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Exact solution for heat transfer free convection flow of Maxwell nanofluids with graphene nanoparticles

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Abstract. This article focuses on the flow of Maxwell nanofluids with graphene nanoparticles over a vertical plate (static) with constant wall temperature. Possessing high thermal conductivity, engine oil is useful to be chosen as base fluid with free convection. The problem is modelled in terms of PDE's with boundary conditions. Some suitable non-dimensional variables are interposed to transform the governing equations into dimensionless form. The generated equations are solved via Laplace transform technique. Exact solutions are evaluated for velocity and temperature. These solutions are significantly controlled by some parameters involved. Temperature rises with elevation in volume fraction while Velocity decreases with increment in volume fraction. A comparison with previous published results are established and discussed. Moreover, a detailed discussion is made for influence of volume fraction on the flow and heat profile.

1. Introduction

The technological implementations of graphene based materials are of great importance. Graphene can be produced by a number of methods i.e. chemical vapor deposition, tearing of carbon nanotubes and many more. Due to its continuous electrical conducting behavior and higher electron moment inside it, it became encouraging nano-material. They exhibit high thermal conductivity 3000 to 5000 Wm-1 K-1[1, 2]. Heat transfer phenomenon is the necessitate process in industrial technologies. The development of more efficient heat transfer system is limited by using the fluids having low thermal conductivities. Heat management has the highest potential to upgrade this thermal performance. A solution to this problem is given by Choi and Eastman [3] in 1999, adding nanoparticles to increase the thermal conductivities of the carrier fluid. The nanoparticles possess upgraded thermal conductivities contrast to the carrier fluids. In the recent years, a number of captivating analysis (experimental and theoretical) has been done related to this area [4-6].

Viscous fluids are examined for exact solutions in a number of literatures but very less work is done on exact solutions of non-Newtonian fluids, remarkably for Maxwell fluids. Usually numerical methods are used to solve non-Newtonian problems [7, 8], because of their complexity in obtaining exact solutions. Furthermore, such investigations are done very rare for convection flow of Maxwell nanofluids. Due to elastic property and viscosity effects together a subclass of rate type i.e. Maxwell fluid model is appointed in this work. A review and some captivating studies of Maxwell fluid flow problems can be found in [9-10]. On the basis of above applications and literature, the exact solution of

heat transfer analysis in flow of Maxwell nanofluid with graphene nanoparticles with natural convection is of tremendous importance. Thus, to find the exact solution and observe the influence of nanoparticles on the flow and heat of present problem is the main focus here. Exact solutions of the problem are evaluated by Laplace transform method. The results are mapped graphically using Mathcad and explored.

2. Mathematical formulation

Consider unsteady Maxwell fluid at rest over a vertical flat plate in $(x, y) -$ plane under free convection. At first, having constant wall temperature T_∞ both the plate and fluid are static. At time $t = 0^+$, the temperature of the plate is upgraded to a constant value T_w and touches a value T_∞. The fluid flows due to temperature gradient which produces buoyancy force. The governing equations are:

$$\rho_{nf}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial u(y,t)}{\partial t} = \mu_{nf}\frac{\partial^2 u(y,t)}{\partial y^2} + \left(1 + \lambda_1 \frac{\partial}{\partial t}\right)(\rho\beta)_{nf}\, g\left(T - T_\infty\right),\tag{1}$$

$$(\rho c_p)_{nf}\frac{\partial T(y,t)}{\partial t} = k_{nf}\frac{\partial^2 T(y,t)}{\partial y^2}\tag{2}$$

The BC's are:

$$u\left(y,0\right) = 0;\ u(0,t) = 0;\ T\left(y,0\right) = T_\infty;\ T\left(0,t\right) = T_w;$$
$$u\left(\infty,t\right) = 0;\ T\left(\infty,t\right) = T_\infty.\tag{3}$$

The Hamilton and Crosser model is used for thermal conductivity:

$$\frac{k_{nf}}{k_f} = \frac{\left(k_s + 2k_f\right) - 2\phi\left(k_f - k_s\right)}{\left(k_s - 2k_f\right) + \phi(k_f - k_s)}.\tag{4}$$

Interpolating the non-dimensional elements:

$$u = \frac{u}{U_0},\quad y = \frac{yU_0}{\nu},\quad t = \frac{tU_0^2}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty},\tag{5}$$

