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# Vibration analysis of circular plates in contact with fluid: A numerical approach

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**Abstract.** A mathematical formulation to obtain the non-dimensionalized added virtual mass incremental factor for uniform circular plate in contact with fluid is presented. Based on Rayleigh's quotient, the natural frequencies can be evaluated using the added virtual mass incremental factors. A suitable approximation is used to describe the effect of non-dimensionalized added virtual mass incremental factors in the vibration analysis of circular plates. The numerical results obtained from the present formulation are compared with the known results reported in the literature and a good agreement was found. The maximum error percentage has been predicted from the present formulation is 0.005%.

## 1. Introduction

Numerical analysis of fluid structure interaction problems have wide range of applications in engineering field. The interaction problems can be considered in the signalling problem of submarines, vibration analysis of ship structures, dams, nuclear reactors and so on. The prediction of natural frequency changes is significant for designing structures in contact with fluid because the natural frequencies in fluid are different from those in air. The numerical approaches such as finite element method as well as boundary element method can handle complex boundary conditions of structures and fluids.

Circular plates in contact with fluid have recently been studied. The vibration analysis of circular plates in contact with fluid has been investigated by Kwak [1]. Based on Fourier - Bessel series approach, the non-dimensionalized added virtual mass factor is evaluated using integral transformation technique. It is also studied the influence of fluid on the natural frequencies in this work. Jeong [2] studied the hydroelastic vibration of two identical circular plates coupled with a bounded fluid. An analytical method based on Fourier - Bessel series approach and Rayleigh Ritz method is used in this



work. By using Navier – Stoke’s equations, finite element analysis of fluid flows coupled with structural interactions has been described in [3]. Large displacement fluid structure interaction problems has been solved by Heil [4] using Newton’s method. It is also reported the importance of consistent stabilisation in the numerical simulation of fluid structure interaction problems. Kwak and Kim [5] investigated the axisymmetric vibration of circular plates in contact with fluid. The research is performed to study the effects of fluid on the natural frequencies by evaluating the added virtual mass incremental factor using integral equation method. The natural frequencies of circular plates are evaluated by using added virtual mass incremental factor. Amabili and Kwak [6] considered the free vibration analysis of circular plates in contact with liquids by revising the Lamb problem. The natural frequencies of the circular plate are calculated for both simply supported and clamped boundary conditions. Based on perturbation technique and the Henkel transformation method, the effect of surface waves on free vibrations of circular plates resting on a fluid surface has been presented in [7]. The frequency and mode shape parameters for plates in vacuo are evaluated in this study. Moreover, the frequency parameters for circular plates having free edge, simply supported and clamped boundary conditions are solved by using characteristic equations. The fluid structure interaction problem with vibration of circular plates for varying thickness has been analysed by Liu and Chang [8]. Based on non-linear properties of structures, the fluid structure interaction problem has been solved in [9]. A combined finite element/ boundary element method for fluid structure interaction problems has been presented by Opstal et.al. [10]. The nonlinear flutter behavior of orthotropic composite rectangular plates under aerodynamic pressures and transverse excitations has been presented by Chen and Li[11]. The Hamilton's principle is employed to derive the nonlinear governing equations of motion based on the third-order shear deformation theory and the von-Karman nonlinear strain–displacement relationship. The Galerkin method is utilized to discretize the equations of motion to a set of nonlinear ordinary differential equations. Numerical simulations are conducted to investigate the flutter characteristics and nonlinear dynamics of the composite laminated plate. Shao et.al [12] studied the flutter and thermal buckling behavior of laminated composite panels embedded with shape memory alloy (SMA) wires. The classical plate theory and nonlinear von-Karman strain-displacement relation are employed to investigate the aero elastic behavior of the smart laminated panel. The effects of ply angle of the composite panel, SMA layer location and orientation, SMA wires temperature, volume fraction and prestrain on the buckling, flutter boundary, and amplitude of limit cycle oscillation of the panel are analyzed in this research. The stability and nonlinear vibration of a composite laminated plate with simply supported boundary conditions in subsonic compressible airflow subjected to transverse periodic external excitation are investigated by Yao and Li [13]. The equation of motion of the plate is established by using the von Karman’s nonlinear plate theory. The linear potential flow

theory considering the compressibility of the airflow is adopted to formulate the aerodynamic model. The critical divergence velocity and the flutter velocity of the plate are obtained by analyzing the natural frequencies of the linear system.

