ACTIVE SWAY CONTROL OF A GANTRY CRANE SYSTEM
(SIMULATION USING LQR CONTROLLER)

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JUDUL: **ACTIVE SWAY CONTROLOF A GANTRY CRANE (SIMULATION USING LQR CONTROLLER)**

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This thesis is submitted as partial fulfillment of the requirements for the award of the Bachelor of Electrical Engineering (Electronics)

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Specially dedicated to
My beloved parents, brothers and sisters

"I hereby acknowledge that the scope and quality of this thesis is qualified for the award of the Bachelor Degree of Electrical Engineering (Electronics)"

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ABSTRACT

This project is about the development of Active Sway Control of a Two-Dimensional Gantry Crane (2D – Gantry Crane) System using Linear Quadratic Regulator (LQR) controller. This project analyzes on how to reduce the sway angle of the rode when it is released from certain position. This system is a 2D system because the rode's movement is only along the x and y axes. The 2D Gantry Crane system consists of several elements such as the cart, rode, actuator, payload and controller. The movement of the cart will cause the swaying motion of the rode because the speed of the cart is directly proportional with the sway angle of the rode. If the speed of the cart is increased, the sway angle also increases. The swaying motion will decrease the efficiency of the crane system. In order to increase the efficiency of the crane system, the crane will need an efficient controller to reduce the sway angle. The rode of the gantry crane will be controlled by using LQR controller. LQR is derived by a set of linear differential equation and the cost is described by a quadratic functional. The main objective of LQR controller is to obtain the best performance of dynamic system at the minimum cost. Performance of the system focuses on the sway angle caused by the rode's movement and the Power Spectral Density (PSD) of the sway angle response.

ABSTRAK

Projek ini bertujuan menghasilkan kawalan ayunan dari kren gantri dua-dimensi (2D – kren gantri) menggunakan pengawal Linear Quadratic Regulator (LQR). Projek ini menganalisis tentang bagaimana mengurangkan sudut ayunan batang pemegang dan beban ketika dilepaskan dari kedudukan yang tertentu. Sistem ini adalah sistem 2D kerana pergerakan batang beban adalah sepanjang paksi x dan y sahaja. Sistem kren gantri 2D ini terdiri daripada beberapa elemen seperti batang, pemegang, troli dan beban. Gerakan troli akan menyebabkan gerakan ayunan batang pemegang kerana kelajuan troli adalah berkadar terus dengan sudut ayunan batang pemegang. Jika kelajuan troli bertambah, sudut ayunan juga meningkat. Gerakan ayunan akan mengurangkan kecekapan sistem kren. Bagi meningkatkan kecekapan sistem kren, kren memerlukan pengawal yang cekap supaya dapat mengurangkan sudut ayunan. Batang pemegang kren akan dikawal menggunakan pengawal LQR. LQR diterbitkan oleh satu persamaan linear dan kos diwakili oleh fungsi kuadratik. Objektif utama pengawal LQR ialah bagi mengenal pasti prestasi sistem dinamik pada kos yang minima. Prestasi sistem ditentukan berdasarkan sudut ayunan yang terhasil disebabkan pergerakan batang pemegang kren dan Power Spectral Density (PSD) respon sudut ayunan.

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LIST OF ABBREVIATIONS

LQR	Linear Quadratic Regulator
PSD	Power Spectral Density
M	Mass of trolley (kg)
m	Mass of payload (kg)
x	Position of trolley (m)
$\dot{x}$	Speed of trolley (m/s)
$\ddot{x}$	Acceleration of trolley (m/s^2)
l	Length of rope
θ	Load swing angle (rad)
$\ddot{\theta}$	Angular acceleration of the load swing (rad/s^2)
θ_i	Initial angle of rope (rad)
g	Gravity acceleration (m/s^2)

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CHAPTER 1

INTRODUCTION

In our environment, there is a necessity to transfer objects such as machine, cargo, mechanical components and equipments from one place to another, whether there are far or not. In the workplace, for example, at construction or industrial sites, ports, railway yards and other similar locations, special equipment is needed to transport the materials. These materials are usually heavy, large and hazardous, thus cannot be handling by workers. In order to make the work easier, cranes have been used to lift, move, position or place machinery, equipment and other large objects. [6]

Cranes are widely used for transportation of heavy material in factories, warehouse, shipping yards, building construction and nuclear facilities. In order to lift heavy payloads in factories, in building construction, on ships and elsewhere, cranes usually have very strong structures.

Crane system tends to be highly flexible in nature, generally responding to commanded motion with oscillations of the payload and hook. The response of this system to external disturbances such as wind is also oscillatory in nature. The swaying phenomenon leads to not only lack in efficiency of the crane, but also cause safety problem in a complicated working environment.

