**Chapter 9**
# Case on Quality Engineering: Acceptance Sampling

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## 1 Introduction

Acceptance sampling is an inspection method that are widely use in vast area of industrial application and implementations to inspect a large numbers of items in a short amount of time without decreasing the quality of the items or inspection precision. It is ideal to inspect all the product manufactured in factory line to ensure the quality of the product without nonconformities (unacceptable product quality) but it is almost impossible to inspect every product without consuming very long period of time, labor costs and possibly causing human error because of fatigue and boredom that will reversely make the nonconform product miss the inspection. The method is very useful in the inspection of for example, food [2], computation [3], geo-science [4], production [5] etc.

## 2 Acceptance Sampling

Definition: It is an **inspection method on a large number of production items that are grouped in manufacturing lots/batches** to manage the quality of a product to an established standards agreed by both the manufacturer and consumer.

<u>Acceptance Sampling is very useful when:</u>

- Regular method doesn't work/ not available.
- Large numbers of item must be process in a short amount of time.
- Human error cause by fatigue/boredom due to similar units inspection.
- The cost of "passing defectives" is low
- Cost of the 100% inspection is higher than the cost of passing a non-conforming unit.
- Automated inspection is not available.

<u>Three important steps of sampling</u>:

- Select a number of random sampling items from the entire Lot
- Accept and Rejects Lots (does not improve the quality) “Lot Sentencing”
- Audit Tool

Three aproaches to “Lot Sentencing”:

- Accept without inspection
- 100% inspection
- Acceptance Sampling

**Advantages:**

- Economical
- Improves inspectors job
- Applies to destructive testing
- When entire lots were not accepted, will give motivations for improvement

**Disadvantages**

- Risk of wrong decision
- Requires planning and documentation
- Less information but usually enough
- No assurance the entire lot conforms or does not conform to specs.

## 3 Sampling Plans

***Sampling Plans*** will define and determined the appropriate lot size, sample size, the number of samples and acceptance/rejection criteria in detail.

**4 Types** of Sampling Plans

1. Single sampling
2. Double sampling
3. Multiple sampling
4. Sequential

***N* :** Lot size

***n* :** sample size

***c* :** acceptance number

If c or less non-conforming units are found in the sample, the lot is accepted, else it is rejected

**Single Sampling Plans**

- One sample of ***n*** size is taken from one lot
- Each was examined and classified
- If the number of defectives exceeds the predetermined acceptance number (***c***), the specific lot inspected will be rejected, otherwise it is accepted.

**Double Sampling Plan**

- This sampling plan has a range of tolerance before completely rejected due to exceeded defective numbers.
- In addition to the initial sample, double sampling plan requires rejection numbers: ( $N$, $n_1$, $c_1(Ac)$, $r_1$ $(Re)$).
- It also requires to predetermine the secondary set of sample size, acceptance number and rejection number based on the total defectives observed in both 1st and 2nd sample: $(n_2,c_2,r_2)$
- Decision making:
  - If the quality is very good, $\leq c_1$, accept lot
  - If the quality is very bad, $\geq c_1$, reject lot
  - If between $c_1$ and $r_1$, take a second sample
  - Second sample is accepted if the total nonconformities are $\leq c_2$ or rejected if the total nonconformities are $\geq r_2$

**Multiple Sampling Plans**

- Similar to the double sampling in that successive trials are made, each of which has a set of acceptance, rejection, and inconclusive options.

**Sequential Sampling**

- Items are sampled and inspected one after another and a decision is made after each item is inspected or when there were enough information to conclude the final decision whether the lots are accepted or rejected.

**Conclusions**: All 4 types will give the same results; therefore consider other factors.

**How to Decide?**

- ***Simplicity-*** Single would be best and sequential the poorest.
- ***Administrative costs*** –Least under single and greatest under sequential.
- ***Units inspected-*** Greatest under single and least under sequential.
- ***Information-*** Best under single and poorest under sequential.
- ***Psychological Impact-*** Best under double

**How to form a Lot:**

Things to consider before inspection:

- Lots are form from homogeneous samples (example: the product manufactured using the same machine).
- It is preferable to do sampling on a larger size of lots.
- Lots formed should be in accordance to predetermine material-handling systems used by vendor and consumer facilities.

