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A suitable numerical approximation for the thermal postbuckling behaviour of orthotropic circular plates

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Abstract. In this paper, a numerical study on thermal postbuckling behavior of orthotropic circular plates is presented. The numerical solutions developed in this study are based on von Kàrmàn nonlinear strain - displacement relations. The governing nonlinear differential equations are transformed into linear differential equations. Both simply supported and clamped boundary conditions are considered. The radial edge load of circular plates is evaluated by assuming suitable admissible function for the lateral displacement. The postbuckling loads of orthotropic circular plates are evaluated using the determined linear buckling load and radial edge load. The numerical results obtained from the present investigation are compared with the known results reported in the literature and found in good agreement. The error percentage of the results has been predicted and a maximum error is found out to be 1.37 % for simply supported and 3.08 % for clamped boundary conditions respectively.

1. Introduction

Orthotropic circular plates are commonly used to idealize most of the aerospace, ocean/marine, mechanical and civil structures. Investigations on the thermo mechanical behaviour of the same are gaining important attentions due to their applications and method of evaluation. Vibration behaviour and buckling analysis of the circular plates has been addressed by many researchers using work energy method [1], Rayleigh- Ritz method [2], versatile finite element method [3] and more. The structures are capable of taking additional compressive loads with high deformations. But, if these deformations are tolerable and do not affect the functional requirements, the additional load carrying capacity of these structure, called as the postbuckling load, can be advantageously used in the design process. The amount of deflection can be determined by solving the differential equations of an appropriate plate theory. The stresses in the plate can be calculated from these deflections. Once the stresses are known, failure theories can be applied to determine the capability of plate under a given load [4]. For first time Woinowski [5] introduced numeric results by using Bessel function for the problem of elastic stability of orthotropic circular plates. The thermo elastic buckling behaviour of orthotropic circular plates with composite material properties was also studied [6]. The critical buckling temperatures were evaluated by solving differential equations based on Love Kirchhoff hypothesis and Sander's nonlinear strain displacement relations. The axisymmetric vibration problem of thermally loaded polar orthotropic circular plates with immovable edges was presented using von Kàrmàn plate theory and Hamilton principle [7]. The postbuckling behaviour of moderately thick circular plates with cylindrically orthotropic material properties has been discussed in [8] by using finite element formulation approach.

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In all mentioned studies, these methods need to undergo a complex computational procedure in order to obtain the approximate solutions because of the nonlinear nature of the problem.

The present study makes an attempt to acquire an exact solution for the thermal postbuckling behavior of orthotropic circular plates by proposing a proper approximation. In the succeeding section, the radial edge load and the thermal postbuckling load are calculated by supposing appropriate function for the lateral displacement '*w*'. The foremost benefit of present derivation is it needs only the values of linear buckling load and uniform radial edge load developed due to the lateral displacement '*w*'. Both simply supported and clamped boundary conditions of the circular plate are considered here. The value of Poisson's ratio *v* is taken as 0.3. Besides, the results obtained from the present research are compared with the previously published results evaluated using finite element analysis.

2. Mathematical Formulation

A circular plate of radius '*a*' and of uniform thickness '*t*' under a uniform compressive radial load 'Nr' per unit length at the boundary is considered. The plate exhibits an orthotropy in radial and circumferential directions.

The strain – displacement relations of the circular plate for large lateral axisymmetric displacements based on von Kàrmàn nonlinearities are expressed as

$$\varepsilon_r = \frac{du}{dr} + \frac{1}{2}\left(\frac{dw}{dr}\right)^2 \tag{1}$$

$$\varepsilon_\theta = \frac{u}{r} \tag{2}$$

$$\chi_r = -\frac{d^2w}{dr^2} \tag{3}$$

$$\chi_\theta = -\frac{1}{r}\left(\frac{dw}{dr}\right) \tag{4}$$

where r, θ, $\varepsilon_r, \varepsilon_\theta, \chi_r, \chi_\theta$ are the radial and circumferential coordinates, strains and the curvatures respectively.

