UNCERTAINTY ANALYSIS FOR THE UNKNOWN FUNCTION USING ARTIFICIAL NEURAL NETWORK (ANN) APPROXIMATED FUNCTION

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SUPERVISOR'S DECLARATION

I hereby declare that I have checked this project and in my opinion, this project is adequate in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering.

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STUDENT'S DECLARATION

I hereby declare that the work in this project is my own except for quotations and summaries which have been duly acknowledged. This project has not been accepted for any degree and is not concurrently submitted for award of other degree.

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ABSTRACT

This thesis deals with the finding of uncertainty analysis for the unknown function from experimental data by using Neural Network Approximation. The objective of this thesis is to estimates the uncertainty value for the unknown function where Artificial Neural Network (ANN) approximated function join together with sequential perturbation method will be applied. The thesis describes the uncertainty analysis techniques which are analytical (Newton Approximation) method and numerical (Sequential Perturbation) method to predict the uncertainty value and build up the new function from the experimental data via Fortran program using non-linear regression. The approach in analyzing uncertainty of Nusselt number is approximate the function via ANN using feed-forward and backpropagation network with four inputs and output were randomly generated. Finally, uncertainty outcome through sequential perturbation with ANN will be compare with the outcome using analytical method. Percentage error between both methods shall be compute to prove that uncertainty analysis for unknown function using sequential perturbation with ANN can also be use. From the results, average percentage error between Newton approximation (analytical method) and sequential perturbation (numerical method) retrieved is 5.52395×10^{-4} %. Meanwhile, the average percentage error between actual Nusselt number produced and approximated Nusselt number is 0.955373 %. However the main focus of this study is to determine whether sequential perturbation with ANN approximated function can be apply or not to estimate the uncertainty for the unknown function. The average percentage error between sequential perturbation with ANN and Newton approximation (analytical method) is 3.563%. Therefore, the objective is achieved.

ABSTRAK

Tesis ini membincangkan pencarian analisis ketidakpastian untuk fungsi yang tidak diketahui dari data eksperimen dengan menggunakan pendekatan Jaringan Neural Tiruan. Objektif tesis ini adalah untuk menganggarkan nilai ketidakpastian untuk fungsi yang tidak diketahui dimana Jaringan Neural Tiruan (ANN) dianggarkan fungsi bergabung dengan kaedah gangguan berjujukan akan digunakan. Tesis menghuraikan teknik-teknik analisis ketidakpastian yang mana analisis (*Newton Approximation*) dan berangka (*Sequential Perturbation*) meramalkan nilai ketidakpastian dan membina fungsi baru dari data percubaan melalui program *Fortran* menggunakan regresi tidak linear. Pendekatan dalam mengkaji ketidakpastian nombor *Nusselt* menghampiri fungsi melalui ANN menggunakan *feed-forward* dan rangkaian *backpropagation* dengan empat input dan output secara rawak dibina. Akhirnya, hasil ketidakpastian melalui usikan berjujukan dengan ANN akan dibandingkan dengan hasil menggunakan kaedah analisis. Ralat peratusan antara kedua-dua kaedah akan dikira untuk membuktikan analisis ketidakpastian itu untuk fungsi yang tidak diketahui menggunakan usikan berjujukan ANN boleh digunakan. Dari keputusan, ralat peratus purata antara penghampiran Newton (kaedah analisis) dan usikan berjujukan (kaedah berangka) ialah 5.52395×10^4 %. Sementara itu, ralat peratus purata antara nombor Nusselt sebenar dan nombor Nusselt dihasilkan ialah 0.955373%. Bagaimanapun, tumpuan utama kajian ini adalah untuk menentukan sama ada fungsi usikan berjujukan dengan ANN boleh digunakan atau tidak atas menganggar ketakpastian untuk fungsi yang tidak diketahui. Ralat peratus purata antara usikan berjujukan dengan penghampiran ANN dan NEWTON (kaedah analisis) ialah 3.563%. Oleh itu, objektif dicapai.

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LIST OF SYMBOLS

a,b	Input vector
Re	Reynolds number
Pr	Prandtl's number
H/D	Twist ratio
ϕ	Nanofluids concentration in percent
Nu	Nusselt number

LIST OF ABBREAVIATIONS

ANN	Artificial neural network
NA	Newton approximation
SP	Sequential perturbation

CHAPTER 1

INTRODUCTION

1.1 PROJECT BACKGROUND

Uncertainty analysis is the main idea to estimates error measurement in the final results. It is also the process of identifying, quantifying and combining the errors. Uncertainty analysis involves determining the uncertainty in model predictions that results from imprecisely known variables or parameters. Uncertainty analysis provides a methodical approach to estimating the accuracy of the results. In other word, uncertainty analysis means how to estimate the tolerance from the final results. In uncertainty analysis, the computational procuders which are analytical method and numerical method. This project is expected to use the numerical method such as Sequential Perturbation technique because it is easier to implement when the data is automated via computer program.

