiation, can be approximately predicted by developing an algorithm where the domain is discretized by Taylor series central difference scheme and finally difference equations have been solved by Jacobi method. Like any other investigation, the analysis has its limiting assumptions. For example, the fin material is homogeneous, its thermal conductivity is the same in all directions, and it remains constant. Similarly, heat conduction in the fin is assumed to be one-dimensional. In the later stage, efficiency and fluid temperature then computed based on the temperature distribution. The result show that the total heat loss to ambient strongly effected by convection and radiation also ambient temperature. The present of irradiation is to increase thermal load on the exchanger and hence the efficiencies decreases. The increasing of irradiation parameter is more significant at lower value of convection and radiation parameter. It has also been demonstrated that the increase of longitudinal heat conduction parameter, N_F, will increase the total heat dissipated to ambient. The effect of convection is to dissipate

more heat and hence the efficiencies is more when compared with $N_C = 0$. This is more pronounced at lower values of N_R and larger values of N_G. For the range of parameters tested, the efficiencies have increased marginally with increase of N_C.

It can be seen that the inclusion of convection is quite significant and helps to predict the effectiveness of the radiator under actual working condition.

5.3 RECOMMENDATIONS FOR THE FUTURE RESEARCH

The progress of space environmental systems involves higher and higher level of installed power and proper heat rejection devices whose mass and dimensions are a governing feature linked to launch costs and technological constraints of space systems. Taking into account that the radiator can be up to the 40% of the overall mass, it is clear that the radiator is the component to focus optimization efforts on. In the present study, the prediction of temperature gradient will be more accurate with two or three dimension conduction analysis. This is more significant at the higher size or volume of fin itself. Although the value of fluid temperature is slightly higher than fin base temperature, this difference temperature should be considered to maximize the prediction of fin performance. Varies of convection heat transfer coefficient, h, should be analyzed rather than constant value of h. It is because h much dependent on temperature different along the fin. At the condition where fin thickness is not too small compared to their length and height, fin thickness has to be considerable. Next, the boundary condition at the tip fin should dissipate heat whether by convection or radiation or both of them. Evidently, the focus of the past studies has been fins of uniform thickness. It is known from studies of pure convective fins that profiles shapes such as trapezoidal, triangular, and concave parabolic can offer a considerable saving in material.

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APPENDICES A1

Project Planning (Gantt chart): Final Year Project 1

Work Progress	Week														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Title of the project, problem statement and project objective is given and overview about project is obtained.															
Study the related chapter (heat transfer, finite difference method and FORTRAN Software).															
Obtain information about previous research (method used and result).															
Governing equation for radiating fin to predict temperature gradient															
Complete the program coding to compute temperature gradient using finite difference method.															
Execute and obtain data from FORTRAN Software and present in graph.															
Report Writing (Chapter 1, 2, 3) (Introduction, Literature review, Methodology)															
Submit draft thesis and slide presentation															
Final year project 1 presentation															

Planning Progress Actual Progress

APPENDICES A2

Project Planning (Gantt chart): Final Year Project 2

Work Progress	Week															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Study the related chapter (heat transfer and dimensionless parameter)																
Governing equations to evaluate efficiency and fluid temperature.																
Complete the program coding to compute efficiency and fluid temperature.																
Execute and obtain data from FORTRAN Software and present in graph.																
Make a comparison with previous result to validate project data.																
Make the correction, discussion, abstract and conclusion base on the project result.																
Report Writing (Chapter 4 and 5), revise and verify all chapter with supervisor.																
Final year project 2 presentation.																
Submit thesis report.																