The numerical analysis procedures are based on numerous mathematical computations and are highly time consuming. This makes the designers to search for alternative solution procedures with less mathematical complexity and computational efforts. In the present study, the vibration analysis of circular plates in contact with fluid is studied by using suitable approximations and the added virtual mass incremental factors for uniform thickness circular plate are evaluated. The use of numerical formulation can minimise the amount of time spent using experimental techniques to find many design alternatives. It will help in solving large industrial and mechanical problems.

## 2. Mathematical formulation

Consider a circular plate in contact with fluid of radius ' $a$ ' and thickness ' $h$ '. Let ' $F$ ' denotes the fluid domain, ' $S_a$ ' denotes the surface between the fluid and rigid wall, ' $S_b$ ' denotes the surface between the fluid and the plate, and ' $S_\infty$ ' denotes the surface at infinity respectively.

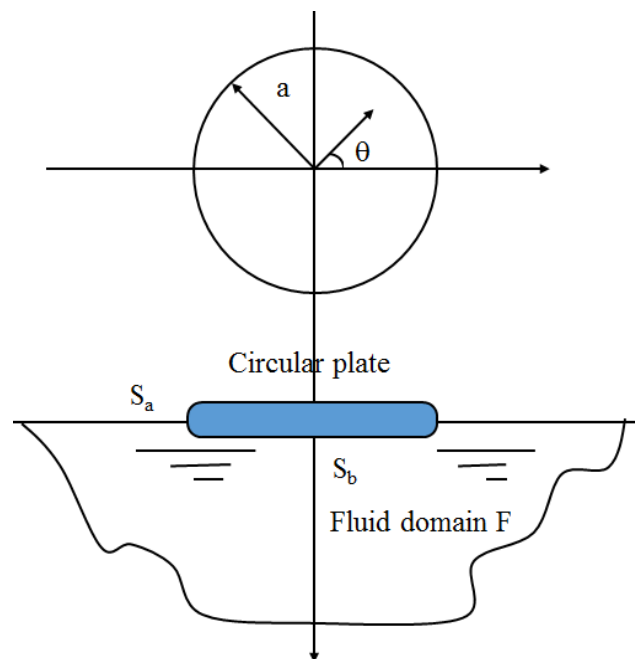


Figure 1. A circular plate in contact with fluid

The differential equation of motion for the circular plate in contact with fluid can be written as

$$D\nabla^4 w + \rho_p \frac{\partial^2 w}{\partial t^2} = 0 \quad (1)$$

where  $w$  is the deflection of the plate,  $D = \frac{Eh^3}{12(1-\nu^2)}$  is the flexural rigidity,  $E$  is the Young's modulus,  $\rho_p$  is the mass density of the circular plate and  $\nu = 0.3$  is the Poisson's ratio.

Suppose that the contacting fluid is incompressible, inviscid and irrotational. Because of the irrotational behaviour of the fluid, there should be a velocity potential, which can be represented as

$$U(r, \theta, z, t) = \phi(r, \theta, z)T(t) = \phi(r, z)T(t) \quad (2)$$

where the spatial velocity potential  $\phi(r, z)$  satisfies the Laplace equation

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0 \text{ in fluid domain F} \quad (3)$$

The condition of the rigid wall on  $S_a$  can be written as

$$\left. \frac{\partial \phi(r, z)}{\partial z} \right|_{z=0} = 0 \text{ on } S_a \quad (4)$$

Furthermore, the interaction between the fluid and the plate can be represented as

$$\left. \frac{\partial \phi(r, z)}{\partial n} \right|_{z=0} = -W(r) \text{ on } S_b \quad (5)$$

where  $W(r)$  is the mode shape of the plate vibrating in contact with the fluid.