Previously, all cranes were manually operated. But manual operation became difficult when cranes became larger, faster and higher. Due to this, efficient controllers are applied into the cranes system to guarantee fast turn over time and to meet safety requirement [3].

1.1 TYPE OF CRANE

A crane consists of a hoisting mechanism such as hook and a support mechanism such as trolley girder. The hoisting mechanism has two main functions. It deposits the payload at the target destination and avoids the obstacle in the path by lifting and lowering the payload. The function of the support mechanism is to move the suspension point around the crane workspace [3].

There are many types of crane that been used for these purposes, such as tower crane, overhead crane, boom crane, gantry crane and others[6]. Crane can be classified based on the degree of freedom which the support mechanism offers the suspension point. There are 3 major types of crane system:

(a) Gantry (overhead) crane

(b) Rotary (tower) crane

(c) Boom crane

1.2 GANTRY CRANE

A gantry crane is composed of a trolley moving in a girder along a single axis. In some gantry crane, the girder is mounted on the second set of orthogonal railings, adding another degree of freedom of the horizontal plane. Gantry crane is commonly used in factories, Figure 1.1[3].

Figure 1.1: Gantry Crane

1.3 TOWER CRANE

Tower crane is commonly used in construction as shown in figure 1.2. The girder rotates in the horizontal plane about a fixed vertical axis. The trolley that holds the load can move in radial position over the girder. The load is attached to the trolley by using a set of cables.

Figure 1.2: Rotary or Tower Crane

1.4 BOOM CRANE

For the boom crane, a boom is attached to a rotating base. The rotational movement of the base along with the elevation movement of the boom places the boom tip over any point in the horizontal plane. The load hangs from the tip of the boom by a set of cables and pulleys. The radial and vertical positions of the load can be changed by manipulating the elevation angle of the boom. Boom cranes are very common on ships and in the harbors. Figure 1.3 shows the example of a boom crane.

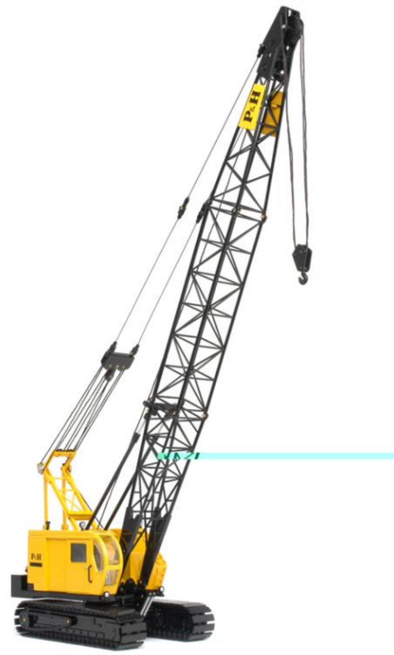

Figure 1.3: Boom Crane

1.5 GANTRY CRANE

There are three main components in a gantry crane which are trolley, bridge and gantry. Figure 1.4 shows a typical gantry crane normally available at ports. Trolley with a movable or fixed hoisting mechanism is the load lifting component. It moves on and parallel to a bridge which is rigidly affixed to a supporting structure called gantry. The gantry extends downward from the bridge to the ground where it can be mobilized on wheels or set of tracks. The motion of the gantry on the ground, the trolley on the bridge and the hoisting of the payload provide the 3 degrees of freedom of the payload.

Figure 1.4: Industrial Gantry Crane

This type of system tends to be highly flexible in nature, generally responding to commanded motion with oscillations of the payload and hook. The response of these systems to external disturbances, such as wind, is also oscillatory in nature. The swaying phenomenon introduces not only less in efficiency of the crane, but also cause safety problem in the complicated working environment.

Gantry crane is similar to an overhead crane, except that the bridge for carrying the trolley or trolleys is rigidly supported on two or more legs running on fixed rails or other runway. To implement the operation, the crane operator will seat inside the cart, and move the cart with the load hanged with it, so that the load can achieve the desired location. A real crane may allow a cart movement of 80 to 90 meters [8], regarding on the desired load location. Figure 1.5 and Figure 1.6 show the illustration of overhead crane and gantry crane respectively.

Figure 1.5: Overhead crane

Figure 1.6: Gantry Crane

1.6 GANTRY CRANE ACCIDENT REPORT

For a crane operator, an experience causing by a crane's accidents can be frightening them. There are many cases and incident regarding on the crane's accidents. For example, in April 1993, the crane becomes unbalanced during two separate incidents at DOE sites in United States of America, which is in Hanford Site and Bryan Mound Site. The first incident occurred in 28th april 1993, where a crane becomes unbalanced while the boom was being lowered. The second incident occurred 2 days later, on 30th

April 1993, which while loading the load, the weight of the load caused the crane to tip forward [1]. From these incidents, guidelines have been suggested in using the cranes. Some of the guidelines are:

(i) the weight of load must be checked.