**Random Sampling**

- Selection of items for inspection should be chosen at random
- If random samples are not used, ***bias*** can be introduced
- If judgment methods are used to select the sample, the statistical basis of the acceptance-sampling procedure is lost
- Location and random number table
- Stratification

**Non-Accepted Lots**

- Sorted at the next process
- Rectified prior to next process

Returned to producers for rectification is the best solution and usually leads to improved quality

## 4 The Operation Characteristic (OC) curve

**Operation Characteristic (OC) curve** is an excellent tools to evaluate lots quality.

- Measures the performance of a sampling plan
- Plots the percentage of probability for users to make a decision either to accept the lot or reject it.

- Shows the probability that a lot submitted at a certain level of defectives will be either accepted or rejected

This curve plots the percentage of lots accepted (Y-axis) versus the percentage of defectives (X-axis). **The OC curve is the main tool for exhibiting Lot Acceptance Sampling Plan in illustration to investigate its properties.** Example for Single Sampling Plan is as shown in Figure 1.

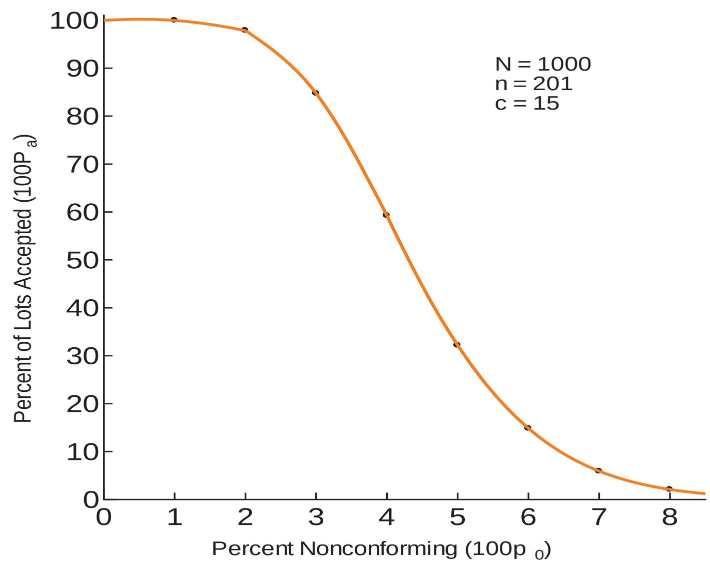


**Figure 1** OC Curve for the Single Sampling Plan *N = 1000, n = 201, c = 15*

To calculate the OC curve, the Poisson Probability Distribution is a good approximation method for almost all sampling plans, and the formula used to calculate Poisson distribution is as follow. The value used for the OC curve is the cumulative values according the count of nonconformities.

$$P(c) = \frac{(nP_0)^c}{c!} e^{-nP_0} \qquad (1)$$

Where;

c = count of nonconformities

$nP_0$ = average count

$e$ = 2.718281

Here, larger c number will resulted to more ideal OC curves where the acceptable lot percentage is drawn nearer to 100 % acceptance. However, larger c means that the quality of acceptance sampling become low.

# 4 Example

## 4.1 Single Sampling Plan

Example 1. A restaurant want evaluates the sales and purchase agreement with its vendor using the single sampling plan N = 1500, n = 110, and c =3. Construct the OC curve using about 8 points.

Solution: In order to plot OC curves, there are several steps needed.

**Step 1**: Assume $P_0$ value,

**Step 2**: Calculate $nP_0$ value,

**Step 3**: Calculate $Pa$ value using equation (1). Ex; $Pa = P_0 + P_1 + P_2 + P_3 = P_2$ *or less*

Here, $P_0$ is representing the value when $c = 0$, $P_1$ is the value when $c = 1$, $P_2$ when $c = 2$ and so on.

