The numerical integration to solutions for the double-slip model for the deformation and flow of granular materials

NOR ALISA MOHD DAMANHURI[1*] & NOR AIDA ZURAIMI MD NOAR[1]

[1]Faculty of Industrial Sciences and Technology, Universiti Malaysia Pahang, 26300 UMP Gambang, Kuantan, Pahang

Abstract

This poster presents a numerical integration to solutions for the plane-strain rigid-perfectly plastic deformation of a granular material in a container satisfying the stress-equilibrium conditions, the Coulomb yield criterion and the double-slip kinematic equations.

Introduction

The problem of modelling fully developed dense granular flows using continuum mechanics is complex and challenging. Stress fields within granular flows can be described by coupling the equations of linear momentum with the Coulomb–Mohr yield condition. This research is to develop a numerical method to find approximations to solutions of the double-slip model for the deformation of granular materials.

Methodology

We consider a rigid/plastic ideal soil and metal in a state of plane strain. The stress components $\sigma_x, \sigma_y, \tau_{xy}$ referred to a rectangular Cartesian coordinate system (x, y) satisfy the Coulomb yield condition

$$(\sigma_x + \sigma_y)\sin\phi + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = 2c\cos\phi \qquad (1)$$

Where the cohesion *c* and the angle of internal friction ϕ are constants, and the equilibrium equations are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \qquad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0. \qquad (2)$$

Equations (1) and (2) are hyperbolic, with characteristics

$$\frac{dy}{dx} = \tan\left(\psi - \frac{\pi}{4} - \frac{\phi}{2}\right), \quad \frac{dy}{dx} = \tan\left(\psi + \frac{\pi}{4} + \frac{\phi}{2}\right), \quad (3)$$

which are termed the $\alpha -$ and $\beta -$ lines respectively, where

$\tan 2\psi = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$ and ψ is the angle of inclination of the direction of the algebraically greater principal stress to the x -axis. If the $\frac{\partial}{\partial s_\alpha}, \frac{\partial}{\partial s_\beta}$ denote differentiation along the $\alpha -$ and $\beta -$ lines respectively, then the relations along the characteristics may be written as

$$\cos\phi \frac{\partial p}{\partial s_\alpha} + 2\left(p\sin\phi + c\cos\phi\right)\frac{\partial \psi}{\partial s_\alpha} = 0$$

$$\cos\phi \frac{\partial p}{\partial s_\beta} - 2\left(p\sin\phi + c\cos\phi\right)\frac{\partial \psi}{\partial s_\beta} = 0$$

Where

$$p = -\frac{1}{2}\left(\sigma_x + \sigma_y\right).$$

Results

Applications to a variety of plane strain problems

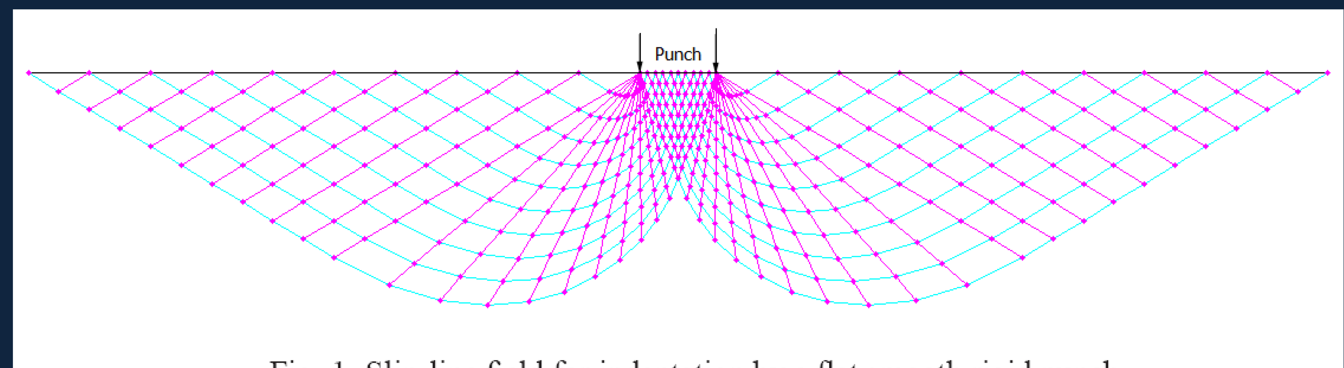


Fig. 1. Slip-line field for indentation by a flat smooth rigid punch.