(ii) crane operations should be supervised by qualified personnel.

(iii) crane operators must be familiar with their equipment.

(iv) crane operators must be trained and qualified to operate their equipment.

Although the guidelines have been sketched in order to prevent the accident, the other factors also must be considered so that the probability of accidents occurs is small or reduced at an acceptable value. There are many factors that have to be considered such as the braking systems, hydraulic and pneumatic components, electrical equipment, operational aids, operating mechanisms, lifting devices, determining load weight, recognizing immediate and potential hazards, control systems and others. In term of control systems, the important issue is how to control the load swing. This is important in order to have a faster operation while maintaining the safety.

In 14th January 2004 at approximately 1300 hours, a sling of dunnage, a cargo spreader and pulley frame that were attached to the No. 2 after gantry crane on *Tasman Independence* fell from their suspended position onto the quay. There was damage to the crane, cargo gear equipment and the quay. Figure 1.7 shows pictures about the accident.

Figure 1.7: Gantry crane's accident

1.7 NEED FOR THE STUDY

From the previous works, it seems that most researchers have given a lot of efforts in developing a control algorithms and designing controllers that can be used and realized in nature. This includes the study related on how to reduce the vibration, especially in crane, where the controllers that been designed are mostly to control the load swing. Since this is relatively simple and well defined problem in dynamics and control, it is surprising that, it has not been solved exactly, where an exact solution is here understood to be a control strategy that guarantees complete success in a finite time. Most of the crane controllers that have been developed until now have been far from satisfactory. Once tested in actual operation, there found to be ineffective and thus were left unused. This may due to the standard control feedback strategies that are not well suited to this problem. Therefore, the problem of controller synthesis for a crane is still under consideration.

Regarding on this matter, in this study, it seems interesting if multiple point of view can be taken in modeling the crane. For this purpose, gantry crane has been chosen in order to achieve the aim. This will involves in determining the relation between the cart' mass, load's mass and the load swing, in order to looking after the effect of the cart and load's mass to the load oscillation. Because the operation of the gantry crane is related with the movement of the cart and load, the effect that cause a vibration will be, whether from the acceleration that been applied at the cart, or the load and cart's inertia that been exists because the movement of these objects.

1.8 ADVANTAGES OF GANTRY CRANE

There are several advantages of a gantry crane such as:

(i) It has mechanical advantage to move loads beyond the normal capability of a human

(ii) It can be used both to lift and lower materials and to move them regarding to the type of crane

(iii) It provides effective way to transfer load (faster and easier)

1.9 DISADVANTAGES OF GANTRY CRANE

Besides that, a gantry crane system also has some disadvantages:

(i) Swaying motion (cause harmness)
(ii) Can't efforts lifting heavy load
(iii) Heavier load needs huge crane and big mechanical systems

1.10 PROBLEMS STATEMENT

In real life, the usage of gantry crane is less in efficiency. This is due to the movement of the cart/trolley. We want to transfer loads as fast as possible, but the increasing of the speed will also increase the swaying movement of the rode and will reduce the efficiency and safety.

Besides the sway angle problem, size of the gantry crane itself is also a major problem. Size of a gantry crane is depending on the size and weight of the load. Bigger load during transfer motion is good and save transfer duration rate and decrease the cost, but the bigger load will need huge crane. Huge crane will need an addition to the cost.

Finding the right and ideal tuning gains are also the major problem for this project. Mistake in finding the right gains will cause the controller to act in less efficiency. So, good gains will give the best performances of the controller

1.11 OBJECTIVE

i. To reduce the sway angle

When the cart is moving, the pendulum will move and will produce the sway angle. The swaying motion of pendulum will cause harmness to environment. So, a controller is needed to reduce the sway angle of the pendulum

ii. To develop effective controller using LQR

Linear quadratic regulator (LQR) controller will be developed in order to reduce the sway angle

iii. To transfer load as fast as possible without causing oscillations

Cart that carry load will increase the sway angle. So, we need to develop controller that will decrease the transfer time and reduce the sway angle at the same time

iv. To study the performances of the controller of the rope

Analyze performance of the developed controller and make improvement from time to time in order to increase the efficiency of the controller

1.12 SCOPE OF PROJECT

There are several scopes of this project:

i. Project understanding about the overall project, the overall idea about the system and controller.

ii. Analysis of 2 Dimensional gantry crane systems. This includes the system performances, mathematical model, problems, and other related tasks.

iii. Study and analysis of Linear Quadratic Regulator (LQR) controller including the implementation process of the controller and mathematical algorithm.

iv. Develops the effective LQR controller using MATLAB and analyze the performances.

v. Performances analysis in MATLAB and improve the performances by manipulating the controller.

1.13 THESIS OUTLINE

Thesis outline is about the overall outline and structure of this thesis. This thesis consists of five chapters; chapter 1, introduction, chapter 2, literature review, chapter 3, methodology, chapter 4, result and discussion and chapter 5, conclusion.