**Step 4**: Plot point using $100P_a$ and $100P_0$

**Table 1** Probabilities of Acceptance for the Single Sampling Plan: $n = 110, c = 3$

| $P_0$ | $nP_0$ | $P_a$ | $100P_0$ | $100P_a$ |
|---|---|---|---|---|
| | | | 0 | 100 |
| 0.01 | 1.1 | 0.9742 | 1 | 97.42 |
| 0.02 | 2.2 | 0.8193 | 2 | 81.93 |
| 0.03 | 3.3 | 0.5803 | 3 | 58.03 |
| 0.04 | 4.4 | 0.3594 | 4 | 35.94 |
| 0.05 | 5.5 | 0.2017 | 5 | 20.16 |
| 0.06 | 6.6 | 0.1051 | 6 | 10.51 |
| 0.07 | 7.7 | 0.0518 | 7 | 5.18 |
| 0.08 | 8.8 | 0.0244 | 8 | 2.44 |

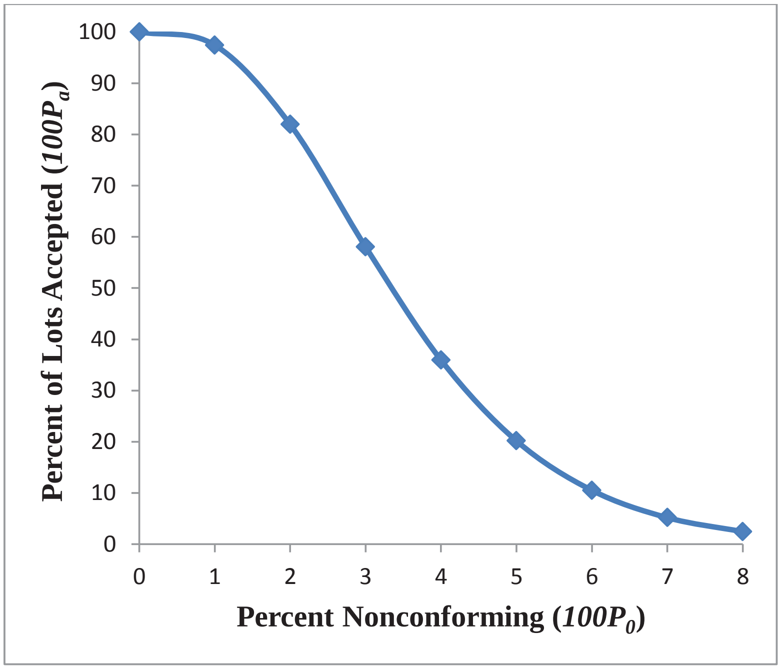


**Figure 2** OC curve for Single Sampling Plan

*N = 1500, n = 110, c = 3*

<u>Example 2.</u> You are the manager in charge of your plant shipping and storage, you need to determine the average outgoing quality where the known incoming lots from your assembly line have an average defective rate of 3%. Your plan is to sample 80 units of every 1000 in a lot. The number of defects in the sample is not to exceed 3. Draw the OC curve for the plan you developed.

<u>Solution:</u> *N = 1000, n= 80, c = 3*. Follow the same steps as Example 1 and plot the OC curves for Single Sampling Plan.

**Table 2** Probabilities of Acceptance for the Single Sampling Plan: $n = 80$, $c = 3$

| $P_0$ | $nP_0$ | $P_a$ | $100P_0$ | $100P_a$ | $P_a$ (assume $c = 2$) |
|---|---|---|---|---|---|
| 0.01 | 0.8 | 0.9909 | 1 | 99.09 | 0.9439 |
| 0.02 | 1.6 | 0.9211 | 2 | 92.11 | 0.7216 |
| 0.03 | 2.4 | 0.7787 | 3 | 77.87 | 0.4436 |
| 0.04 | 3.2 | 0.6025 | 4 | 60.25 | 0.2289 |
| 0.05 | 4.0 | 0.4334 | 5 | 43.34 | 0.1032 |
| 0.06 | 4.8 | 0.2942 | 6 | 29.42 | 0.0419 |
| 0.07 | 5.6 | 0.1906 | 7 | 19.06 | 0.0157 |
| 0.08 | 6.4 | 0.1189 | 8 | 11.89 | 0.0055 |

