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Analytical solution for unsteady second grade fluid in presence of non-coaxial rotation

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Abstract. The analytical solution of non-coaxial rotation for second grade fluid is discussed in this paper. The effect of heat and mass transfer through an oscillating vertical disk is also been considered. The Laplace transform technique is used to obtain the solution of velocity, temperature and concentration profiles. The velocity profiles are obtained in two parts of velocity, there are primary and secondary velocities. All pertinent parameters that involved in these profiles are plotted and analyzed graphically. It is found that, the primary velocity first decreases and then increases whereas the secondary velocity is only increased by increasing second grade parameter. From the observation, the primary velocity is higher compared to the secondary velocity for all parameter studied. The solutions of this problem are satisfied the initial and boundary conditions.

1. Introduction

The flow due to non-coaxial rotation of a disk and a fluid at infinity has been considered by a number of researchers. This is because non-coaxial rotation has wide applications in boundary layer control, food processing, mixer machines, stirrers with a two-axis kneader, cooling turbine blades, jet engines, thermal syphon tubes, geophysical flows, pumps and vacuum cleaners. Berker [1] studied the flow between two disks and a fluid at infinity that are rotating non-coaxially. The researcher implied possible to derive an exact solution for such flow. Coirier [2] discussed the fluid flows which driven by a disk and a fluid at infinity that are rotating non-coaxially at slightly different angular velocities. Flow due to non-coaxial rotation of a porous disk and a fluid at infinity was also investigated by Erdogan [3,4]. The researcher found the exact solutions of the governing equation. Recently, Guria *et al* [5] analytically studied the combined effect of Hall currents and the slip condition on the unsteady conducting viscous flow due to noncoaxial rotations of a porous disk and a fluid at infinity. Maji *et al* [6] obtained an exact solution for the unsteady flow due to non-coaxial rotation of a porous disk and a fluid at infinity through porous medium. Mohamad *et al* [7] theoretically examined the unsteady viscous fluid due to non-coaxial rotation over an isothermal oscillating vertical plate. Further, heat and mass transfer in unsteady non-coaxial rotating flow of viscous fluid over an infinite vertical disk was analyzed by Mohamad *et al* [8].

Besides that, some researchers have considered the effect of non-coaxial rotation on Non-Newtonian fluid flows. Hayat *et al* [9] analytically analyzed the flow of a second grade fluid bounded by a porous disk and induced by non-coaxial rotation of the disk. Then, Hayat *et al* [10] extended previous work done by Hayat *et al* [9] to study the effect of a uniform transverse magnetic field on the fluid flows by assuming the fluid is electrically conducting. Recently, Mohamad *et al* [11] studied the effects of non-coaxial rotation, magnetic field and porosity on incompressible second grade fluid mixed convection flow.

2. Formulation of the problem and its solution

Consider a Cartesian coordinate system with an incompressible non-conducting viscous fluid and the heat transfer occurs due to free convection. The x-axis is taken in upward direction along the disk and the z-axis is taken normal to the plane of the disk. The axes of rotation for both disk and fluid are assumed to be in the plane $x = 0$. Initially, at $t = 0$ the disk and fluid at infinity are rotating about z'-axis in the common angular velocity Ω with constant temperature T_∞ and constant mass diffusion C_∞. After time $t > 0$, the disk suddenly executes to oscillate and the rotate about z-axis with uniform angular velocity Ω while the fluid at infinity continues to rotate about z'-axis with the same angular velocity as that of the disk. Meanwhile, the temperature of the disk is raised to constant temperature T_w and constant concentration C_w. The distance between axes of rotation is equal to ℓ. The governing equation of momentum is written as

$$\frac{\partial F}{\partial t} + \Omega i \left(F - \Omega \ell\right) = \left(\upsilon - \frac{\alpha_1 i \Omega}{\rho}\right) \frac{\partial^2 F}{\partial z^2} + \frac{\alpha_1}{\rho} \frac{\partial^3 F}{\partial z^2 \partial t} + g_x \beta_T \left(T - T_\infty\right) + g_x \beta_C \left(C - C_\infty\right), \quad (1)$$

together with energy and mass equations

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial z^2}, \quad (2)$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2}, \quad (3)$$