Chapter 1 discussed about introduction of the thesis and project. An overview of the system and control algorithm will be explained including problem statement, project objective, project scopes and others.

Chapter 2 discussed about literature review of previous thesis, papers and works done by other people that related to my thesis. Literature review will cover about gantry crane system and controller algorithm.

Chapter 3 discussed about work methodoly that of this project. Each step to finish the project and thesis will be explained detail including the project flow chart, modelling gantry crane system and deriving control algorithm.

Chapter 4 consists of experimental results and result analysis. All the discussion and analysis will be explained in this chapter including the comparison of the data.

Chapter 5 summerizes the overall conclusion of the project including the system limitation and suggestion for system improvement.

CHAPTER 2

LITERATURE REVIEW

2.1 GANTRY CRANE

A crane has been used to move a load from one location to another. When the load moves, the crane must be controlled so that the load reaches the location needed with less or without swinging. In operating the crane, the swing that occurs is seemly like a pendulum motion, which is at a certain level, the operation must be stopped until the swing or oscillation become disappears. For example, in the manufacturing industries, many lifting cycles has been performed, and this required an accurate and efficient equipment to do all of this work[6]. Furthermore, designed controller is not only used to control the sway of the cranes, but also to control load positioning[17].

In general, because of several reasons, either because of inertial coupling effect between different motions of the moving components, the oscillation or vibration in machine is usually nonlinear. This will lead to be unsuitable condition for practical purpose because the machine is having a behavior different from the designer intend[9]. For certain mechanical system where precise motion is needed, this will lead to a serious problem [10].

The main objective of this project is to design robust, fast, and practical controllers for gantry cranes. The controllers are designed to transfer the load from point to point as fast as possible and, at the same time, the load swing is kept small during the transfer process and completely vanishes at the load destination. Moreover, variations of the system parameters, such as the cable length and the load weight, are also included. Practical considerations, such as the control action power, and the maximum acceleration and velocity, are taken into account [7].

To ensure that the developed control algorithm is appropriate and suite with the focused problem, one aspect that cannot be ignored is the model of gantry crane itself. Some researchers take the characteristic of pendulum as their model to derive dynamic equation that representing the gantry crane. At this point, more attention is needed because all of the processes forward will be based on the developed model. Therefore, this point becomes an interest for researchers to do a work related on this, and they have come out with their suggested model, where a lot of consideration and factors have been taken on their models.

One of the works regarding on modeling to calculate the dynamics response of structure to moving loads, that been implemented by Wu and members [11]. They have taken a mobile gantry cranes as a model, and the dynamics response characteristics has been simulated, and then an improvement has been implemented to it. In order to improve the model, the model has been divided into two parts, the static framework and the moving sub-structure and the finite element techniques have been used in order to model the system.

The payload dynamics has been examined under three different situations in order to determine the effect of the length of the pendulum, the effect of the mass of the carriage and the load, and the effect of carriage speed[12]. The equation of motion has been derived by using Hamilton's principles and operational calculus is used to determine the vibration of the beam, and hence to get the dynamics of suspended load.

Several works has been done in develop a new practical system such as using numerical solution to find a way with many non-ideal dynamics effect of the crane[13]. The model that been used is gantry crane, which is in practice, the operator, which combining intuition, experience and skill, will locate the load hanging on the cable by stopping the trolley somewhat short of the target position and then letting the load move to that location by a further movement of the trolley. This makes sense on how to develop the automatic control based this situation, which combining understanding, quantification, automation and then optimization. In other words, the gantry a controller has to learn from the previously unknown dynamics response in the first part of the motion exactly how to terminate the motion, i.e. self adapting, even the system dynamics become more complex.

2.2 LINEAR QUADRATIC REGULATOR

The linear quadratic regulator (LQR) is a well-known design technique that provides practical feedback gains [5]. Linear Quadratic Regulator (LQR) theory are used to achieve enhanced closed loop performance and stability characteristics with full state feedback[15]. . LQR method determines the feedback gain matrix that minimizes cost function, *J* in order to achieve some compromise between the use of control effort, the magnitude, and the speed of response that will guarantee a stable system[16].

For the derivation of the linear quadratic regulator, assume that the plant to be written in state-space form as: $\dot{x} = Ax + Bu$

And that all of the n states x are available for the controller. The feedback gain is a matrix K of the optimal control vector

$$u(t) = -Kx(t)$$

so as to minimize the performance index

$$J = \int_0^\infty x(t)^T Qx(t) + u(t)^T Ru(t)dt$$

Where Q is a positive-definite (or positive-semidefinite) Hermitian or real symmetric matrix and R is a positive-definite Hermitian or real symmetric matrix. Note that the second term on the right-hand side of the Equation 3 accounts for the expenditure of the energy of the control signals. The matrices Q and R determine the relative importance of the error and the expenditure of this energy. In this problem, assume that the control vector **u(t)** is unconstrained.