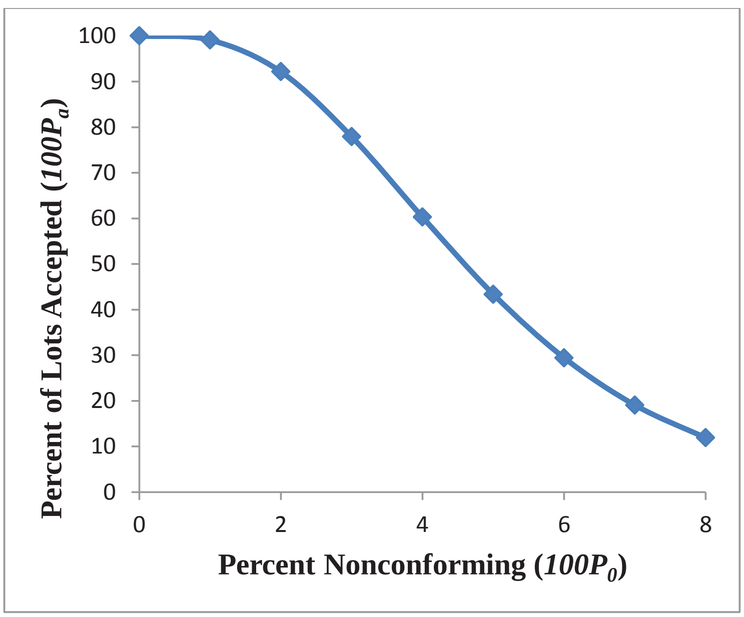


**Figure 3** OC curve for Single Sampling Plan

N = 1000, n = 80, C = 3.

### 4.2 Double Sampling Plan

Example 3. A Double Sampling Plan has a lot size of $N = 1000$, first sampling size $n_1 = 80$, first acceptance number $c_1 = 1$, first rejection number $r_1 = 4$, second sampling size $n_2 = 150$, second acceptance number $c_2 = 5$, second rejection number $r_2 = 6$.

Solution.

**Step 1**: Determine the equations to construct OC curve.

**Rule 1**: If there is one or less nonconforming number on first sampling plan, the lot is accepted. And it could be expressed as,

$$(P_a)_1 = (P_1 \text{ or less})_1 \qquad (2)$$

**Rule 2**: To construct the equation for the second sample, the number of non-conforming units in the first sample must be less than 4 and between 1 to 4 ( $1< \boldsymbol{c} < 4$ ). If the rule of the nonconforming units are as mentioned, second sample can be construct to accept the lot under the following rules.

**Rule 2.1**: Two (2) non-conforming units on the first sample ***and*** three (3) or less non-conforming units on the second sample, **or**

**Rule 2.2**: Three (3) non-conforming units on the first sample **and** two (2) or less non-conforming units on the second sample.

The combination of both Rule 2.1 and 2.2 could be express as shown below,

$$(P_a)_2 = (P_2)_1(P_3 \text{ or less})_2 + (P_3)_1(P_2 \text{ or less})_2 \qquad (3)$$

Where the "**and**" are expressed as multiply and when "**or**" are expressed, add are applied. IMPORTANT: This equation is unique to this case only. When first and second acceptance and rejection number differs, different set of equation are needed.

Note that the number of nonconforming units is equal to or less than the second acceptance number. Also, when the $r_1$ and $r_2$ are not given, by default they are equal to $c_2 + 1$

**Step 2**: To obtain the probability of acceptance for sampling plan, both equations are combined and the combination could be calculated as below,

$$(P_a)_{\text{combined}} = (P_a)_1 + (P_a)_2 \qquad (4)$$

**Step 3**: Repeat the step of Single Sampling plan above and calculate all the $P_a$ values for both first and second sample.