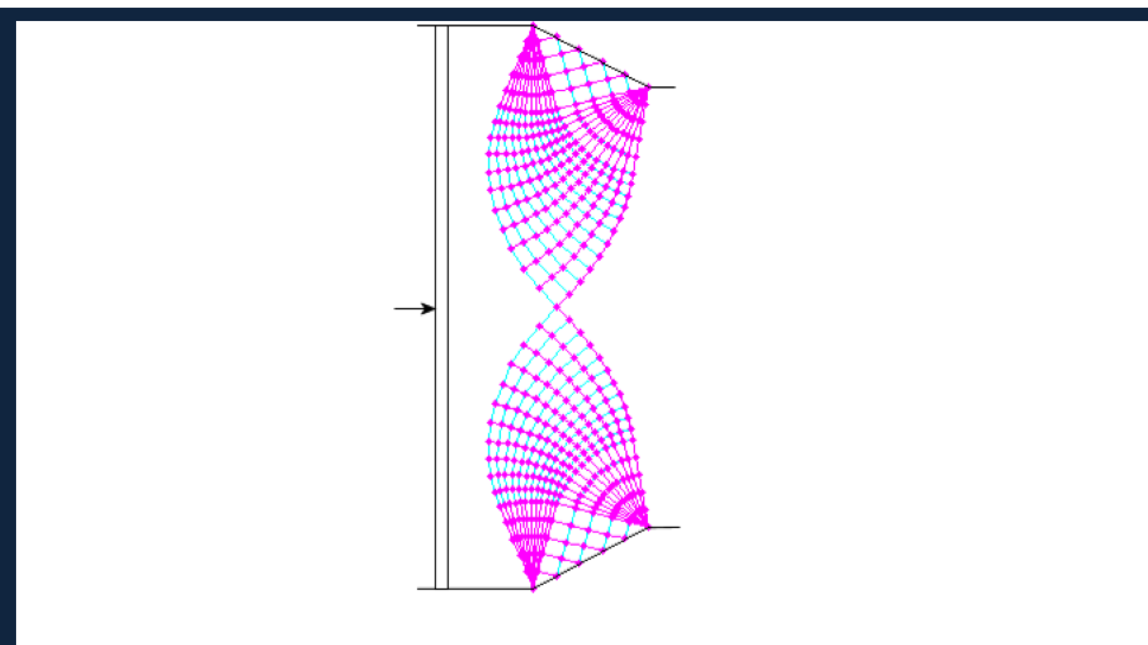


Fig. 2. Slip-line field and plastic region for flow through a smooth wedge-shaped die.

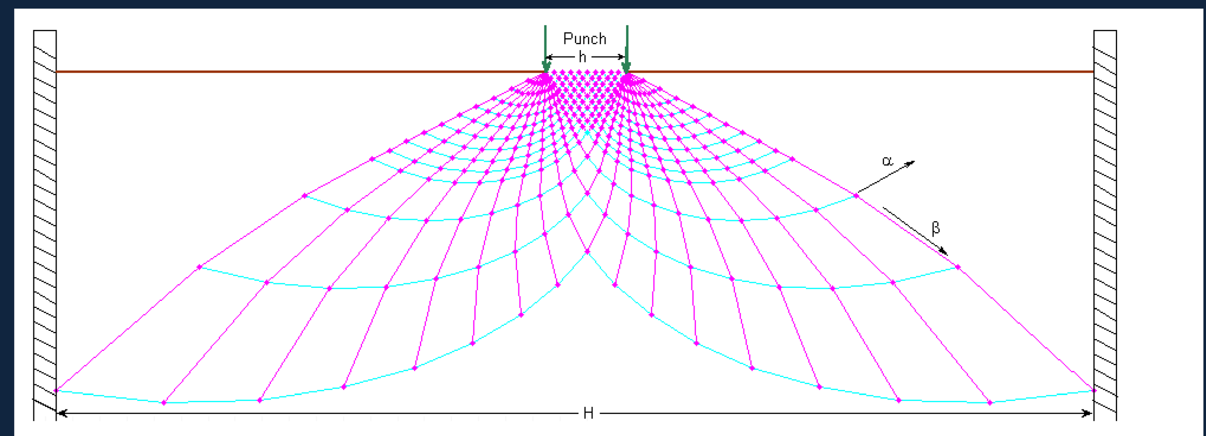


Fig. 3. Slip-line field for indentation by a rigid punch for small h/H and a smooth container walls.