The linear control law given by Equation (2) is the optimal control law. Therefore, if the unknown elements of the matrix K are determined so as to minimize performance index, the $u(t) = -Kx(t)$ is optimal for any initial state $x(0)$. The determination of state feedback gains in design methods are easier to obtain by using LQR method[16]. The block diagram showing the optimal configuration is shown in figure 2.1 :

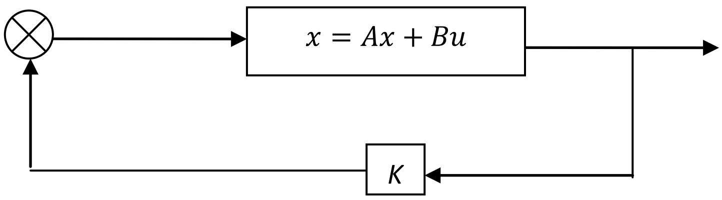

Figure 2.1 : LQR block diagram

2.2.1 BASIC BLOCK DIAGRAM OF LQR SYSTEM

A plant with state space model as :

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

y is the output that would like to be controlled. LQR objective is to regulate / make the output y to zero with minimum input. This objective is achieved by designing a feedback gain K which can minimize the cost function

$$J = \int [y'(t)Qy(t) + u'(t)Ru(t)]dt$$

Q and R is the matrix weighing, and both of this matrix must be positive-definite symmetric. This cost function is in the form of a quadratic equation and this is the origin of a Linear Quadratic Regulator. Figure 2.2 shows basic block diagram of LQR system in MATLAB.

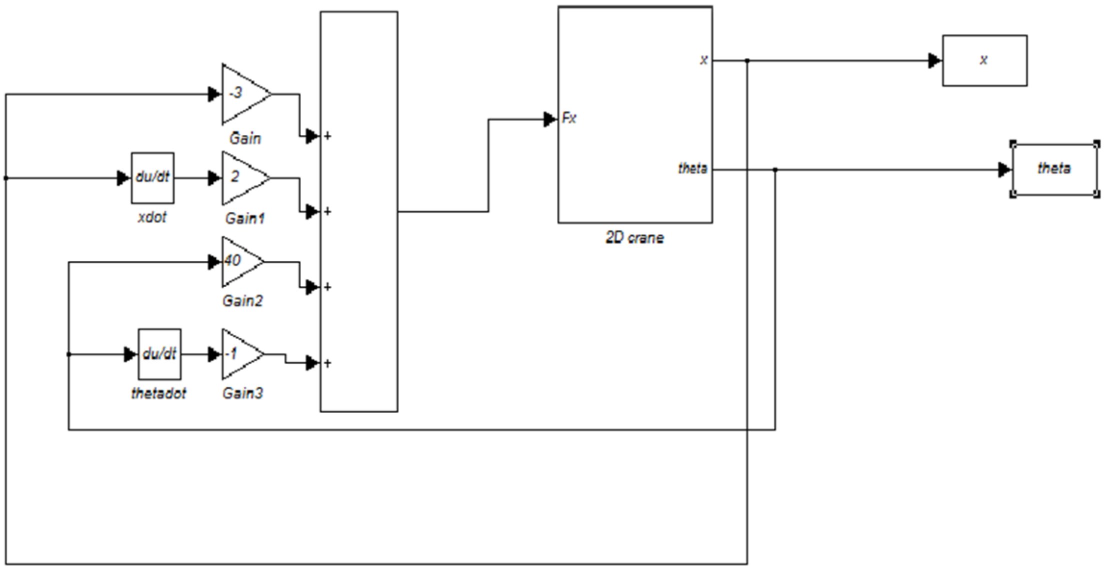

Figure 2.2: Basic LQR system

CHAPTER 3

METHODOLOGY

Work methodology is a work progress that used in construction to describe a document that gives specific instructions on how to perform a work related task, operate a plant or equipment. This chapter will discuss about work methodology and process to complete the experiment or the task.

Overview about the gantry crane system is important before starting the project. It will give an overall idea on analysis about the system. Developing mathematical model of the system is also important in order to analyze the system process and performances.

Effective controller can be developed by understanding the control algorithm (LQR). In this chapter, LQR controller will be analyzed and developed using MATLAB.

Analyze and investigate the controller performances to the system is important in order to achieve best performances. The developed controller will be improved from time to time according to the system's performances. In term of work methodology, it can be summarized as in figure 3.1.