subjected to initial and boundary conditions

$$\begin{gathered} F\left(z,0\right) = \Omega \ell; \quad \forall z > 0, \quad F\left(\infty, t\right) = \Omega \ell; \quad \forall t > 0, \\ F\left(0,t\right) = UH\left(t\right) \cos\left(\omega t\right) \quad \text{or} \quad F\left(0,t\right) = U \sin\left(\omega t\right); \quad \forall t > 0, \end{gathered} \quad (4)$$

$$T\left(z,0\right) = T_\infty; \forall z > 0, \;\; T\left(0,t\right) = T_w; \forall t > 0, \;\; T\left(\infty,t\right) = T_\infty; \forall t > 0, \quad (5)$$

$$C\left(z,0\right) = C_\infty; \forall z > 0, \;\; C\left(0,t\right) = C_w; \forall t > 0, \;\; C\left(\infty,t\right) = C_\infty; \forall t > 0, \quad (6)$$

where $F = F(z,t) = f(z,t) + ig(z,t)$ is a complex velocity, $f(z,t)$ is a primary velocity, i is an unit vector in the vertical flow direction, $g(z,t)$ is a secondary velocity, υ is the kinematic viscosity, α_1 is normal stress moduli, ρ is the density of fluid, g_x is gravitational acceleration, β_T is the volumetric coefficient of thermal expansion for temperature, $T(z,t)$ is functions of temperature, β_C is the volumetric coefficient of mass transfer, $C(z,t)$ is functions of concentration, k is the thermal conductivity, C_p is the specific heat capacity, D is the mass diffusivity, U is the amplitude of the disk oscillations, $H(t)$ is a Heaviside function, ω is a frequency of oscillation, $sin()$ and $cos()$ are trigonometric functions for sine and cosine cases. In order to non-dimensionalize equation (1) until equation (6), introduce the following non-dimensional variables

$$F^* = \frac{F}{\Omega \ell} - 1, \quad z^* = \sqrt{\frac{\Omega}{\upsilon}} z, \quad t^* = \Omega t, \quad \omega^* = \frac{\omega}{\Omega}, \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}, \quad C^* = \frac{C - C_\infty}{C_w - C_\infty}, \quad (7)$$

and obtained the non-dimensional equations (momentum, energy, mass, initial and boundary conditions) as follow

$$\frac{\partial F}{\partial t}+iF=m_1\frac{\partial^2 F}{\partial z^2}+\alpha\frac{\partial^3 F}{\partial z^2\partial t}+GrT+GmC, \tag{8}$$

$$\frac{\partial T}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial z^2}, \tag{9}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial z^2}, \tag{10}$$

$$\begin{gathered}F\left(z,0\right)=0;\forall z>0,\quad F\left(\infty,t\right)=0;\forall t>0,\\ F\left(0,t\right)=-1+U_0H\left(t\right)\cos\left(\omega t\right)\quad\text{or}\quad F\left(0,t\right)=-1+U_0\sin\left(\omega t\right);\forall t>0,\end{gathered} \tag{11}$$

$$T\left(z,0\right)=0;\forall z>0,\quad T\left(0,t\right)=1;\forall t>0,\quad T\left(\infty,t\right)=0;\forall t>0, \tag{12}$$

$$C\left(z,0\right)=0;\forall z>0,\quad C\left(0,t\right)=1;\forall t>0,\quad C\left(\infty,t\right)=0;\forall t>0. \tag{13}$$

Hence, the non-dimensional parameters are

$$\begin{gathered}m_1=1-\alpha i,\quad \alpha=\frac{\alpha_1\Omega}{\rho\upsilon},\quad Gr=\frac{g_x\beta_T\left(T_w-T_\infty\right)}{\Omega^2\ell},\quad Gm=\frac{g_x\beta_C\left(C_w-C_\infty\right)}{\Omega^2\ell},\\ \Pr=\frac{\mu c_p}{k},\quad Sc=\frac{\upsilon}{D},\quad U_0=\frac{U}{\Omega\ell},\end{gathered} \tag{14}$$

where m_1 is a constant parameter, α is the second grade parameter, Gr is the Grashof Number, Gm is the modified Grashof Number, Pr is the Prandtl Number, Sc is the Schmidt Number and U_0 is the non-dimensional parameter of amplitude of the plate oscillations. Analytical solutions for velocity of the fluid $F_c(z,t)$ (subscripts c refers to cosine oscillations of the disk), temperature $T(z,t)$ and concentration $C(z,t)$ are obtained by Laplace transform technique and they can be expressed in the following form as follow (after simplification)