By considering the above mentioned strain – displacement relations, strain energy U of the plate with orthotropic material properties [9] can be represented as

$$U = \frac{1}{2}\int_0^{2\pi}\int_{r_1}^{r_2}\left[C_1\varepsilon_r{}^2 + C_2\,\varepsilon_\theta{}^2 + C_{12}\,\varepsilon_r\varepsilon_\theta + D_1\,\chi_r{}^2 + D_2\,\chi_\theta{}^2 + D_{12}\,\chi_r\chi_\theta\right] r\,dr\,d\theta \tag{5}$$

where $C_1 = \frac{E_r h}{1-\nu^2}$, $C_2 = \frac{E_\theta h}{1-\nu^2}$, $C_{12} = \nu_r C_2 = \nu_\theta C_1$, $D_1 = \frac{E_r h^3}{12\left(1-\nu^2\right)}$, $D_2 = \frac{E_\theta h^3}{12\left(1-\nu^2\right)}$ and $D_{12} = \nu_r D_2 = \nu_\theta D_1$.

After substituting the values of $C_1, C_2, D_1, D_2, C_{12}, D_{12}, \varepsilon_r$ and ε_θ in (5), the improved equation can be written as

$$U = \frac{1}{2}\int_0^1\left(\frac{d^2w}{dr^2}\right)^2 + \beta\frac{1}{r^2}\left(\frac{dw}{dr}\right)^2 + 2\nu\frac{\beta}{r^2}\left(\frac{d^2w}{dr^2}\right)\left(\frac{dw}{dr}\right)dr \tag{6}$$

where $\beta = E_\theta / E_r$ be the orthotropic parameter and $E_\theta \neq E_r$.

The numerical expression for work done W by the external load N_r per unit length at the boundary of the element [9] is $W = \frac{1}{2}\int_0^{2\pi}\int_{r_1}^{r_2}\overline{N}_r\left(\frac{dw}{dr}\right)^2 r\,dr\,d\theta$ (7)

By applying the numerical expression of $\overline{N_r}$, equation (7) can be marked as

$$W = \frac{\lambda\beta}{2}\int_0^1 \frac{d^2w}{dr^2}\,dr \tag{8}$$

The total potential energy of the plate can be simply expressed as

$$\Pi = U - W \tag{9}$$

Following the same procedure described in [10], numerical values of linear buckling load can be determined by solving equation (9) for both boundary conditions. The tension parameters can be measured based on the nonlinear strain – displacement relations suggested in equations (1), (2), (3) and (4). Due to the improvement of the edgewise buckled surface of the circular plate, the equations (1) and (2) are used to express N_r.

$$N_r = \frac{\beta t}{1-\nu^2}\left[\frac{du}{dr} + \frac{1}{2}\left(\frac{dw}{dr}\right)^2 + \nu\frac{u}{r}\right] \tag{10}$$

As reported by Berger's approximation [11], the second invariant of the strains are neglected or $\varepsilon_r << \varepsilon_\theta$, the expression for N_r can be stated as $N_r = \frac{\beta t}{1-\nu^2}\left[\frac{du}{dr} + \frac{1}{2}\left(\frac{dw}{dr}\right)^2\right]$ (11)

The exact numerical results are obtained for simply supported immovable edges, u = 0 but $\frac{dw}{dr} \neq 0$ and for clamped edge, u = 0 and $\frac{dw}{dr} = 0$. The uniform radial edge load developed due to large deflections in circular plates gained in non – dimensional form as

$$N_{r_T} = \frac{12}{\beta\left(1-\nu^2\right)}\int_0^1 \left(\frac{dw}{dr}\right)^2 dr \tag{12}$$

By considering appropriate admissible function for the lateral displacement '*w*', the uniform radial edge tensile load N_{r_T} developed in the circular plate can be attained from equation (12) which satisfies the geometric boundary conditions. The admissible function $F = b_0\left(1 - \left(\frac{r}{a}\right)^2\right)^n$ for the lateral displacement '*w*' is considered.