For a free surface,

$$\phi, \frac{\partial \phi}{\partial r}, \frac{\partial \phi}{\partial z} \rightarrow 0 \text{ as } r, z \rightarrow \infty \text{ on } S_\infty \quad (6)$$

Using Rayleigh's quotient, we can write

$$f_a^2 \alpha \left( \frac{V_p}{T_p^*} \right)_{air} \text{ and } f_l^2 \alpha \left( \frac{V_p}{T_p^*} + T_l^* \right)_{fluid} \quad (7)$$

where  $f_a$  is the natural frequency of the plate,  $f_l$  is the natural frequency of the plate in contact with the fluid,  $T_p^*$  and  $V_p$  are the reference kinetic energy and maximum potential energy of the circular plate and  $T_l^*$  is the reference kinetic energy of the fluid due to the motion of the plate.

The relation between reference and maximum kinetic energies can be written as

$$T_{max} = T^* \omega^2 \quad (8)$$

where  $\omega$  is the frequency in radians per second. The expression (8) is adapted from [14].

Based on the assumption that the dynamic loading of fluid has an insignificant effect on mode shapes, the following equation can be obtained from eqn (7).

$$f_l = \frac{f_a}{\sqrt{1+\beta}} \quad (9)$$

Here,  $\beta$  is called the added virtual mass incremental factor (AVMI factor) can be defined as the ratio of the kinetic energy of the plate due to the motion of the plate over the kinetic energy of the circular plate itself.

Therefore,  $\beta$  can be written as

$$\beta = \frac{T_l^*}{T_p^*} = \Gamma \frac{\rho_l}{\rho_p} \left( \frac{a}{h} \right) \quad (10)$$

where  $\Gamma$  is called the non-dimensionalized added virtual mass incremental factor (NAVMI factor),  $\rho_l$  is the density of the fluid and  $\rho_p$  is the mass density of the circular plate.

The reference kinetic energy due to the presence of fluid can be evaluated by the velocity potential as

$$T_l^* = -\frac{1}{2} \rho_l \int_0^{2\pi} \int_{-\infty}^{\infty} \frac{\partial \phi(r, \theta)}{\partial n} \phi(r, 0) r dr d\theta \quad (11)$$

The reference kinetic energy of the plate can be written as

$$T_p^* = \frac{1}{2} \rho_p \int_0^{2\pi} \int_0^a h W^2(r) r dr d\theta \quad (12)$$

where  $W(r)$  is the mode shape of the plate.

Suppose that the variable thickness is of the form

$$h(r) = h_0 f(r) ; f(r) = 1 - \mu r^2 \quad (13)$$

where  $h_0$  is the thickness of the center of the plate and  $\mu$  is the taper parameter of the varying curve.

Besides, suppose that

$$w(r, t) = W(r)T(t) = W(r)e^{i\omega t} \quad (14)$$

Therefore, the NAVMI factor can be evaluated from eqn (10) by calculating  $T_l^*$  and  $T_p^*$  and  $f_l$  can be evaluated from eqn (9). By using the NAVMI factor, the natural frequencies can be evaluated.

### 3. Numerical results and discussions

The present study investigates the NAVMI factor of uniform thickness circular plate in contact with fluid. The equations are solved by using a suitable admissible function with boundary conditions. The following admissible function is considered in the present study.

$$F = b_0 \left( 1 - \left( \frac{r}{a} \right)^2 \right)^n$$

It should satisfy the boundary conditions.

$$\text{At } r = 0, \frac{\partial w}{\partial r} = 0; \quad \text{At } r = a, w = 0, \frac{\partial w}{\partial r} = 0$$

In this study, the following parameters for uniform circular plate are used.

$$a = 1\text{m}; h_0 = 0.5\text{m}; \rho_p = 2.44 \times 10^3 \text{ kg / m}^3, \rho_f = 1000 \text{ kg / m}^3$$

The following table represent the values of NAVMI factors for a uniform thickness circular plate. In this table, 'n' represents the order of the mode number and 'N' denotes the number of terms used in the interpolation function to approximate the mode shape. The present numerical results are compared with the numerical results obtained from the known literature. It can be say that the results match well with the known results.

