

**TWO WHEELS MOBILE ROBOT CONTROL USING LINEAR QUADRATIC  
REGULATOR AND POLE PLACEMENT CONTROLLER**

**DONNY ANAK JU**

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**Faculty of Electrical & Electronic Engineering  
University Malaysia Pahang**

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## DECLARATION

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Signature : \_\_\_\_\_

Author : DONNY ANAK JU

Date : 23 NOVEMBER 2009

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## ABSTRACT

There is various type of controller that able to use in control a balancing of two wheels mobile robot. In term to control the two wheels mobile robot, there are four criteria's that need to consider. The four criteria's that control the balancing of two wheels mobile robot are position, speed, angle, and angle rate. From state space equation, two wheels mobile robot is presented using MATLAB application. Linear quadratic regulator (LQR) and Pole place are designed to control the balancing of two wheels mobile robot. Disturbance is applied to angle rate to test the balancing of robot. Simulation on MATLAB application is analyzed and the LQR and Pole placement controller performance are compared to find out the best controller for two wheels mobile robot model.

## ABSTRAK

Pelbagai pengawal yang mampu untuk mengawal keseimbangan robot dua tayar pendulum. Dalam mengawal robot dua tayar pendulum, ada empat kriteria yang perlu dipertimbangkan. Empat kriteria yang digunakan untuk mengawal keseimbangan robot dua tayar pendulum ialah kedudukan, kelajuan, sudut, dan kadar sudut. Daripada persamaan matematik, robot dua tayar pendulum direka menggunakan aplikasi MATLAB. “Linear Quadratic Regulator” dan “Pole Placement” direka untuk mengawal keseimbangan robot dua tayar pendulum. Gangguan diaplikasikan pada kadar sudut untuk menguji keseimbangan robot dua tayar pendulum. Simulasi dalam MATLAB dianalisis untuk mengenalpasti antara dua pengawal “Linear Quadratic Regulator” dan “Pole Placement” yang lebih bagus untuk mengawal keseimbangan robot dua tayar pendulum.

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## CHAPTER 1

### INTRODUCTION

#### 1.1 BACKGROUND

This project is based on controller design and modeling the inverted pendulum robot system in process to control a balancing two wheel of robot. Linear Quadratic Regulator (LQR) controller and Pole Placement controller are developed using mathematical equation to get the feedback to control the speed, position, angle, and angle rate of the inverted pendulum robot using MATLAB application.

The LQR tuning algorithm in robot model is applied to control a balancing two wheel of robot. The performance measure to be minimized contains output error signal and differential control energy. The LQR receives error signal only and it does not need to feedback full states. The Q matrix can be determined from the roots of the characteristics equation. Once the poles for the closed-loop system are assigned, the existence criteria of the LQR controller are derived.

The pole placement is develop based on linear state space model for two wheel of robot and equation  $u(t) = -K_x(t)$ . Using MATLAB application, the pole placement tuning algorithm is applied by consider the value of state feedback gain matrix K and control signal  $u$  by place all closed-loop poles at desired location.

## **1.2 PROBLEM STATEMENT**

As research had been done, there are lots of controllers that are used. But there are some features not same as LQR and pole placement controller features in others controller in process to design a robust control for two wheel of mobile robot.

### **1.2.1 Current controller and software**

There are a lot of controllers which can be used to control the speed of the motor such as Proportional Integral Derivatives (PID) and Fuzzy Logics. For the software, many companies have developed various type of software that related to engineering in this day like MATLAB, Visual Basic and Lab view. However, the problems are:-

The controller like PID need has percentage of overshoot and take some time to its stabilizing the system. It also has the time settling that are can reach more than 1 sec. this will affect the effectiveness of the system

Software like Lab view has problem in term of control system because we need to download separately from the manufacturer's website the control toolkit. Then after that we could use it for the control system. The toolkit is like the add-ons to the Lab view software. For the Visual Basic, we cannot do the simulation to know the result before we do it in the real-time.



### **1.2.2 Problem solving**

Linear quadratic regulator is the more effective controller because it regulates the error to zero and it does not have percentage of overshoot and time settling. So it can stabilize the system quicker than PID.

Pole placement is the most effective controller because of their settling time. Pole placement able to stabilize the system faster compare to others controller. The tuning method is simple and easy to understand.

MATLAB is choose because it is friendly user and do not use complex coding to run it. Just drag and drop the function to use it. It also can simulate the linear system before use in real-time using SIMULINK. In SIMULINK, we need to create the system using block diagram in MATLAB.