Planning Progress Actual Progress

APPENDIX B1

Table 6.1: Temperature distributions at ψ = 0.2323, N_C = 16.78, N_R = 0.1738 and N_G = 0

Dimensionless distance, ξ	Dimensionless temperature, θ		
	Present solution	**Experimental data by Cobble**	**Numerical solution by Cobble**
0.00	1.0000	1.0000	1.0000
0.01	0.9836		
0.02	0.9675		
0.03	0.9517		
0.04	0.9363		
0.05	0.9211		
0.06	0.9062		
0.07	0.8917		
0.08	0.8774		
0.09	0.8634		
0.10	0.8497		
0.11	0.8362		
0.12	0.8231		0.8238
0.13	0.8101		
0.14	0.7975		
0.15	0.7851		
0.16	0.7729		
0.17	0.7610		
0.18	0.7493		
0.19	0.7379		
0.20	0.7266		
0.21	0.7156		
0.22	0.7049		
0.23	0.6943		
0.24	0.6840		
0.25	0.6739	0.6659	0.6622
0.26	0.6640		
0.27	0.6543		
0.28	0.6448		
0.29	0.6355		
0.30	0.6263		
0.31	0.6174		
0.32	0.6087		

0.33	0.6001		
0.34	0.5918		0.5704
0.35	0.5836		
0.36	0.5756		
0.37	0.5678		
0.38	0.5601		
0.39	0.5526		
0.40	0.5453		
0.41	0.5382		
0.42	0.5312		
0.43	0.5243		
0.44	0.5177		
0.45	0.5112		
0.46	0.5048		
0.47	0.4986		
0.48	0.4925		
0.49	0.4866		
0.50	0.4809	0.4712	0.4789
0.51	0.4752		
0.52	0.4698		
0.53	0.4644		
0.54	0.4592		
0.55	0.4542		
0.56	0.4493		
0.57	0.4445		
0.58	0.4398		
0.59	0.4353		
0.60	0.4309		
0.61	0.4267		
0.62	0.4225		
0.63	0.4185		0.4309
0.64	0.4146		
0.65	0.4109		
0.66	0.4073		
0.67	0.4037		
0.68	0.4003		
0.69	0.3971		
0.70	0.3939		
0.71	0.3909		
0.72	0.3880		
0.73	0.3852		
0.74	0.3825		
0.75	0.3799	0.3868	0.3941
0.76	0.3774		
0.77	0.3751		

0.78	0.3728		
0.79	0.3707		
0.80	0.3687		
0.81	0.3668		
0.82	0.3650		
0.83	0.3633		
0.84	0.3617		
0.85	0.3602		
0.86	0.3589		
0.87	0.3576		
0.88	0.3565		
0.89	0.3554		0.3721
0.90	0.3545		
0.91	0.3536		
0.92	0.3529		
0.93	0.3523		
0.94	0.3518		
0.95	0.3513		
0.96	0.3510		
0.97	0.3508		
0.98	0.3507		
0.99	0.3507		
1.00	0.3507	0.3611	0.3648

APPENDIX B2

Table 6.2: Fin efficiency at $N_C = 1$, $N_G = 0$ and $\psi = 0.8$

Radiation Parameter, N_R	**Fin Efficiency, η**	
	Present solution	**Ngunyen and Aziz (1990)**
0.0	0.9778	0.6968
0.1	0.6153	
0.2	0.4873	0.4679
0.3	0.4154	
0.4	0.3677	0.3631
0.5	0.3331	
0.6	0.3066	0.303
0.7	0.2853	
0.8	0.2679	0.2638
0.9	0.2532	
1.0	0.2406	0.2365
1.1	0.2297	
1.2	0.2201	
1.3	0.2115	
1.4	0.2039	
1.5	0.197	
1.6	0.1907	
1.7	0.185	
1.8	0.1798	
1.9	0.1749	
2.0	0.1705	
2.1	0.1663	
2.2	0.1624	
2.3	0.1588	
2.4	0.1554	
2.5	0.1522	
2.6	0.1491	
2.7	0.1463	
2.8	0.1436	
2.9	0.141	
3.0	0.1386	
3.1	0.1363	
3.2	0.1341	
3.3	0.1319	