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# Thermal radiation effect on MHD Flow and heat transfer analysis of Williamson nanofluid past over a stretching sheet with constant wall temperature

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**Abstract.** In this paper, the boundary layer flow of Williamson nanofluids past over a stretching sheet in the presence of thermal radiation effect is analyzed. Using similarity transformations, the governing equations are reduced to a set of nonlinear ordinary differential equations (ODEs). These equations are solved numerically by with shooting method. The effects of Williamson parameter, magnetic parameter, radiation parameter, Prandtl number on velocity, temperature and concentration fields are shown graphically and discussed. It is found that the rate of heat transfer is higher for Williamson nanofluid compared to the classical viscous fluid. Also, the comparisons with existing results are provided in the literature.

## 1. Introduction

Due to the characteristic of low thermal conductivity, the heat transfer fluids such as water, engine oil and ethylene glycol have limited heat transfer capabilities. There are several ways have been carried out to increase the performance of convective heat transfer for these fluids such as changing flow geometry, boundary conditions, or by increasing thermal conductivity. It is also true that metals have higher thermal conductivities than fluids. Thermal conductivity can be increased by adding metals to the base fluids. The resultant fluids are termed as nanofluids. This classical idea was first introduced by Choi [1]. After the first work of Choi [1], many other researchers have made their useful investigations that involved the nanoparticles [2-3]. The studies of non-Newtonian fluids are remarkably enhanced during the last few years because of their important applications in industrial processes. To describe the behavior of pseudoplastic fluids, different models have been proposed in the literature. One of the most popular pseudoplastic fluids is Williamson fluid. It is potentially applied in the biological engineering to estimate the occurrence of mass and heat transfer in the vessels such as diffusion of nutrients in blood, hemodialysis and others [4].

Nadeem and Hussain [5] analyzed the two-dimension flow of heat transfer on Williamson nanofluids. They found that Lewis number appeared only in volume fraction equation when the velocity transformation depends on thermal diffusivity; if not then Schmidt number appeared, and vice versa. They used the homotopy analysis method to obtain the solutions. Prasannakumara et al. [6] presented the chemical reaction effects on Williamson nanofluids slip over a stretching sheet in the presence of embedded porous medium. In their results, the skin coefficient is higher in the presence of velocity, thermal and solutal slip for Williamson parameter. The same work was extended by Krishnamurthy et al. [7] by considering the factor of boundary layer flow of MHD and melting heat transfer. Besides, the effect of thermal radiation and magnetic field with the presence of peristaltic



motion flow Williamson nanofluids in a tapered asymmetric channel was carried out by Kothandapani and Prakash [4].

The effects of thermal radiation and magnetic field have giving significant impact on nanofluids flow. This issue has been discovered by Sheikholeslami et al. [8] who had presented the heat transfer analysis on two phase model with the effect of thermal radiation on MHD nanofluid flow. Recently, an analysis inspects the numerical investigation of MHD flow of Williamson fluid model over a sheet with variable thickness by Salahuddin et al. [9]. An investigation on MHD steady flow of viscous copper-water nanofluids due to a rotating disk has been done by Hayat et al. [10]. Besides, the use of electrically conducting fluids under the influence of magnetic fields in various industries has led to a renewed interest in investigating hydromagnetic flow and heat transfer in different geometries.

Also, stretching sheet happens when the velocity at the boundary is away from a fixed point. Also, it is wide-ranging applications and important in some engineering processes such as polymer extrusion, glass fiber production, manufacturing of foods and paper. The rate of heat transfer for stretching sheet is the crucial for the final quality of the product (Sarif et al. [11]). Gorla and Giresha [12] had studied the dual solutions for the stagnation point flow and convective heat transfer of a Williamson nanofluid past a stretching or shrinking sheet.

As a continuation, the main purpose of the present work is to investigate the thermal radiation and magnetic effects on heat transfer flow of Williamson nanofluid over a stretching sheet. Constant wall temperature (CWT) is considered on the sheet. Through the similarity transformation, the governing nonlinear equations are reduced into ordinary differential equations then solved numerically using a shooting method.