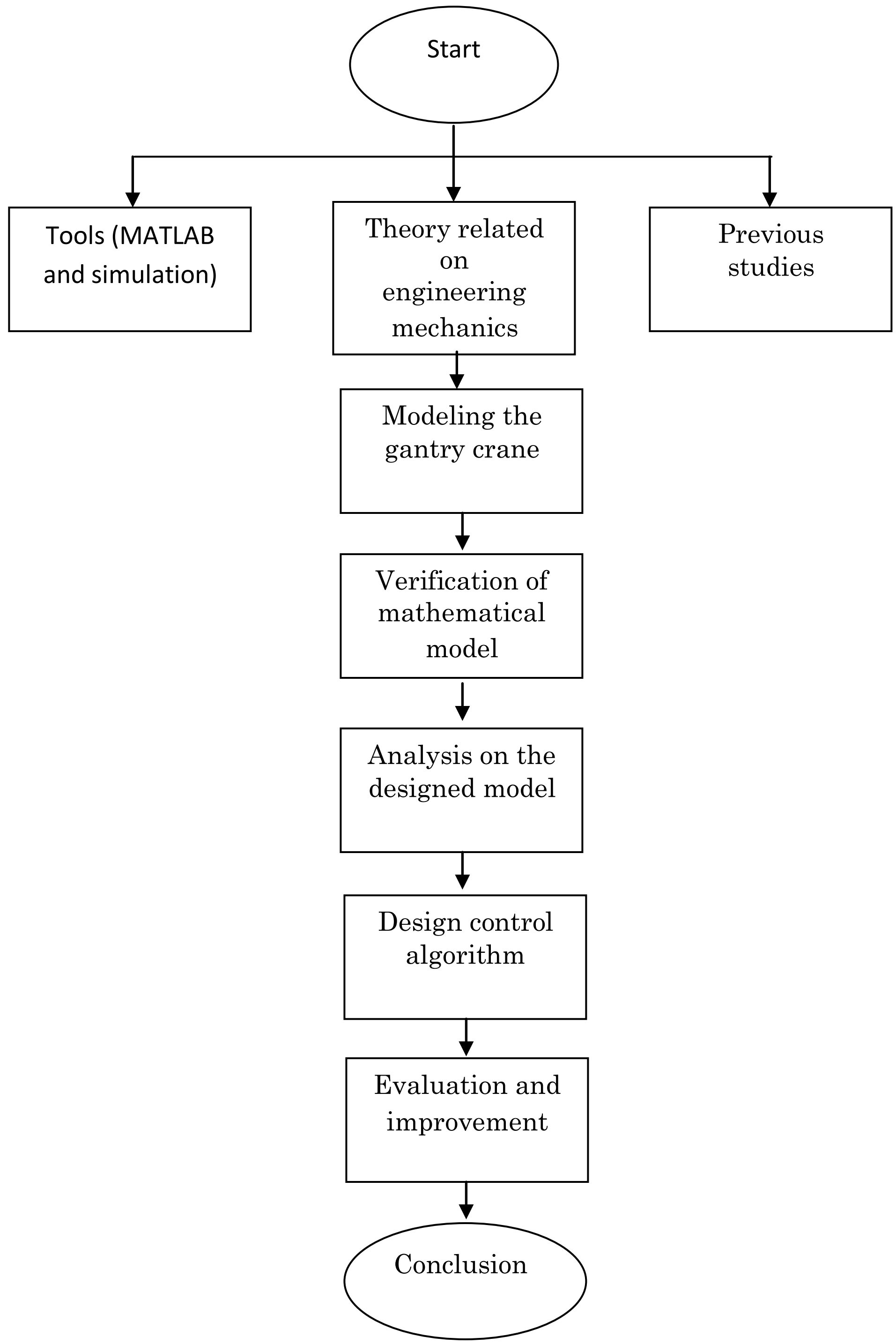

Figure 3.1: Project flow chart

3.1 GANTRY CRANE MODEL DESCRIPTION

Figure 3.2 shows the model of gantry crane in 3-Dimensional

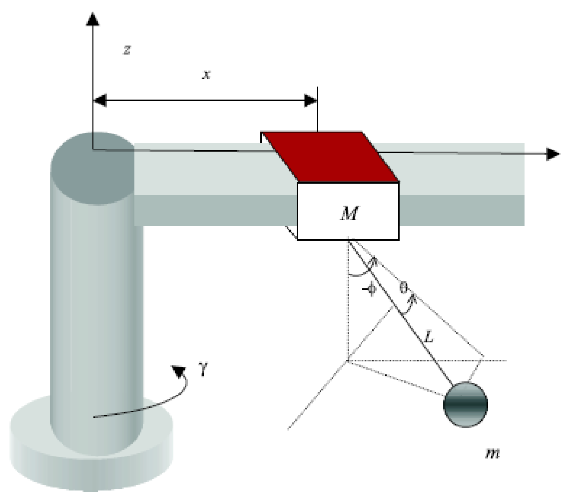

Figure 3.2: Gantry crane in 3-Dimensional

Figure 3.3 shows the model of a gantry crane, where,

x = Horizontal position of the trolley (m)

l = Length of the rope (m)