Using Laplace transforms technique to find solution of Eq (6) and (7):

$$\phi_5\frac{\partial^2 \bar{\theta}(y,q)}{\partial y^2} - \phi_4 \Pr q\bar{\theta}(y,q) = 0,\ \bar{\theta}(0,q) = \frac{1}{q},\ \bar{\theta}(y,q) \to 0\ at\ y \to \infty.\tag{6}$$

$$\phi_1(1 + \lambda q)q\bar{u}(y,q) = \phi_2\frac{\partial^2 \bar{u}(y,q)}{\partial y^2} + \phi_3(1 + \lambda q)Gr\bar{\theta}(y,q),\ \bar{u}(0,q) = 0,\ \bar{u}(y,q) \to 0, y \to \infty.\tag{7}$$

Where

$$\lambda = \frac{\lambda_1 U_0^2}{\nu_f},\ Gr = \frac{\nu_f g\beta_f\Delta T}{U_0^3},\ \Pr = \frac{(\mu c_p)_f}{k_f},\ \phi_1 = (1 - \phi) + \phi\left(\frac{\rho_{CNT}}{\rho_f}\right),\ \phi_2 = \frac{1}{(1 - \phi)^{2.5}},$$

$$\phi_3 = (1 - \phi) + \phi\left(\frac{\rho_{CNT}\beta_{CNT}}{\rho_f\beta_f}\right),\ \phi_4 = (1 - \phi) + \phi\frac{(\rho c_p)_{CNT}}{(\rho c_p)_f},\ \phi_5 = \frac{\left(k_s + 2k_f\right) - 2\phi\left(k_f - k_s\right)}{\left(k_s - 2k_f\right) + \phi(k_f - k_s)},$$

The solutions of the partial differential (6) and (7) are:

$$\bar{\theta}(y,q) = \frac{1}{q}\exp\left(-y\sqrt{b_1}\sqrt{q}\right),$$

$$\theta(y,t) = erfc\left(\frac{y\sqrt{b_1}}{2\sqrt{t}}\right). \tag{8}$$

$$\bar{u}(y,q) = -a_1\frac{(\lambda q+1)}{q^2\left[a_0\lambda q+(a_0-b_1)\right]}\exp\left(-y\sqrt{a_0 q(\lambda q+1)}\right)+a_1\frac{(\lambda q+1)}{q^2\left[a_0\lambda q+(a_0-b_1)\right]}\exp\left(-y\sqrt{b_1}\sqrt{q}\right),$$

(9)

Where $a_0 = \dfrac{\phi_1}{\phi_2}$, $a_1 = \dfrac{\phi_3}{\phi_2}Gr$.

Using the inverse Laplace transform and convolution theorem, Eq. (9) emits:

$$u(y,t) = -\frac{a_1}{a_0\lambda}\int_0^t \frac{[\frac{a\lambda-1}{a^2}H(t-s)+\frac{1}{a}(t-s)}{+\frac{1-a\lambda}{a^2}\exp(-a(t-s))]}{f_1(y,(t-s))}\,ds + \frac{a_1}{a_0\lambda}\int_0^t \frac{\frac{a\lambda-1}{a^2}H(t-s)+\frac{1}{a}(t-s)+}{\frac{1-a\lambda}{a^2}\exp(-a(t-s))]}\frac{y\sqrt{b_1}\exp\left(-\frac{y^2 b_1}{4t}\right)}{2t\sqrt{\pi t}}\,ds. \tag{10}$$

2.2 Nusselt number

The practical quantity Nusselt number is:

$$Nu = -\frac{k_{nf}}{k_f}Re\frac{\partial \theta}{\partial y}\bigg|_{y=0} = \frac{k_{nf}}{k_f}Re\frac{\sqrt{b_1}}{\sqrt{\pi}\sqrt{t}}$$

$$= \frac{\left(k_s+2k_f\right)-2\phi\left(k_f-k_s\right)}{\left(k_s-2k_f\right)+\phi(k_f-k_s)}Re\frac{\sqrt{b_1}}{\sqrt{\pi t}}.$$

Where Re_x is the Reynold's number. $Re_x = \dfrac{xU_0}{v_f}$.

## 3. Numerical results and discussions

Thermo physical properties of graphene and Engine oil are given in Table 1. The quantity of physical interest of Nusselt number is evaluated. The effect of volume fraction ϕ on velocity and temperature is displayed in Figures 1-2. Fig. 1[a] and 1[b] shows the impact of nanoparticles volume fraction ϕ on temperature and velocity. Temperature is an increasing function of volume fraction ϕ. Increasing the amount graphene nanoparticles in the nanofluids increases its thermal conductivity and the fluids temperature elevates. Fig 1 [b] shows the alteration of the flow of Maxwell nanofluid with ϕ. It is tackled that velocity decreases with elevating volume fraction ϕ. Increasing the amount of nanoparticles makes the fluid denser, so its velocity reduces. The present results are compared with a prior literature Chandran *et al*. [11] in Figure 2. In limiting case (When $\phi=0$ and $\lambda=0.00001$), the present result is in consistency with the prior result Chandran *et al*. [11].