## **1.3 OBJECTIVES**

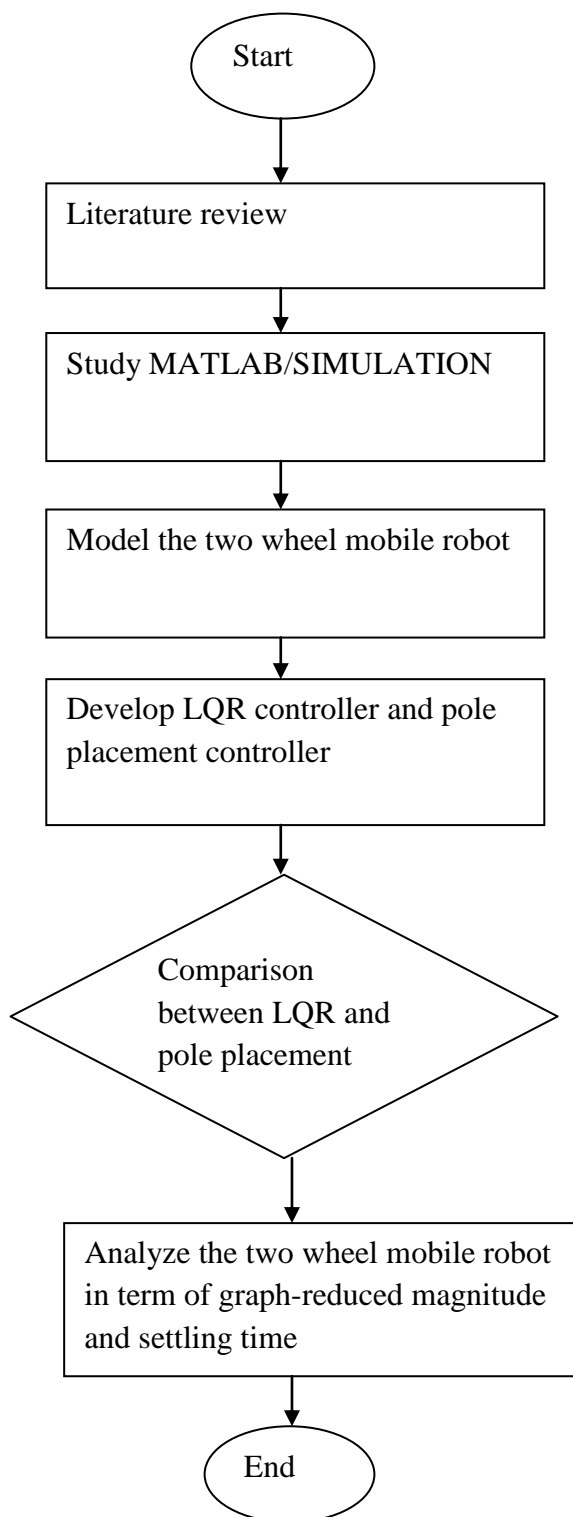
- i. To develop Pole Placement and Linear Quadratic Regulator for balancing control of two wheels mobile robot.
- ii. To compare and analyze performance of Pole Placement and Linear Quadratic Regulator using MATLAB/SIMULINK.

## 1.4 SCOPES

In order to achieve this project, there are several scopes had been outlined:

- i. Two wheel mobile robot model design
- ii. Disturbance applied at angle rate.
- iii. Simulation using MATLAB application of two wheel mobile robot using pole placement and LQR technique and analysis.
- iv. To choose the optimal value of feedback gain in order to grab the stability of the system.
- v. To compare the robustness control between LQR and pole placement controller in term of settling time.

## 1.5 METHODOLOGY



## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 INTRODUCTION.

This chapter includes the study of linear quadratic regulator (LQR), Pole Placement controller, and two wheels mobile robot.

#### 2.2 LINEAR QUADRATIC REGULATOR (LQR).

Linear quadratic regulator or LQR is commonly used technique to find the state feedback gain for a closed loop system. This is the optimal regulator, by which the open-loop poles can be relocated to get a stable system with optimal control and minimum cost for given weighting matrices of the cost function. On the other hand, by using the optimal regulator technique, that freedom of choice is lost for both discrete-time and continuous-time systems, because, in order to get a positive-definite Riccati equation solution, there are some areas where the poles cannot be assigned <sup>[1]</sup>.

A crucial step in the LQR design process is the selection of the quadratic weighting matrices. These matrices determine the Kalman steady state gain and ultimately the state response <sup>[2]</sup>. Algorithms which aid in the selection of the quadratic weights based on some specified criteria are very desirable since they eliminate a trial and error weight selection process. The LQR problem can be solved for either the continuous or discrete time case. Each method yields an optimal gain. These gains are not interchangeable.

The problem presented by the infinite horizon LQR formulation is given a linear n-dimensional state variable system of the form:  $\dot{x}(t) = Ax(t) + Bu(t)$  Compute the m-dimensional control input vector  $u(t)$ , such that the performance index is minimized. Quadratic weighting matrices Q and R are selected by the designer to give appropriate state responses. It is well known that the solution is  $u(t) = -Kx(t)$ , where the Kalman gain, **K**, is given by  $K = R^{-1}B^T P$ , with **P** being the solution to the algebraic Riccati equation,  $0 = A^T P + PA - PBR^{-1}B^T P + Q$ .