**Table 3** Probabilities of Acceptance for the Double Sampling Plan: $N = 1000$, $n_1 = 80$, $c_1 = 1$, $r_1 = 4$, $n_2 = 150$, $c_2 = 5$, $r_2 = 6$.

| | $P_0$ | First Sample | | Combined Sample | | |
|---|---|---|---|---|---|---|
| | | $nP_0$ | $100P_a$ | $nP_0$ | $P_a$ | $100P_a$ Combined |
| $P_0$ | 0.01 | 0.8 | 80.87 | 1.5 | 0.1653 | 97.41 |
| $P_1$ | 0.02 | 1.6 | 52.49 | 3.0 | 0.2256 | 75.05 |
| $P_2$ | 0.03 | 2.4 | 30.84 | 4.5 | 0.1257 | 43.41 |
| $P_3$ | 0.04 | 3.2 | 17.12 | 6.0 | 0.0454 | 21.65 |
| $P_4$ | 0.05 | 4.0 | 9.157 | 7.5 | 0.0126 | 10.42 |
| $P_5$ | 0.06 | 4.8 | 4.773 | 9.0 | 0.0029 | 5.069 |
| $P_6$ | 0.07 | 5.6 | 2.441 | 10.5 | 0.0006 | 2.501 |
| $P_7$ | 0.08 | 6.4 | 1.229 | 12.0 | 0.0001159 | 1.241 |
| $P_8$ | 0.09 | 7.2 | 0.6122 | 13.5 | 2.040E-05 | 0.6142 |
| $P_9$ | 0.10 | 8.0 | 0.3019 | 15.0 | 3.394E-06 | 0.3022 |
| $P_{10}$ | 0.11 | 8.8 | 0.1477 | 16.5 | 5.389E-07 | 0.1477 |

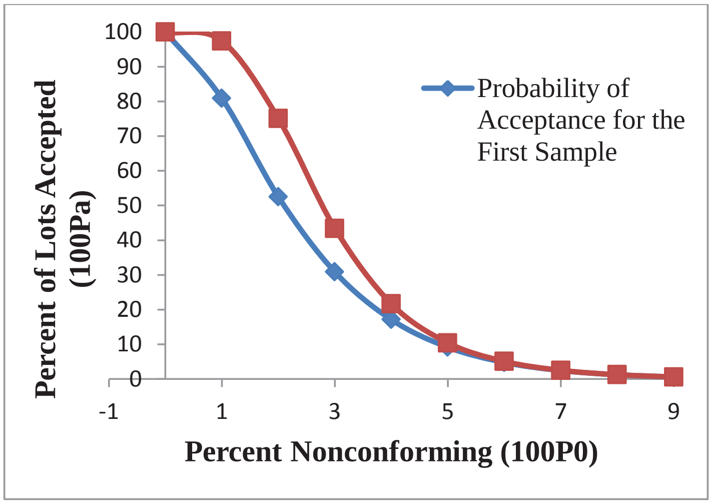


**Figure 4** OC curves for Double Sampling Plan: $N = 1000$, $n_1 = 80$, $c_1 = 1$, $r_1 = 4$, $n_2 = 150$, $c_2 = 5$, $r_2 = 6$.

### 4.3 Multiple Sampling Plans

The OC curve for multiple sampling plan is construct on same basic technique as Double Sampling Plans but more complex on the level of sampling and combined sample.

## 5 Case Study

A bakery purchases fruits from a local farm to be used in preparing the filling for their cakes. Sometimes the fruits are fresh and ripe. But, sometimes they over ripe or not ripe enough. The bakery owner has decided that they need an agreement with the farmers to a predetermined quality of fruits using sampling plan for the transaction. The company has agreed that local farm has to prepare an acceptance sampling of at least four levels of Multiple Sampling Plans. Justify the need of Multiple Sampling Plans, assume the lot size, sample size, acceptance and rejection number. State the rule of the Sampling plan and derive the equation unique to this situation. Then, construct the OC curve for the Sampling Plans.

## 6 References

1. Dale H., Besterfield, 2014, Quality Improvement Ninth Edition, Pear Education, Inc., ISBN 10: 1-292-02230-2, ISBN 13: 978-1-292-02230-7
2. Edgar Santos-Fernández, K. Govindaraju, Geoff Jones. A new variables acceptance sampling plan for food safety. Food Control, 44 (2014), pp. 249-257.
3. Ritwik Bhattacharya, Biswabrata Pradhan, Anup Dewanji.Computation of optimum reliability acceptance sampling plans in presence of hybrid censoring. Computational Statistics and Data Analysis, 83 (2015), pp. 91-100.