θ = Sway angle of the rope (rad)

M = Mass of the trolley (kg)

m = Mass of the payload (kg)

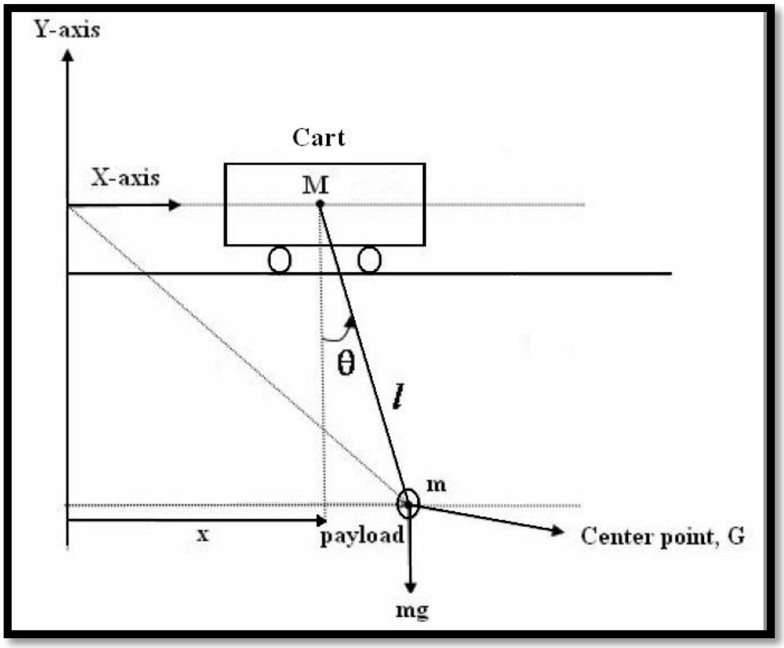

Figure 3.3: Model of Gantry Crane

Below are the assumptions made to simplify the process system modeling :

(i) Ignored the trolley friction force

(ii)The trolley and the payload can be considered as point masses

(iii) Ignored the tension force that may cause the hoisting rope elongate

(iv) The trolley and the payload are assumed to move in two dimensional only, x-y plane

The kinetic and potential energy of the whole system are given by :

Kinetic Energy, $T = T_{trolley} + T_{payload}$

$$= \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m(\dot{x}^2 + \dot{l}^2 + l^2 \dot{\theta}^2 + 2\dot{x}\dot{l}\sin\theta + 2\dot{x}l\dot{\theta}\cos\theta$$

Potential energy of the beam,

$U = mgy_m$

$= -mgl\cos\theta$

Using the Lagrangian approach, the following equation is derived as

$L = T - U$

$$= \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m(\dot{x}^2 + \dot{l}^2 + l^2 \dot{\theta}^2 + 2\dot{x}\dot{l}\sin\theta + 2\dot{x}l\dot{\theta}\cos\theta + mgl\cos\theta$$

Let the generalized forces corresponding to the generalized displacements $\bar{q} = \{x, \theta\}$ be $\overline{F} = \{F_x, 0\}$. Constructing the Lagrangian $L = T - U$ and using Lagrangian's equations [2]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_j}\right) - \frac{\partial L}{\partial qi} = F_j$$

Where, i = 1, 2

We will obtain the equation of motion for the gantry crane system. Firstly, the equation of motion associate with the generalized coordinate $q = x$ can be derived as below [2].

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F_x$$

$$\frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial \dot{x}} = M\dot{x} + \frac{1}{2}m(2\dot{x} + 2\dot{l}\sin\theta + 2l\dot{\theta}\cos\theta)$$

$$= M\dot{x} + m\dot{x} + m\dot{l}\sin\theta + ml\dot{\theta}\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = M\ddot{x} + m\ddot{x} + m(\ddot{l}\sin\theta + \dot{l}\dot{\theta}\cos\theta) + m[\dot{l}\dot{\theta}\cos\theta + l\ddot{\theta}\cos\theta + l\dot{\theta}(-\sin\theta)\dot{\theta}]$$

$$= (M + m)\ddot{x} + m(\ddot{l}\sin\theta + \dot{l}\dot{\theta}\cos\theta) + m(\dot{l}\dot{\theta}\cos\theta + l\ddot{\theta}\cos\theta - l\theta^2\sin\theta)$$

$$= (M + m)\ddot{x} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) + 2m\dot{l}\dot{\theta}\cos\theta + m\ddot{l}\sin\theta$$

Thus,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - 0 = F_x$$

$$F_x = (M + m)\ddot{x} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) + 2m\dot{l}\dot{\theta}\cos\theta + m\ddot{l}\sin\theta$$

Secondly, the equation of motion associate with the generalized coordinate $q = \theta$ is as below [2]