The supposed algebraic function satisfies the given boundary conditions.

(i) *Simply supported:* At r = 0, $w' = 0$; At r = a, $w = 0$

(ii) *Clamped:* At r = 0, $w' = 0$; At r = a, $w = 0$, $w' = 0$.

The values of n = 1 and n = 2 represent the simply supported and clamped boundary conditions, for the function *F* respectively.

If the plate is heated to a temperature ΔT from the stress free plate, an equivalent uniform radial edge compressive load N_r is developed in the plate. When the temperature becomes the critical temperature (ΔT_r), the plate just buckles because of the critical uniform radial edge compressive load $N_{r_{cr}}$ developed. If the temperature ΔT is further raised, lateral displacements of the plate take place and an additional uniform radial edge tensile load N_{r_T} is developed because of the large lateral displacements. This N_{r_T}, for a particular central (maximum) lateral displacement, allows the plate to take more thermal load beyond the critical load or in other words the plate can withstand more equivalent uniform radial edge compressive load N_r beyond $N_{r_{cr}}$. Therefore, the total equivalent compressive uniform radial edge load carrying capacity of the circular plate $\left(N_{r_{NL}}\right)$, namely post buckling load, will be mathematically represented as

$$N_{r_{NL}} = N_{r_T} + N_{r_{cr}} \tag{13}$$

or in the non-dimensional form as

$$\overline{N}_{r_{NL}} = \overline{N}_{r_{Cr}} + \overline{N}_{r_T} \text{ or } \frac{\overline{N}_{r_{NL}}}{\overline{N}_{r_{cr}}} = 1 + \frac{\overline{N}_{r_T}}{\overline{N}_{r_{cr}}} \tag{14}$$

where each term in equation (12) is non – dimensionalised as

$$\overline{N}_r = \frac{N_r a^2}{D} \tag{15}$$

in which D is the plate flexural rigidity $D = \frac{Eh^3}{12\left(1-\nu^2\right)}$.

The ratio of the radial edge tensile load to the linear buckling load parameters, which represent the postbuckling load (γ), can be calculated from equation (14).

$$\gamma = \frac{N_{r_{NL}}}{N_{r_{cr}}} = 1 + c\left(\frac{b_0}{t}\right)^2 \tag{16}$$

When the values of $\overline{N}_{r_T}$ and $\overline{N}_{r_{cr}}$ are known, the postbuckling performance of the orthotropic circular plates can be evaluated. As the values of b_0/t varies, the corresponding postbuckling loads (γ) are evaluated for various values of orthotropic parameter β ranging from 1.2 to 2.0

3. Numerical results and discussions

The thermal postbuckling behavior of circular plates with orthotropic material properties are presented by determining the linear buckling and radial edge loads for numerous values of β. The appropriate admissible function for the lateral displacement '*w*', which satisfies the boundary conditions, is supposed. The radial edge tensile load and the thermal postbuckling load carrying capacity for different β values ranging from 1.2 to 2.0 in steps of 0.2 are tabulated.

The post buckling loads for both simply supported and clamped orthotropic circular plates with the values of b_0/t are included in Tables 1 and 2 which give a clear idea about the thermal postbuckling results. The numerical values attained from the present study are compared with the results from [9] for various β values. It can be observed that the given numerical results are match well with those obtained by [9] which applying the finite element approach. The maximum error percentage from the reference is noticed as 1.37% for simply supported and 3.08% for clamped boundary conditions. It is assumed in Berger's approximation that the strain energy due to the second variant of the middle surface strains can be neglected could be the reason for the much higher values for the simply supported and clamped circular plates.

Also, it can be seen that the error percentage of simply supported circular plates are less than clamped circular plates. The result shows that some has to be cautious in choosing admissible function for deriving nonlinear differential equations of the plates where geometric nonlinearity is involved.