$$\begin{aligned}F_c\left(z,t\right)=\;&zm_6\int_0^t\int_0^\infty H\left(t-s\right)\cos\left(\omega t-\omega s\right)f_1duds-zm_7\int_0^t\int_0^\infty f_1duds+\\ &zn_6\int_0^t\int_0^r\int_0^\infty\frac{1}{u\sqrt{s}}\exp\left(-\frac{z^2}{4\alpha u}-m_4s-u\right)I_1\left(2\sqrt{m_5us}\right)f_3dudsdr+\\ &\left[U_0H\left(t\right)\cos\left(\omega t\right)-1\right]\exp\left(-\frac{z}{\sqrt{\alpha}}\right)-\frac{m_8}{m_{10}}\int_0^t erfc\left(\frac{1}{2}\sqrt{\frac{\Pr}{r}}z\right)f_2dr+\\ &zm_{11}\int_0^t\int_0^r\int_0^\infty\frac{1}{u\sqrt{s}}\exp\left(-\frac{z^2}{4\alpha u}-m_4s-u\right)I_1\left(2\sqrt{m_5us}\right)f_2dudsdr+\\ &\frac{m_8}{m_{10}}\int_0^t\exp\left(-\frac{z}{\sqrt{\alpha}}\right)f_2dr+\frac{n_3}{n_5}\int_0^t\exp\left(-\frac{z}{\sqrt{\alpha}}\right)f_3dr-\frac{n_3}{n_5}\int_0^t erfc\left(\frac{1}{2}\sqrt{\frac{Sc}{r}}z\right)f_3dr,\end{aligned} \tag{15}$$

$$T\left(z,t\right)=erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}\right), \tag{16}$$

$$C\left(z,t\right)=erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}\right), \tag{17}$$

which is

$$f_1=\frac{1}{u\sqrt{s}}\exp\left(-\frac{z^2}{4\alpha u}-m_4 s-u\right)I_1\left(2\sqrt{m_5 us}\right),\quad f_2=\sinh\left(m_{10}\left(t-r\right)\right)\exp\left(-m_9\left(t-r\right)\right),$$
$$f_3=\sinh\left(n_5\left(t-r\right)\right)\exp\left(-n_4\left(t-r\right)\right), \tag{18}$$

where $m_2=\dfrac{m_1\Pr-1}{\alpha\Pr}$, $m_3=\dfrac{i}{\alpha\Pr}$, $m_4=\dfrac{m_1}{\alpha}$, $m_5=m_4-i$, $m_6=\dfrac{U_0}{2}\sqrt{\dfrac{m_5}{\alpha\pi}}$, $m_7=\dfrac{1}{2}\sqrt{\dfrac{m_5}{\alpha\pi}}$, $m_8=\dfrac{Gr}{\alpha\Pr}$, $m_9=\dfrac{m_2}{2}$, $m_{10}=\sqrt{\dfrac{{m_2}^2+4m_3}{4}}$, $m_{11}=\dfrac{m_8\sqrt{m_5}}{2m_{10}\sqrt{\alpha\pi}}$, $n_1=\dfrac{m_1 Sc-1}{\alpha Sc}$, $n_2=\dfrac{i}{\alpha Sc}$, $n_3=\dfrac{Gm}{\alpha Sc}$, $n_4=\dfrac{n_1}{2}$, $n_5=\sqrt{\dfrac{{n_1}^2+4n_2}{4}}$ and $n_6=\dfrac{n_3 m_7}{n_5}$.

3. Results and discussion

The analytical solutions have been obtained to investigate the combined heat and mass transfer on the free convection flow of an incompressible second grade fluid in non-coaxial rotation past a vertical oscillating disk. In order to understand the physical behavior of non-dimensional parameters such as second grade α, phase angle ωt, Grashof number Gr, modified Grashof number Gm, Prandtl number Pr, time t and Schmidt number Sc, figure 1 until figure 8 are displayed. From this description, figure 1 shows the behavior of α in primary and secondary velocities. The primary velocity decreases for the first time and then increases. This is due to the physical fact of second grade fluid that consist of viscosity and elastic properties. But, the velocity is only increasing in secondary velocity because of less effect on viscosity. The effect of ωt on velocity profiles is presented in figure 2. From this observation, it shows that, the obtained solutions of velocity are satisfied the imposed initial and boundary conditions.