4. Xiaohua Tong, ZhenhuaWang et al.. Designing a two-rank acceptance sampling plan for quality inspection of geospatial data products. Computers & Geosciences, 37 (2011), pp. 1570-1583.
5. Muhammad Al-Salamah. Economic production quantity in batch manufacturing with imperfect quality, imperfect inspection, and destructive and non-destructive acceptance sampling in a two-tier market. Computers & Industrial Engineering, 93 (2016), pp. 275-285.

**Teaching Note**

**Quality Engineering: Acceptance Sampling**

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This case study is suitable for Manufacturing/Production management essentially and Service Operations Management subject generally for (Management) diploma/degree students. This case study designed for level 2 and/or 3 and/or 4 (Comprehension and/or Application and/or Analysis) of Bloom's taxonomy.

Lecturers may ask students to be in small groups and read, discuss and calculate the case for about 30 minutes. Then lecturer may ask the groups to share their findings with the class. At the end lecturer may use a few minutes to conclude the answers. Lecturer need to mention bellow solutions.

## Problem Solution

Students should have a deep understanding about the features and significant advantages and disadvantages of each Sampling Plan so that they could choose the best plan suit each unique case. Notes and examples in the case study are suffice to cover the basics. Step by Step method are mentioned but they should try solve the example by themselves. Starts first by understanding the simple Single Sampling Plan before moving to more complex Double Sampling Plan.

To simplisized things, calculate the Probability of Acceptance ($P_a$) in advance using equation (1) of Poisson Probability Distribution into table in MICROSOFT EXCEL or equivalent software so that calculation will be much easier than having to calculate and re-calculate the same $\boldsymbol{c}$, $\boldsymbol{nP_0}$ and $\boldsymbol{P_a}$ each time.

- **Training**

Student needs to do various type of case to avoid confusion and resulted in doing careless mistake in considering $P_0$, $P_1$ or $P_2$ as equal to the term  for example $P_2$ or less or $P_1$ or less when it is actually equal to the factorial number of $P_2$ or $P_1$.

If it is well understood, the calculation would be much easier.

**Case practice**

In order to solve the case study given in the lecture notes, first the student have to justify the needs of preparing the complex Multiple Sampling Plans in order for the Fresh Pie Company to purchase the apples from the local farm. This would be an open-ended problem where student are given freedom to think and justify the answer.

One of the logical answers to the justification is unlike production with consistent result as in manufacturing company, apples as nonconforming units are too unpredictable where the decaying part or not ripe enough apples sometimes are easily overlook as the operator doing the quality check have to look more into details. This will surely cause a lot of stress, fatigue and boredom and resulted to human error. Compared to that, rejected products in production line in manufacturing factory are easier to detect and predict. Prediction of where the cause of problem, where the problem will usually occurs or when it would probably occurs are much more easier. Thus in order to reduce shipping and re-shipping cost, quality management operator costs, and most important the farm prestiges that ensure repeated future purchase order, Multiple Sampling Plan are selected.

As we are handling fresh and raw food which has a the fresh peak and expiring period, Multiple Sampling Plan is the best method where lots of apples are not immediately rejected but, are going through a couple of levels of inspection which save a lot of time of re-inspection.

Student could also plot all the OC curves for Single, Double, Multiple and Sequential Sampling Plan and make a comparison.

In real practice, the selection of Sampling Plans are generally depends on the producer – consumer agreement and relationship.

To simplify the calculations and the inspector's job, sample size of all the sampling plan should be set to the same value whenever possible.

The steps are

(1) Assume $P_0$ value,
(2) Calculate $(nP_0)_1$, $(nP_0)_2$, $(nP_0)_3$, $(nP_0)_4$ values.
(3) Determine $Pa$ value using four equations and general customized Probability of Acceptance table using Poisson Distribution.
(4) Plot points
(5) Repeat steps 1 to 4 until a smooth is obtained.