**Table1:** Non-dimensionalized added virtual mass incremental factor for uniform circular plate in contact with fluid.

<i>N/n</i>	1	2	3	4	5
1	0.6830 (0.6826)*	-	-	-	-
2	0.6675 (0.6671)*	0.3155 (0.3153)*	-	-	-
3	0.6673 (0.6668)*	0.2833 (0.2829)*	0.2071 (0.2068)*	-	-
4	0.6670 (0.6668)*	0.2812 (0.2808)*	0.1754 (0.1752)*	0.1535 (0.1530)*	-
5	0.6669 (0.6668)*	0.2806 (0.2800)*	0.1676 (0.1670)*	0.1294 (0.1291)*	0.1198 (0.1192)*

\* denotes the values from the known literature [8].

From Table 1, it can be shown that the values of the NAVMI factor decreases with respect to order of the mode numbers. It is clear that the presence of fluid effects the values of NAVMI factor. Moreover, the existence of fluid has a significant impact on the mode of the circular plate. Also, it can be noted that the values of the NAVMI factor in the first mode has higher values compared to the other modes.

**Table 2:** The percentage error of NAVMI factor for uniform circular plate in contact with fluid to the known results.

$N/n$	1	2	3	4	5
1	0.0005	-	-	-	-
2	0.0006	0.0006	-	-	-
3	0.0007	0.0014	0.0015	-	-
4	0.0003	0.0014	0.0011	0.0033	-
5	0.0001	0.0021	0.0036	0.0023	0.0050

Table 2 describes the percentage error of NAVMI factor for uniform circular plates in contact with fluid to the known literature results. The highest percentage error obtained from reference is 0.0050%. Hence it can be say from the tables that the results from the present investigation are in good agreement with those obtained from known literature.

#### 4. Conclusion

A numerical study has been carried out to obtain the non-dimensionalized added virtual mass incremental factor (NAVMI factor). A suitable approximation with the help of an admissible function is used in the present study to analyse the influence of fluid on the circular plate. The numerical values of NAVMI factor is evaluated for various mode numbers and are compared with the known results. The present numerical results match well with the known literature values. Also, it is clear that the presence of fluid effects the values of NAVMI factor.

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#### Nomenclature

- $a$  = Radius of the plate
- $h$  = Thickness of the plate
- F = Fluid domain
- $S_a$  = The surface between fluid and rigid wall
- $S_b$  = The surface between fluid and plate



- $S_\infty$  = The surface at infinity  
 $w$  = Deflection of the plate  
 $D$  = Flexural rigidity  
 $E$  = Young's modulus  
 $\rho_p$  = Mass density of the plate  
 $\rho_l$  = Density of the fluid  
 $\nu$  = Poisson's ratio  
 $U(r, \theta, z, t)$  = Velocity potential  
 $\phi(r, z)$  = Spatial velocity potential  
 $r, \theta, z$  = Radial, circumferential and axial coordinates  
 $\nabla^2$  = Laplace operator  
 $\phi$  = Scalar function  
 $W(r)$  = Mode shape of the plate  
 $f_a$  = Natural frequency of the plate  
 $f_l$  = Natural frequency of the plate in contact with the fluid  
 $T_p^*, V_p$  = Reference kinetic energy and maximum potential energy of the circular plate  
 $T_l^*$  = Reference kinetic energy of the fluid due to the motion of the plate  
 $\omega$  = Frequency in radians per second  
 $\beta$  = Added virtual mass incremental factor (AVMI factor)  
 $\Gamma$  = Non-dimensionalized added virtual mass incremental factor (NAVMI factor)  
 $h_0$  = Thickness of the center of the plate  
 $\mu$  = Taper parameter of the varying curve  
 $n$  = Order of the mode number  
 $N$  = Number of terms used in the interpolation function to approximate the mode shape

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