In figures 3 and 4, three different values of Gr and Gm have been chosen. Here, increase of Gr and Gm indicates the small viscous effect in the momentum equation and consequently, enhances the buoyancy force and causes increase in the velocity profiles. The behavior of Pr and Sc are the same as shown in figures 5 and 7. It can conclude that, increase in Pr and Sc will increase the viscosity and the fluid becomes thick and consequently leads to a decrease in temperature and concentration profiles. Lastly, the temperature and concentration profiles increase when the dimensionless time t increases as plotted in figure 6 and 8. Increase in time will increase the amount of energy transfers into fluid flow and leads these profiles to increase.

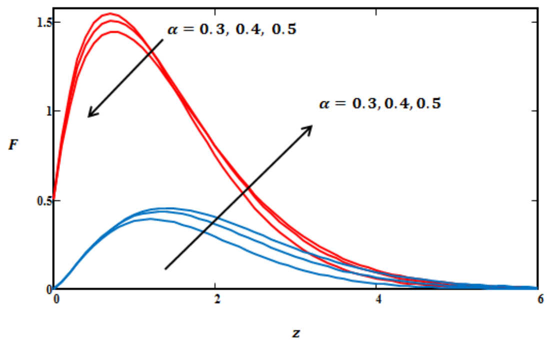

Figure 1. Velocity profiles for different values of α where red line is primary velocity and blue line is secondary velocity.

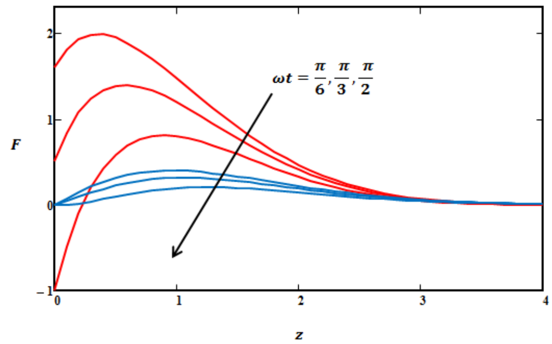

Figure 2. Velocity profiles for different values of ωt where red line is primary velocity and blue line is secondary velocity.

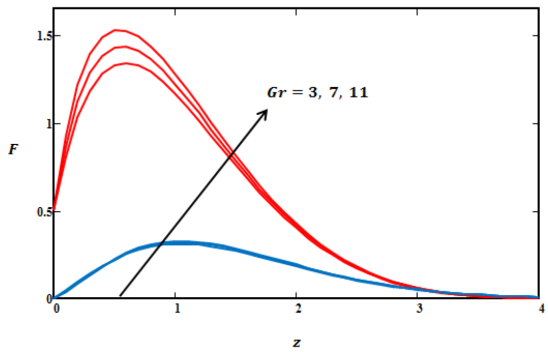

Figure 3. Velocity profiles for different values of Gr where red line is primary velocity and blue line is secondary velocity.

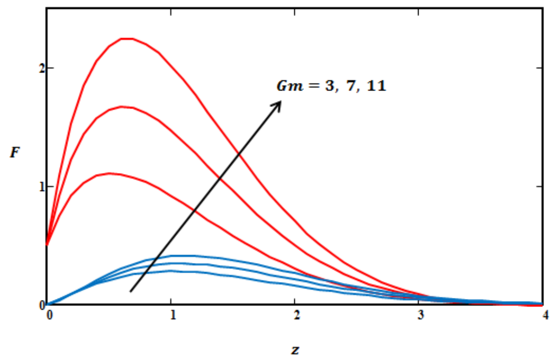

Figure 4. Velocity profiles for different values of Gm where red line is primary velocity and blue line is secondary velocity.

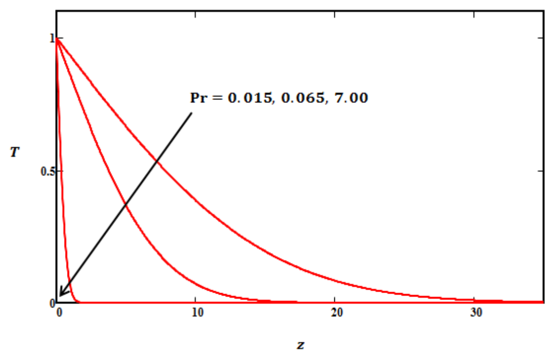

Figure 5. Temperature profiles for different values of Pr.