Example solution;

$N = 3000$

$n_1 = 30, c_1 = 0, r_1 = 4$

$n_2 = 30, c_2 = 2, r_2 = 5$

$n_3 = 30, c_3 = 3, r_3 = 5$

$n_4 = 30, c_4 = 4, r_4 = 5$

Equations for this Multiple Sampling Plan are,

$(P_a)_1 = (P_0)_1$

$(P_a)_2 = (P_1)_1(P_{1\ or\ less})_2 + (P_2)_1(P_0)_2$

$(P_a)_3 = (P_1)_1(P_2)_2(P_0)_3 + (P_2)_1(P_1)_2(P_0)_3 + (P_3)_1(P_0)_2(P_0)_3$

$(P_a)_4 = (P_1)_1(P_2)_2(P_1)_3(P_0)_4 + (P_1)_1(P_3)_2(P_0)_3(P_0)_4 + (P_2)_1(P_1)_2(P_1)_3(P_0)_4 + (P_2)_1(P_2)_2(P_0)_3(P_0)_4 + (P_3)_1(P_0)_2(P_1)_3(P_0)_4 + (P_3)_1(P_1)_2(P_0)_3(P_0)_4$

**Table 1** Probabilities of Acceptance for the Four Level Multiple Sampling Plan

| $P_0$ | $100P_0$ | $nP_0$ | First Sample | Second Sample | Third Sample | Fourth Sample |
|---|---|---|---|---|---|---|
| | | | $100P_a$ | $100P_a$ Combined | $100P_a$ Combined | $100P_a$ Combined |
| 0.01 | 1 | 0.3 | 74.08 | 97.96 | 99.24 | 99.66 |
| 0.02 | 2 | 0.6 | 54.88 | 89.22 | 93.38 | 95.44 |
| 0.03 | 3 | 0.9 | 40.66 | 75.62 | 81.33 | 84.47 |
| 0.04 | 4 | 1.2 | 30.12 | 60.60 | 66.11 | 69.10 |
| 0.05 | 5 | 1.5 | 22.31 | 46.58 | 50.96 | 53.15 |
| 0.06 | 6 | 1.8 | 16.53 | 34.73 | 37.80 | 39.17 |
| 0.07 | 7 | 2.1 | 12.25 | 25.31 | 27.30 | 28.06 |
| 0.08 | 8 | 2.4 | 9.07 | 18.16 | 19.36 | 19.75 |
| 0.09 | 9 | 2.7 | 6.72 | 12.88 | 13.58 | 13.76 |
| 0.1 | 10 | 3 | 4.98 | 9.07 | 9.46 | 9.54 |
| 0.11 | 11 | 3.3 | 3.69 | 6.36 | 6.57 | 6.61 |

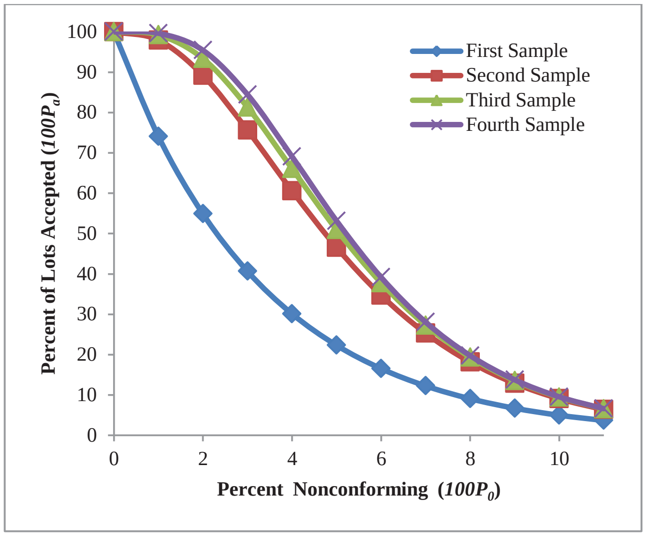


**Figure 1** OC Curve for the Four Level Multiple Sampling Plan