## 2. Mathematical formulation

Let considering that there is a two-dimensional steady flow of an incompressible Williamson nanofluid flow over a stretching surface and the flow region is defined as  $y > 0$ , and the plate is stretched along the  $x$ -axis with the velocity  $U_w = bx$ , where  $b$  is a positive constant. The two-dimensional boundary layer equations governing of the flow are given as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \sqrt{2} \nu \Gamma \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho}, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\rho_p C_p}{\rho C} \left[ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right] - \frac{\partial q_r}{\partial y}, \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}, \quad (4)$$

and corresponding boundary conditions are

$$\begin{aligned} u = U_w(x) = bx, \quad v = 0, \quad T = T_w, \quad C = C_w, \quad \text{at } y = 0 \\ u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad \text{as } y \rightarrow \infty \end{aligned} \quad (5)$$

by following Yasin et al. [13], we look for a solution of Equations (2)-(4) of the form:

$$u = bx f'(\eta), \quad v = -\sqrt{bv} f(\eta), \quad \eta = \left( \frac{a}{v} \right)^{\frac{1}{2}} y, \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \quad (6)$$

The governing equations then can be reduced to ordinary differential equations where by using the following similarity transformations as

$$f'''(\eta) + \lambda f'''(\eta) f''(\eta) + f''(\eta) f'(\eta) - f'^2(\eta) - M f'(\eta) = 0, \quad (7)$$

$$\left( 1 + \frac{4}{3} \right) R \theta''(\eta) + \text{Pr} f(\eta) \theta'(\eta) + \frac{Nc}{Le} \theta'(\eta) \phi'(\eta) + \frac{Nc}{Le.Nbt} \theta'^2(\eta) = 0, \quad (8)$$

$$\phi''(\eta) + Scf(\eta)\phi'(\eta) + \frac{1}{Nbt}\theta''(\eta) = 0. \tag{9}$$

where  $f$ ,  $\theta$  and  $\phi$  are functions of  $\eta$  and prime denotes derivatives with respect to  $\eta$  and  $R = \frac{4\sigma^* T_\infty^3}{\alpha k^*}$ ,

$$\lambda = \sqrt{\frac{2b^3}{\nu}} \Gamma x, M = \frac{\sigma B_o^2}{\rho b}, Pr = \frac{\nu}{\alpha}, Nc = \frac{\rho_p C_p}{\rho C} (C_w - C_\infty), Le = \frac{\alpha}{D_B}, Sc = \frac{\nu}{D_B}, Nbt = \frac{T_\infty D_B (C_w - C_\infty)}{D_T (T_w - T_\infty)}.$$

The corresponding boundary conditions will take the following form:

$$\begin{aligned} f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1, \\ f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0. \end{aligned} \tag{10}$$

The physical quantities of interest are skin friction coefficient  $C_f$  local Nusselt number  $Nu$  and local Sherwood number  $Sh$  which are defined as

$$C_f = \frac{\tau_w}{\rho U_w^2}, Nu = -\frac{xq_w}{T_w - T_\infty} \frac{\partial T}{\partial y}_{y=0} \text{ and } Sh = -\frac{x}{C_w - C_\infty} \frac{\partial C}{\partial y} \tag{11}$$

Where the shear stress  $\tau_w = \mu \left[ \frac{\partial u}{\partial y} + \frac{\Gamma}{\sqrt{2}} \left( \frac{\partial u}{\partial y} \right)^2 \right]$ . Using the non-dimensional variables, we obtain

$$C_f \sqrt{Re} = f''(0) + \frac{\lambda}{2} f''^2(0), \frac{Nu}{\sqrt{Re}} = -\theta'(0) \text{ and } \frac{Sh}{\sqrt{Re}} = -\phi'(0). \tag{12}$$

where  $Re = \frac{b}{\nu} x^2$  is the local Reynold's number.

### 3. Result and discussion

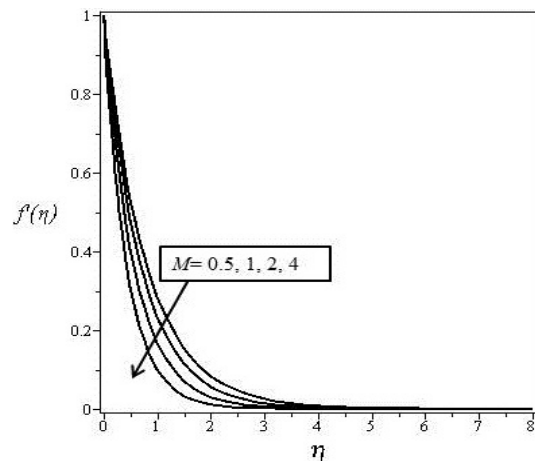
**Table 1:** Comparison of results for viscous case  $-\theta'(0)$  when  $\lambda = \infty, Le=Nbt = 1,$   
 $Pr = R = M = Nc = 0.$