In the uncertainty analysis, there are two case studies which are from known function and unknown function. This project is focus on the unknown function that will be generated from experimental data or measurement data. Using the FORTRAN® software it can generate the new function that have relationships between inputs and outputs from the experimental data. The approximation function can be approach by using Artificial Neural Network (ANN). The Artificial Neural Network (ANN) will be used because of its ability to derive meaning from complicated or imprecise data.

1.2 PROBLEM STATEMENT

Generally, uncertainty analysis always used to calculate for known function such simple function or complex function. The type of method that can be utilise in order to solve a simple function is by analytical method (Newton approximation). However, for the complex function, numerical method (Sequential Perturbation) can be used. In this project the problem statement is how to calculate the uncertainty for unknown function. In fact, it is not easy to solve unknown function if compare it with known function. In this case solver need come out with new idea to solve this problem. First step to calculate the uncertainty, the problem must have a function either simple or complex. However for this case it just only have the data and need to find the uncertainty without the equation or function. As a solution new function from data need to be generate using FORTRAN® and suggested by Neural Network in MATLAB® software to approximate the function.

From this problem statement, major setback here is for the unknown function. Differ from known function, unknown function cannot be solving by any of those proposed methods, neither Newton approximation method nor Sequential Perturbation method. If the function is simple, this function can be solving using analytical method via Newton approximation. Then if the function is complex appropriate to use the numerical method such as Sequential Perturbation method. But in this project, the exact function is unknown and the data obtain from experiment only the source available. The uncertainty analysis for unknown function is not determined in a specific manner. The proposed method using Numerical Sequential Perturbation Method in calculating uncertainty in application where the unknown function is complex (multivariable) and is approximated using Artificial Neural Network (ANN). For further understanding on what uncertainty analysis is all about, refer Figure 1.1.

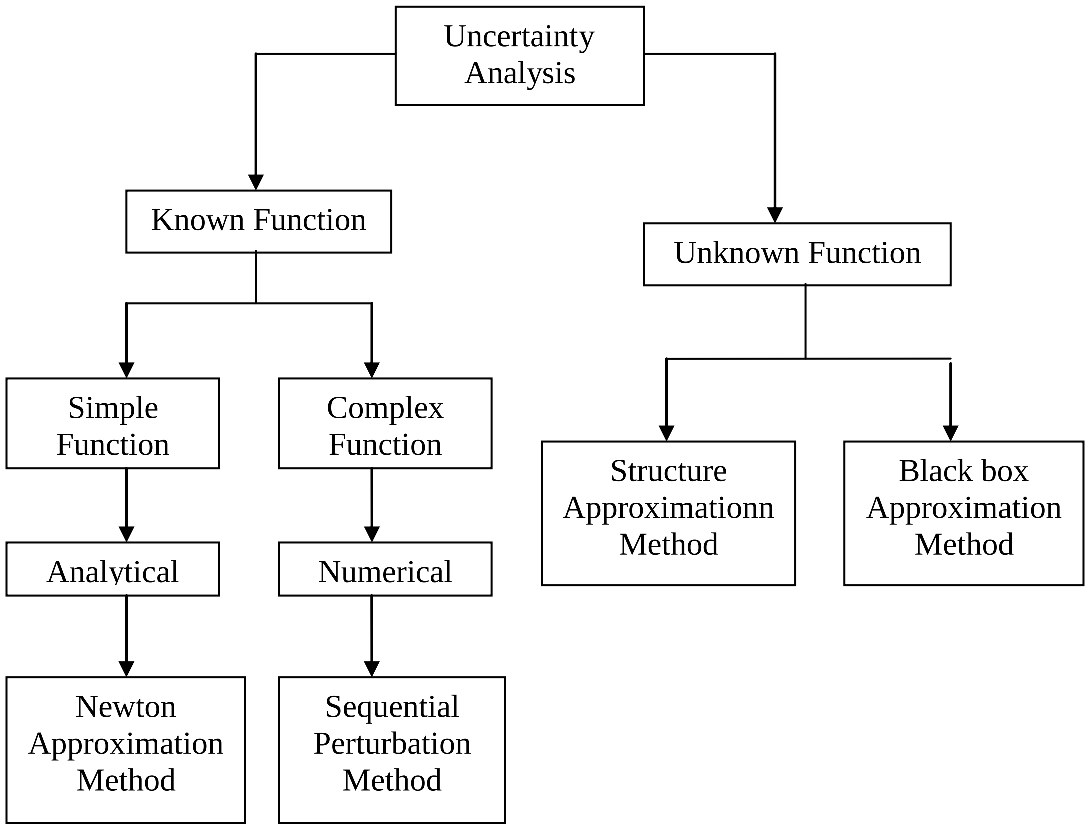

Figure 1.1 : Flow Chart of Uncertainty Analysis

1.3 OBJECTIVE OF THE RESEARCH

The objective of this project is to estimate the uncertainty analysis for the unknown function from experimental data using Neural Network Approximation to approximate the function and using the sequential perturbation to find the uncertainty for the unknown function.