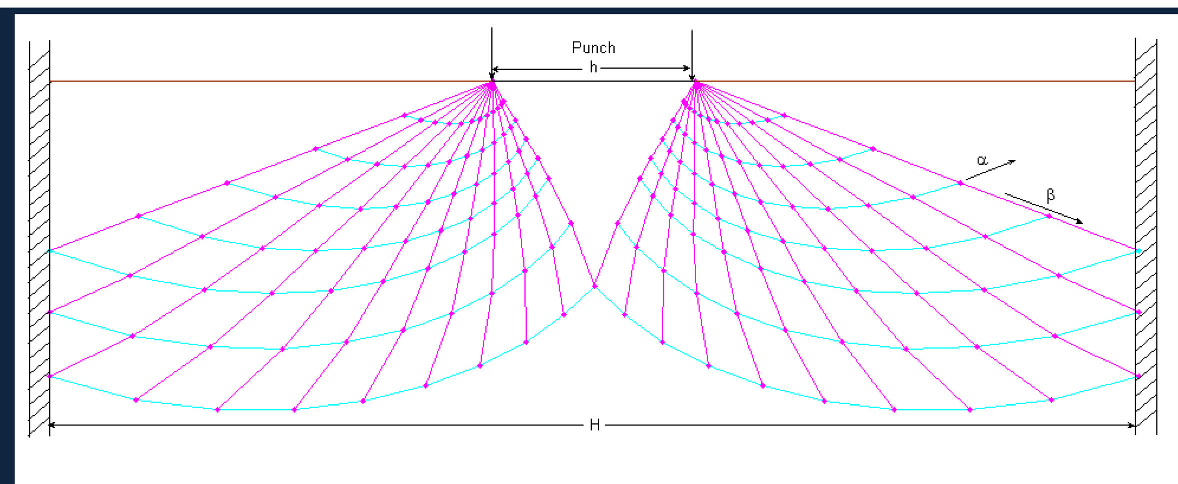


Fig. 4. Slip-line field for indentation for larger h/H and a smooth container walls.

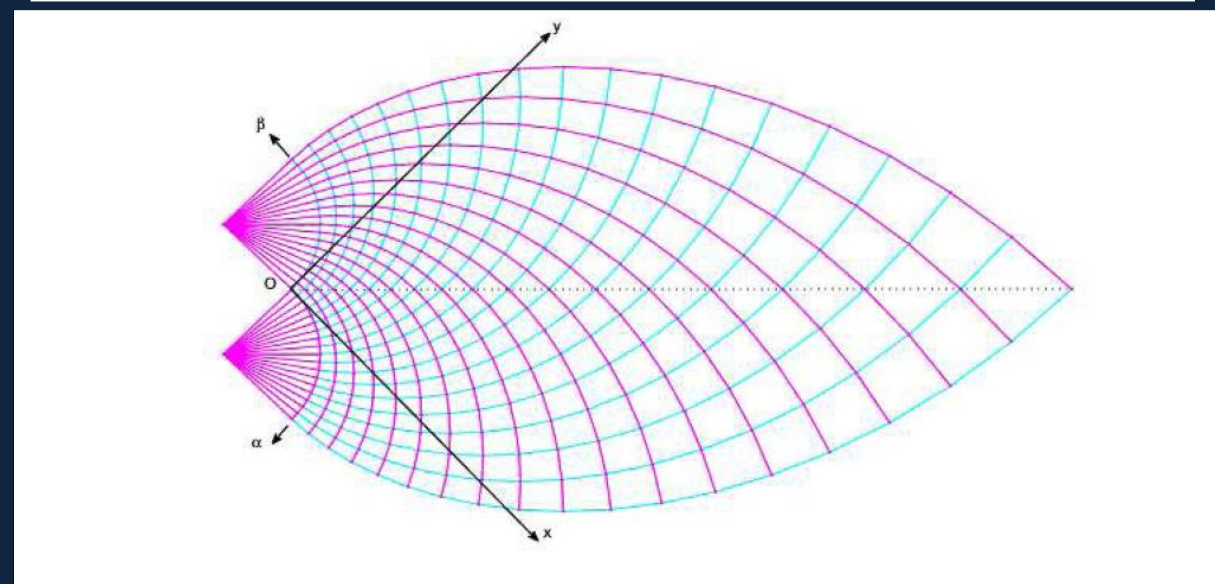
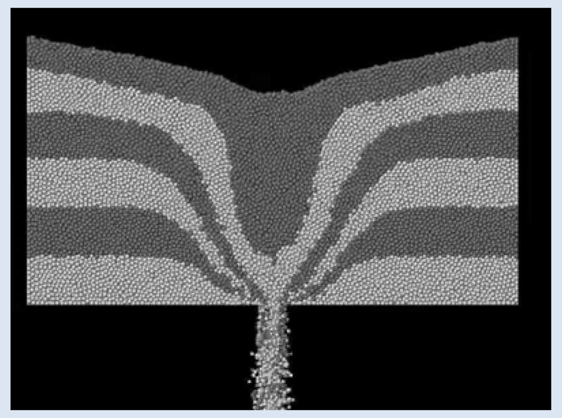


Fig. 5. Slip-line field defined by two slip-lines which are circular arcs of equal radii where

Applications

In the pharmaceutical and food industries, one major roadblock to operational efficiency has been the inability to understand the particulars of granular flow. This research will able to account for these complicated flow issues that promises to help engineers design perfected systems for increased productivity. This could mean some major savings to the industry. The model and computer algorithm can now predict flow paths of various grain types, which will help engineers better design chutes and troughs to prevent blockages and. In understanding how grains flow could helped manufacturers optimising their production cycles. This research is also importance in geological processes, such as plate tectonics, landslides and erosion



Conclusion

The result shows that using the classical method for metal plasticity to the double shearing model and using the numerical method to construct the stress fields have found to agree well in various plane strain problems and made possible to solve the complete boundary value problem.

Novelty

- The double shearing is modelled as a single phase continuum.
- Using the developed numerical algorithm and programming to the model, the stress field in the plastic region for a number of problems may be constructed and these have not been previously solved.

References

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Further Information

noralisa@ump.edu.my