Table 1. Representing the values of postbuckling load 'γ' of simply supported circular plates for the assumed function.

b_0/t	Present results									
	β = 1.2	Error (%)	β = 1.4	Error (%)	β = 1.6	Error (%)	β = 1.8	Error (%)	β = 2.0	Error (%)
0.0	2.2786 (2.2557)*	1.01	2.2621 (2.2466)*	0.69	2.2518 (2.2381)*	0.61	2.2426 (2.2300)*	0.57	2.2354 (2.2224)*	0.59
0.2	2.2879 (2.2661)*	0.96	2.2718 (2.2566)*	0.67	2.2642 (2.2478)*	0.73	2.2507 (2.2393)*	0.51	2.2431 (2.2314)*	0.52

0.4	2.3146 (2.2972)*	0.75	2.3014 (2.2866)*	0.65	2.2875 (2.2767)*	0.48	2.2761 (2.2673)*	0.39	2.2614 (2.2584)*	0.13
0.6	2.3553 (2.3487)*	0.27	2.3476 (2.3363)*	0.48	2.3295 (2.3246)*	0.21	2.3205 (2.3136)*	0.3	2.3198 (2.3032)*	0.72
0.8	2.4536 (2.4203)*	1.37	2.4146 (2.4054)*	0.38	2.4142 (2.3913)*	0.95	2.3817 (2.3780)*	0.16	2.3692 (2.3654)*	0.16
1.0	2.5283 (2.5114)*	0.67	2.5085 (2.4932)*	0.61	2.4976 (2.4761)*	0.87	2.4831 (2.4600)*	0.94	2.4629 (2.4448)*	0.74

* Indicates the reference values taken from [9].

Table 2. Representing the values of postbuckling load 'γ' of clamped circular plates for the assumed function.

b_0/t	Present results									
	$\beta = 1.2$	Error (%)	$\beta = 1.4$	Error (%)	$\beta = 1.6$	Error (%)	$\beta = 1.8$	Error (%)	$\beta = 2.0$	Error (%)
0.0	2.2237 (2.1951)*	1.3	2.2145 (2.1924)*	1.01	2.2095 (2.1894)*	0.92	2.1988 (2.1860)*	0.59	2.1876 (2.1824)*	0.24
0.2	2.2515 (2.2031)*	2.19	2.2416 (2.2003)*	1.88	2.2300 (2.1972)*	1.49	2.2195 (2.1936)*	1.18	2.2084 (2.1899)*	0.85
0.4	2.2865 (2.2271)*	2.67	2.2728 (2.2239)*	2.19	2.2673 (2.2204)*	2.11	2.2529 (2.2164)*	1.65	2.2373 (2.2122)*	1.14
0.6	2.3356 (2.2666)*	3.04	2.3210 (2.2629)*	2.57	2.3122 (2.2587)*	2.37	2.2964 (2.2540)*	1.88	2.2781 (2.2491)*	1.29
0.8	2.3928 (2.3212)*	3.08	2.3799 (2.3167)*	2.73	2.3681 (2.3117)*	2.44	2.3552 (2.3061)*	2.13	2.3333 (2.3002)*	1.44
1.0	2.4557 (2.3902)*	2.74	2.4386 (2.3848)*	2.26	2.4278 (2.3788)*	2.06	2.4153 (2.3720)*	1.83	2.3951 (2.3648)*	1.27

* Indicates the reference values taken from[9].

The complexity of solving related problems are stated in literature earlier using various mathematical methods. This complexity is prompted us to select an innovative substitution based derivation through which postbuckling load can be calculated. The present mathematical analysis is more simple and yet are able to obtain the accurate results.

4. Conclusions

A simple mathematical approximation based on von Kàrmàn nonlinearity to evaluate the thermal postbuckling load of orthotropic circular plate is presented. Both simply supported and clamped boundary conditions are considered. The uniform radial edge tensile load is evaluated by taking suitable assumptions and by introducing stress-free simplifications. The obtained numerical results show a satisfactory agreement with the results attained from the literature with in engineering accuracy.

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