1.4 SCOPE OF WORK

The approach in the uncentainty analysis of the function will be as follows:

i. Neural Network approximated function using MATLAB® software.
ii. Case study is solving the unknown function.
iii. Artificial Neural Network (ANN) using Feed-Forward and Backpropagation network.
iv. Used four input variables and only one output.
v. Compare the result between analytical and numerical method.
vi. Experimental data from experimental study of nanofluids heat transfer coefficient and Nusselt's number.

CHAPTER 2

LITERATURE REVIEW

2.1 UNCERTAINTY ANALYSIS

Uncertainty analysis is a powerful tool for improving the value of experimental work and can be applied during all phases of experimental program and also measure of the 'goodness' of a result. Without such a measure, it is impossible to judge the fitness of the value as a basis for making decisions relating to health, safety, commerce or scientific excellence. Evaluation of uncertainty is an ongoing process that can consume time and resources. It can also require the services of someone who is familiar with data analysis techniques, particularly statistical analysis.

Uncertainty can be described as that portion of the measurement beyond which are not sure of its true value. Each time a measurement is taken (mass, volume length) we rely upon a mechanical or visual point of reference in order to assign the appropriate value. These values, no matter how carefully they are obtained contain some degree of what is referred to as uncertainty.

However, the greatest value of uncertainty is almost certainly obtained when it used during the planning of an experiment. So important is uncertainty analysis is planning phase of an experiment that it was prominently featured in both of the primary conclusions obtained from landmark 1983 symposium on uncertainty analysis sponsored by the ASME Journal of Fluidss Engineering(JFE). Those two conclusions were (KLINE 1985b) :

i. Uncertainty analysis is an essential ingredient in planning, controlling, and reporting experiments. The important thing is that reasonable uncertainty analysis be done. All differences of opinion about appropriate methods are subsidiary to this conclusion.
ii. It is particularly important to use an uncertainty analysis in the planning and checkout stages of an experiment.

In all but the simplest of experiments, the end result of an investigation is not measured directly but rather is determined by calculation from data reduction equation. The end result, and the uncertainty in it are product of the direct measurement of the parameters and in most cases assumed values of material properties or other physical 'constants'. In all phases of experimentation will consider how the uncertainties in these variables propagate through the data reduction equation into the end result.

2.1.1 UNCERTAINTY PROPAGATION EQUATION

The general case of an experimental result, *r,* computed from *J* measured variables $X_{1...J,}$ the data reduction equation is :

$$r = r(X_1, X_2, ..., X_J) \tag{2.1}$$

And the uncertainty in the experimental result given by

$$U^2{}_r = \left(\frac{\partial r}{\partial X_1}\right)^2 U^2{}_{X_1} + ... + \left(\frac{\partial r}{\partial X_J}\right)^2 U^2{}_{X_J} \tag{2.2}$$

Where U_r is the uncertainty in the result, U_{XI} is the uncertainty in the variable X_1 . This is the most general form of the uncertainty propagation equation (Coleman and Steele 1999). When applying the uncertainty propagation equation, the individual uncertainties should all be expressed with the same odds, for example at 95% confidence. In addition. The measured variables and their uncertainties are assumed to be independent of one another.

2.1.2 Analytical (Newton Approximation) Method

The analytical method involves deriving a single formula for the uncertainty in a measurement.

i. Straightforward computation
ii. Becomes unwieldy and eventually impractical as the data reduction procedure becomes increasingly complex.

As a consequence of the Fundamental Theorem of Calculus

$$\frac{\partial R}{\partial x_i} \xrightarrow{\lim} \left[\frac{R(x_i + \Delta x_i) - R(x_i)}{\Delta x_i}\right] \approx \frac{R(x_i + \partial x_i) - R(x_i)}{\partial x_i} \tag{2.3}$$

Where δxi is a finite perturbation in the measured value of xi.
Now, *if we use,* $\delta x \approx ui$ then

$$\left[\frac{\partial R}{\partial x_i}\delta x_i\right]^2 \approx \left[\frac{R(x_i + \delta x_i) - R(x_i)}{\delta x_i}\right]^2 \approx [R(x_1 + u_i) - R(x_i)]^2 \tag{2.4}$$

The uncertainty in R due to the uncertainty in xi can be estimated by perturbing the data reduction formula by ui.