Pr	Nadeem and Hussain [14] by HAM method	Goyal and Bhargava [15] by FEM method	Present study by Shooting method
0.20	0.169	0.1691	0.1698
0.70	0.454	0.4539	0.4539
2.00	0.911	0.9113	0.9113
7.00	-	1.8954	1.8954
20.00	-	3.3539	3.3539

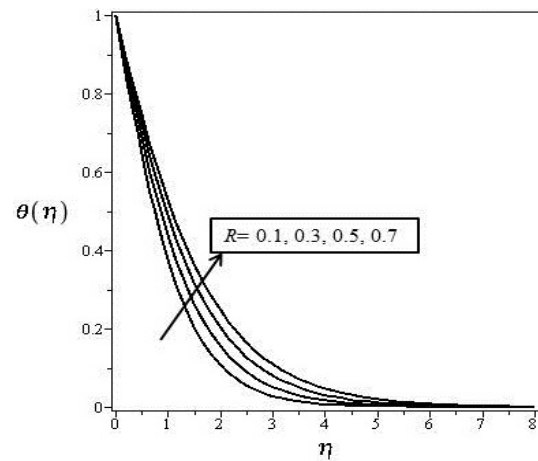
The nonlinear ordinary differential equations are of third order in  $f$ , second order in  $\theta$  and  $\phi$ , are then reduced to simultaneous ordinary equations. With the initial conditions of  $\eta \rightarrow \infty$ , we can get the value of  $f'(\eta), \theta(\eta)$  and  $\phi(\eta)$ . Thus, we may obtain unknown initial conditions  $\eta = 0$  by using Shooting method and assumed the initial values for boundary value problem. In this study, there are no consideration of variation in temperature, velocity and concentration where we took large infinity condition but finite value for  $\eta$ . We chose value at  $\eta_{max} = 8$ , which is sufficient to achieve asymptotic boundary conditions for all parameter values considered. Table 1 show the comparison values of  $-\theta'(0)$  with previous results. It concluded that this method work efficiently and the results presented here are accurate.

Figures 1 and 2 show that the magnetic parameter  $M$  effects on velocity  $f'(\eta)$  and temperature  $\theta(\eta)$  fields, respectively. It is found that velocity  $f'(\eta)$  field decreases with increases in the magnetic parameter  $M$ . The reason is that an increase in magnetic parameter  $M$  means an increase in the

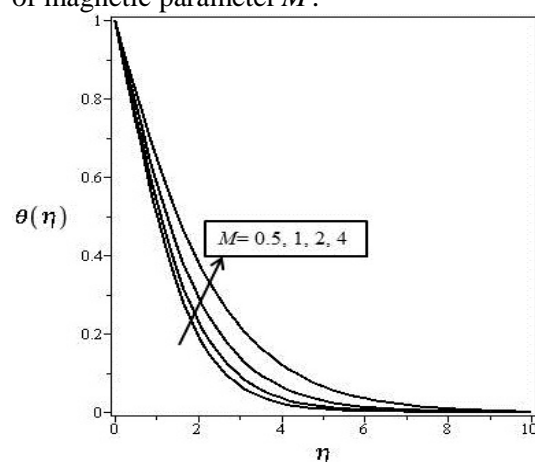
Lorentz force that opposes the fluid motion. Besides, Figure 2 shows that an increase in the magnetic parameter  $M$  leads to an increase in temperature  $\theta(\eta)$  field. Further, this enhancement is very significant near the sheet; however this effect is almost negligible away from the sheet. Figure 3 reveals the temperature profile of effect on radiation parameter. Based on the graph, the result shown that increase the value of  $R$  then increase in temperature profile. The observation shows that temperature increases with an increase in  $R$  because increase the value of radiation parameter provides more heat to fluid that causes an enhancement in the temperature and the thickness of thermal boundary layer. Figure 4, 5 and 6 is plotted to explain the effect of Williamson parameter and Magnetic parameter on skin friction coefficient, local Nusselt number and Sherwood number respectively. The below results show that the skin friction coefficient, local Nusselt number and Sherwood number decreases with increasing the value of  $\lambda$  and  $M$ . Thus, the heat transfer rate at the surface is also decreased since it's represent as local Nusselt number. Figure 7 presents the effect of  $\lambda$  and  $R$  on temperature gradient. The higher the value of  $\lambda$  and  $R$ , the lower the rate of heat transfer as well as boundary layer thickness.