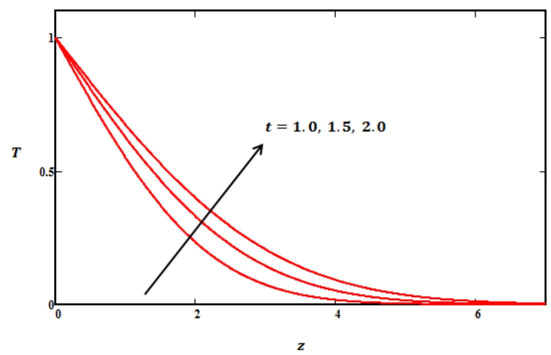

Figure 6. Temperature profiles for different values of t.

4. Conclusion

Analytical solutions of heat and mass transfer with free convection flow of second grade fluid in non-coaxial rotation through an oscillating disk was solved in Section 2. The physical behaviors were also discussed and explained in Section 3 where all the obtained solutions satisfied the

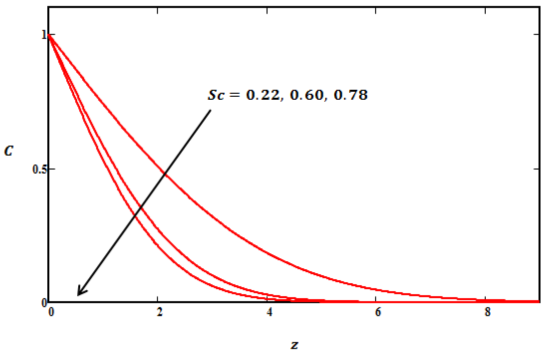

Figure 7. Temperature profiles for different values of Sc.

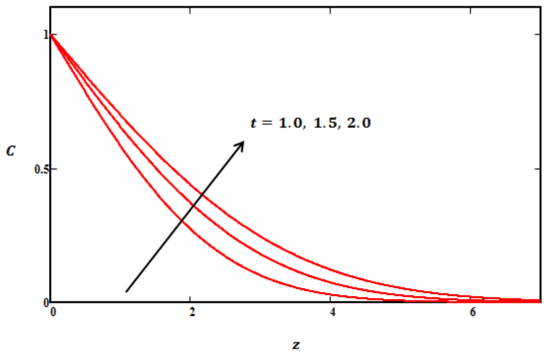

Figure 8. Temperature profiles for different values of t.

imposed initial and boundary conditions. The analytical solutions obtained in this paper provided interesting and complete benchmark to verify numerical schemes for different complex flow situations.

5. Acknowledgments

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References

[1] Berker R 1963 *Hand Book of Fluid Dynamics* vol. VIII/3 (Berlin: Springer) pp 87
[2] Coirier J 1972 *J. Mech.* **87** pp 317-340
[3] Erdogan M E 1976 *Trans. ASME J. Appl. Mech.* **43** pp 203–204
[4] Erdogan M E 1977 *Rev. Roum. Sci. Tech. Mec. Appl.* **22** pp 171-178
[5] Guria M, Kanch A K, Das S and Jana R N 2010 *Meccanica* **45** pp 23-32
[6] Maji S L, Manna G, Guria M and Jana R N 2010 *Chem. Eng. J.* **197** pp 791–803
[7] Mohamad A Q, Khan I, Ismail Z and Shafie S 2016 *SpringerPlus* **5** pp 2090
[8] Mohamad A Q, Khan I, Shafie S, Isa Z M and Ismail Z 2017 *Neural Comput Appl* **2017**
[9] Hayat T, Ellahi R, Asghar S and Siddiqui A M 2004 *Appl. Math. Model.* **28** pp 591-605
[10] Hayat T, Ellahi R and Asghar S 2007 *Chem Eng Commun* **194** pp 37-49
[11] Mohamad A Q, Khan I, Zin N A M, Isa Z M, Shafie S and Ismail Z 2017 *AIP Conf. Proc. Int. Conf.* vol 1830 (AIP Publishing) pp 020044