Table 1. Thermophysical properties of graphene and Engine oil.

model	$\rho(kg/m^3)$	$c_p(kg^{-1}/k^{-1})$	$k(Wm^{-1}k^{-1})$	$(\beta \times 10^{-5}k^{-1})$
Graphene	2250	2100	2500	21
Engine oil	884	1910	0.144	70
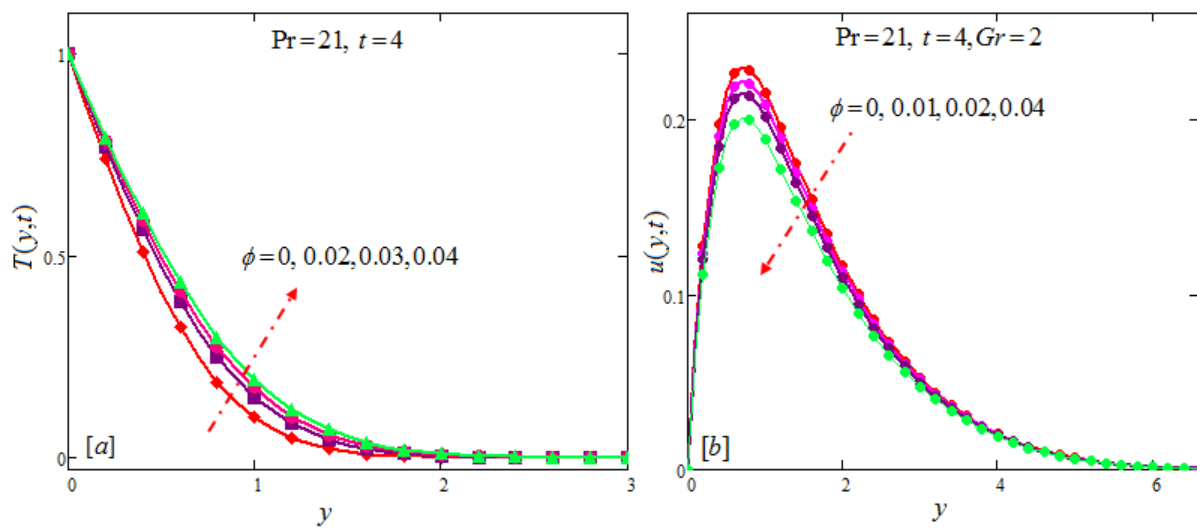
Figure 1: Temperature and velocity profiles for different values of volume fraction.
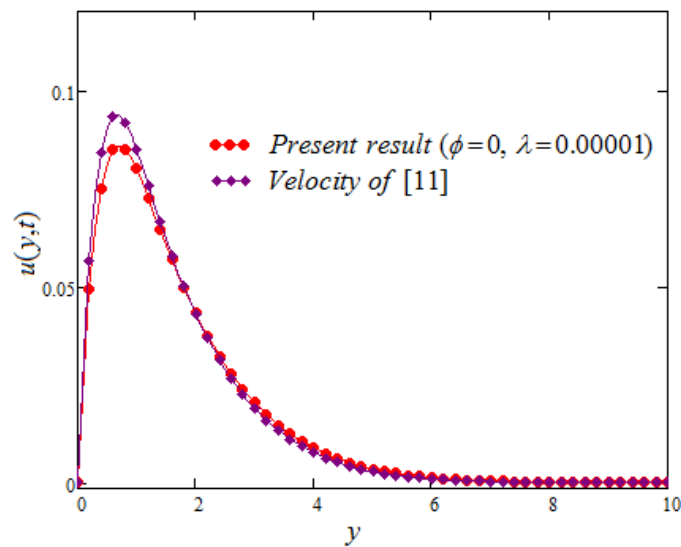
Figure 2. Comparison of present Velocity with previous result.

4. Conclusions

In this strive, exact solution for unsteady free convection flow of graphene based Maxwell nanofluid is achieved via Laplace transform method. For an increase in volume fraction ϕ, there is a significant increase in temperature of the fluid. Velocity decreases with elevating amount of nanoparticles. The results are in consistency with prior published result [11].

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