The gain vector  $K = R^{-1}B^T P$  determines the amount of control fed back into the system. The matrix R and Q, will balance the relative importance of the control input and state in the cost function (J) being optimized with a condition that the elements in both Q and R matrices are positive values. The size of Q matrix depends on the size of the system's state matrix and R matrix is dependent on the number of control input to the system <sup>[3]</sup>.

Using MATLAB, the algebraic Riccati equation is solved and the control gain K is evaluated for different values of Q and R weighting matrices. The response of the system is simulated as well. The weighting matrix R is a scalar value as there is only one control input to the system. The values in the Q matrix are adjusted according to the

required response of the system a higher value of the weightings indicates the importance of the states <sup>[4]</sup>.

### 2.3 POLE PLACEMENT CONTROLLER.

There a lot of research that already done using pole placement controller. Pole placement is one of the robust controllers that able to control various kind of system model.

The position of poles defines the stability of a system. According to linear control theory (Dorf & Bishop, 2001), the poles of the system can be arbitrarily placed in the complex plane if the Controllability matrix is of full rank. This matrix is defined by,  $C = [B \ AB \ A_2B \ A_3B]$ . The control law for the Pole-placement controller is given as  $u = -Kx$  where  $u$  is the control voltage,  $x$  is the state parameters and  $K$  is the state feedback gain matrix <sup>[5]</sup>.

Pole-placement control gives designers the option of relocating all the closed loop poles of the system. This is in contrast with the classical design using Bode plots and frequency response methods, whereby the designer can only hope to achieve a pair of complex conjugate poles that are dominant <sup>[6]</sup>.

The theory of Pole-placement control might look trivial, but because all other poles and zeroes may fall anywhere, meeting the design specification becomes a matter of trial and error. With the freedom of choice rendered by state feedback comes the responsibility of selecting the poles judiciously. The poles of the system have to be placed carefully as there are obvious costs that are associated with shifting pole

locations. As a result, several simulation trials using MATLAB have to be performed to attain the best pole location that gives the desired response while not straining the control input <sup>[7]</sup>.

In term to use pole placement controller, root-locus control schemes for processes with time delays must be developed. The control design methodologies are based on classical feedback control structures with simple pole placement. The overall behavior control system is adapted along the root loci of the system in each control horizon. The tuning gain can be enlarged to improve the performance of the control system. The optimal tuning gain is repeatedly updated by an optimization technique when the estimated model is confirmed. Every value of pole of locus must be tune to match the modeling system to get a better result <sup>[8]</sup>.

This control scheme combines a system estimation algorithm with pole-placement control design to produce a control law with self-tuning capability. A parametric model with a prediction outputs is adopted for modeling the controlled system <sup>[9]</sup>. Then, a system estimation algorithm which applies a prediction error is employed to identify the parameters of the model. It is shown that the implementation of the estimation algorithm including a time-varying inverse logarithm step size mechanism has an almost sure convergence.

Pole-placement control and prevents the closed-loop control system from occurring unstable pole-zero cancellation. An analysis is provided that this control scheme guarantees parameter estimation convergence and system stability in the mean squares sense almost surely <sup>[6]</sup>. To get the best design of pole placement, MATLAB application is the best choice idea to define the locus of pole to make sure the system can be stabilize well.

## 2.4 BALANCING TWO WHEEL MOBILE ROBOTS.

The inverted pendulum problem is not uncommon in the field of control engineering. The uniqueness and wide application of technology derived from this unstable system has drawn interest from many researches and robotics enthusiasts around the world. In recent years, researchers have applied the idea of a mobile inverted pendulum model to various problems like designing walking gaits for humanoid robots, robotic wheelchairs and personal transport systems.

The paper ‘Cooperative Behavior of a Wheeled Inverted Pendulum for Object Transportation’ presented by Shiroma et al. in 1996 shows the interaction of forces between objects and the robot by taking into account the stability effects due to these forces. This research highlights the possibility of cooperative transportation between two similar robots and between a robot and a human <sup>[10]</sup>.

In the research titled ‘Comparative Study of Control Methods of Single – Rotational Inverted Pendulum’ conducted by Xu & Duan (1997) showed that the LQR controller fared better than the pole placement controller in balancing an inverted pendulum mounted on a rotation arm. This is because the LQR controller offers an optimal control over the system’s input by taking the states of the system and the control input into account. The arbitrary placement of control poles for Pole-placement controllers might cause the poles to be placed too far into the left-hand plane and cause the system susceptible to disturbances <sup>[11]</sup>.

On a higher level, Sugihara et al. (2002) modeled the walking motion of a human as an inverted pendulum in designing a real time motion generation method of a humanoid robot that controls the centre of gravity by indirect manipulation of the Zero