Equation (1) becomes

$$u_R = \left[D_1^2 + D_2^2 + \dots\dots\dots + D_n^2\right]^{1/2} \tag{2.5}$$

Where

$$D_1 = R(x_1 + u_{1,} x_2 \dots\dots\dots x_n) - R(x_{1,} x_2 \dots\dots\dots x_n) \tag{2.6}$$

$$D_2 = R(x_1, x_{2,} u_2 \dots\dots\dots x_n) - R(x_{1,} x_2 \dots\dots\dots x_n) \tag{2.7}$$

$$D_n = R(x_1, x_2 \dots\dots\dots x_{n,} + x_n) - R(x_{1,} x_2 \dots\dots\dots x_n) \tag{2.8}$$

The uncertainty in *R* is estimated by *sequentially* perturbing the input values *xi* by their respective uncertainty.

The analytical method is a linear approximation for δ_y can be made,Which is valid when tsx^- is small and neglect the higher order term in equation

$$\delta_y \approx \left(d_y / d_x\right) \ tsx^- \quad (2.9)$$

For small deviations from the value of $\bar{x}$, this slope predicts an acceptable, relationship between tsx^-; and δ_y . The derivative term is a measure of the sensitivity changes in x. Since the slope of the curve can be different for different value of x, it is important to evaluate the slope using a representative value. Applying this analysis to the errors that contribute to the uncertainty in x, written as u_x. The uncertainty in x will be related to the uncertainty in the resultant

$$U_y = \left(d_y / d_x\right) u_x \quad (2.10)$$

Consider a result R which is determined throught some function relationship between independent variables, $x_{1,} x_2 .... x_L$ defined by.

$$R = f_1 \{x_{1,} x_2 .... x_n\} \quad (2.11)$$

Where L is the number of independent variables involved.Each variables will contain some measure of uncertainty that will effect the result. The best estimate of the true mean value R would be stated as

$$R = R^- \pm U_R \quad (P\%) \quad (2.12)$$

Where the sample mean of R is found from

$$R = f_1 \{x_{1,} x_2 .... x_n\} \quad (2.13)$$

And uncertainty R¯ is found from

$$R = f_1 \{U_{x1}, U_{x2} \ldots\ldots\ldots U_{xn}\} \tag{2.14}$$

The RSS form can be derived from the linearized approximation of the Taylor series expansion of the multivariable function. A general sensitivity index ɵi, result from taylor series expansion is

$$e_i = \delta_R \qquad i=1,2,\ldots\ldots\text{L} \tag{2.15}$$

The Propagation of uncertainty in the variables to the result will yield an uncertainty estimate given by

$$U_R = \pm\left[\sum (e_i U x_i)^2\right]^{1/2} \quad (P\%) \tag{2.16}$$

2.1.3 Numerical (Sequential Perturbation) Method

The sequential perturbation technique is easy to implement when the data reduction procedure is automated via a computer program.

i. Uncertainty estimate is approximate, not exact as in the analytical method.
ii. Is simple to implement, and allows for evolution of the *model* underlying the data reduction.

A complex dynamic system is one consisting of multiple elements, where the future state of the system is determined by a function f of its current state,

$$s(t+1) = f(s(t)) \tag{2.17}$$

Where $s(t)$ is the state of the system at time t.

The typical feature of interest of complex dynamic systems is their asymptotic behavior as $t \to \infty$. The set of states towards which a system converges under these conditions is known as an *attractor*. Attractors may be fixed points, limit cycles, or non-repeating `chaotic' attractors. Systems may contain single or multiple attractors. The set of initial states of a system that converge to a given attractor forms the *basin of attraction* of that attractor.

The Perturbation Analysis pattern requires a dynamic system, consisting of:

i. a finite set of elements, each of which may take a discrete or continuous value; and
ii. A deterministic updating function.

The sequential perturbation method is a numerical approach can also be used to estimate the propagation of uncertainty .Referred to as sequential perturbation it is generally the preffered method when direct partial differentiation is too cumbersome or intimdating.or the number of variables involved is large.The method is straightforward and uses a finite difference method to approximate the derivaties. *(Figliola & Beasley 2006)*

i. Based on measurement for the independent variables under some fixed operating condition, calculate a result $R = f(x_1, x_2, x_3........x_n)$ This value fixes the operating point for the numerical approximation.
ii. Increasing the independent variables by their respective uncertainty and recalculate the result based on each of these new value.

$$R_1 + = f(x_1 + ux_1, x_2........x_n) \tag{2.18}$$

$$R_2 + = f(x_1 x_2, ux_2.........x_n) \tag{2.19}$$

$$R_3 + = f(x_1, x_2, x_3.........x_n + ux_L) \tag{2.20}$$

iii In the similar manner decrease the independent variables by their respectives uncertainties and recalculates the result based on each of these new values.call these values $R^{-}{}_{1}$.