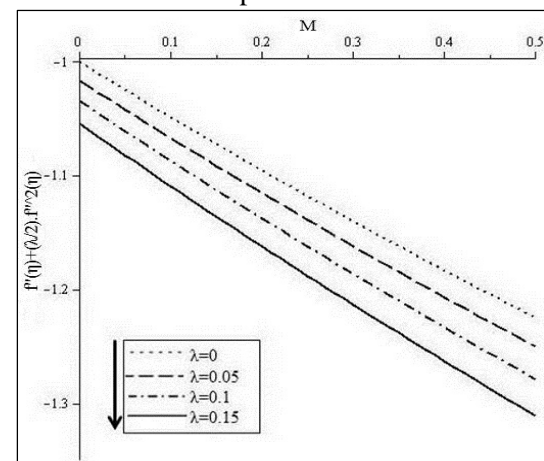
**Figure 1:** Velocity profile for various values of magnetic parameter  $M$ .



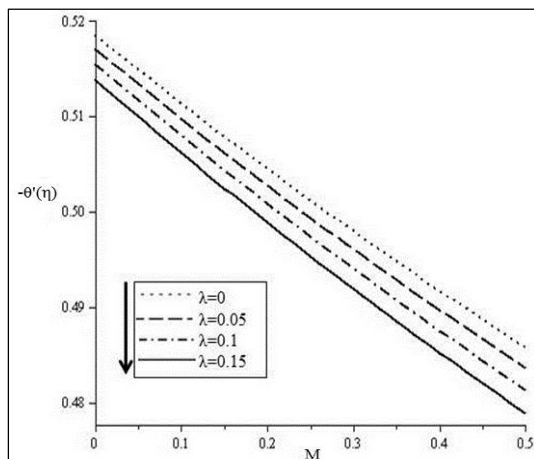
**Figure 3:** Temperature profile for various values of radiation parameter  $R$ .



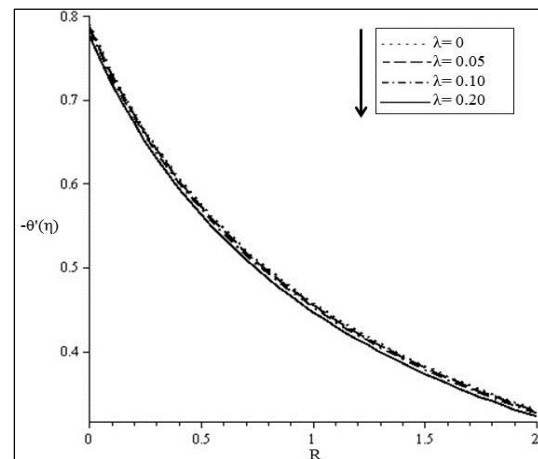
**Figure 2:** Temperature profile for various values of magnetic parameter  $M$ .



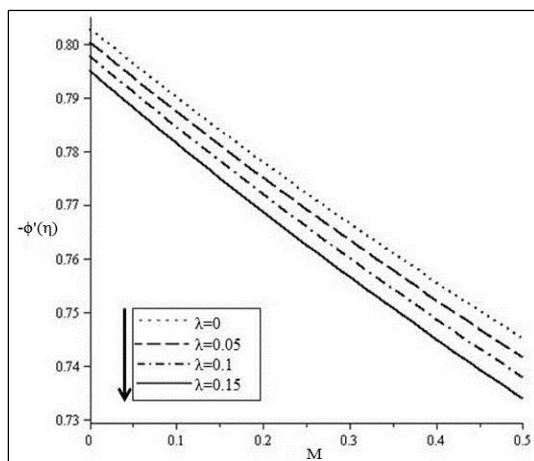
**Figure 4:** The effect of  $\lambda$  and  $M$  on skin friction coefficient.



**Figure 5:** The effect of  $\lambda$  and  $M$  on temperature gradient.



**Figure 7:** The effect of  $\lambda$  and  $R$  on temperature gradient.



**Figure 6:** The effect of  $\lambda$  and  $M$  on concentration gradient.

#### 4. Conclusion

The thermal radiation effect on MHD flow and heat transfer analysis of Williamson nanofluids are studied numerically. The governing non-linear equations are solved by using Shooting method. Numerical outcome for velocity profiles, surface heat transfer rate and nanoparticle volume fraction have been obtained with various ranges of boundary condition and different acceptable values of flow pertinent parameters. The important findings in this research are summarized as follows:

- Boundary layer thickness increase for temperature and concentration profile but velocity distribution decreases with increase in magnetic parameter.
- Temperature profile increases with the increasing values of radiation parameter but decreases the rate of heat transfer.
- All the physical quantities decrease at all the time as  $\lambda$  and  $M$  increase in the Williamson nanofluid.
- The value of temperature gradient decrease at all the time as  $\lambda$  and  $R$  increase.

#### Acknowledgement

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