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}}) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \theta} = \frac{1}{2}m[2\dot{x}\dot{l}\cos\theta + 2\dot{x}l\dot{\theta}(-\sin\theta)] + mgl(-\sin\theta)$$

$$= \frac{1}{2}m(2\dot{x}\dot{l}\cos\theta - 2\dot{x}l\dot{\theta}\sin\theta) - mgl\sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2}m(2l^2\dot{\theta} + 2\dot{x}l\cos\theta)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{1}{2}m\left[2l^2\ddot{\theta} + 2\dot{l}\dot{\theta}(2l) + 2\ddot{x}l\cos\theta + 2\dot{x}\dot{l}\cos\theta + 2\dot{x}l\dot{\theta}(-\sin\theta)\right]$$

$$= \frac{1}{2}m(2l^2\ddot{\theta} + 4l\dot{l}\dot{\theta} + 2\ddot{x}l\cos\theta + 2\dot{x}\dot{l}\cos\theta - 2\dot{x}l\dot{\theta}\sin\theta)$$

Thus,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{1}{2}m(2l^2\ddot{\theta} + 4l\dot{l}\dot{\theta} + 2\ddot{x}l\cos\theta + 2\dot{x}\dot{l}\cos\theta - 2\dot{x}l\dot{\theta}\sin\theta) - \frac{1}{2}m(2\dot{x}\dot{l}\cos\theta - 2\dot{x}l\dot{\theta}\sin\theta) + mgl\sin\theta = 0$$

$$2l^2\ddot{\theta} + 4l\dot{l}\dot{\theta} + 2\ddot{x}l\cos\theta + 2\dot{x}\dot{l}\cos\theta - 2\dot{x}l\dot{\theta}\sin\theta - 2\dot{x}\dot{l}\cos\theta + 2\dot{x}l\dot{\theta}\sin\theta + 2gl\sin\theta = 0$$

$$2l\ddot{\theta}+4\dot{l}\dot{\theta}+2\ddot{x}\cos\theta+2g\sin\theta=0$$

$$l\ddot{\theta}+2\dot{l}\dot{\theta}+\ddot{x}\cos\theta+g\sin\theta=0$$

The equations of motion of the gantry crane model associated with the generalized coordinates $\bar{q}=\{x,\theta\}$ can be summarized, respectively as [2]

$$x: \quad F_x=(M+m)\ddot{x}+ml(\ddot{\theta}\cos\theta-\dot{\theta}^2\sin\theta)+2m\dot{l}\dot{\theta}\cos\theta+m\ddot{l}\sin\theta$$

$$\theta: \quad l\ddot{\theta}+2\dot{l}\dot{\theta}+\ddot{x}\cos\theta+g\sin\theta=0$$

3.1.1 LINEARIZATION OF THE SYSTEM

The above derived model is a nonlinear model. The nonlinear model has to be linearized to simplify the progress of the modelling.

For safe operation, two assumptions had been made. First we assume that the swing angle should be kept small [2]

$$\theta\approx 0$$

$$\dot{\theta}\approx 0$$

In this study, we assume that changing the rope length is needed only to avoid obstacles in the path of the load. This change can be considered small too.

$$\dot{l}\approx\ddot{l}\approx 0$$

Using these two assumptions, the simplified equation of motion for the gantry crane system can be obtained.

$$x: \quad F_x = (M+m)\ddot{x} + ml\ddot{\theta} \tag{1}$$

$$\theta: \quad l\ddot{\theta} + \ddot{x} + g\theta = 0 \tag{2}$$

3.1.2 STATE SPACE REPRESENTATION OF THE SYSTEM

After getting the linearized equation, the equations can be written in state space representation.

$$\dot{x} = Ax + Bu$$

Where, $x = [x \quad \theta \quad \dot{x} \quad \dot{\theta}]^T$

$$\dot{x} = [\dot{x} \quad \dot{\theta} \quad \ddot{x} \quad \ddot{\theta}]^T$$

From equation (2),

$$\ddot{x} = -l\ddot{\theta} - g\theta \tag{3}$$

Substituting equation (3) into (1),

$$F_x = (M+m)(-l\ddot{\theta} - g\theta) + ml\ddot{\theta}$$

$$\ddot{\theta} = -\left[\left(\frac{M+m}{Ml}\right)g\theta + \frac{F_x}{Ml}\right] \tag{4}$$

Substituting eq. (4) into (1),

$$F_x = (M+m)\ddot{x} - ml\left[\left(\frac{M+m}{Ml}\right)g\theta + \frac{F_x}{Ml}\right]$$

$$\ddot{x} = \frac{F_x}{M} + \left(\frac{m}{M}\right)g\theta \qquad (5)$$

Equation (4) and (5) can be arranged into the matrix form as below:

$$\dot{x} = Ax + Bu$$

With output equation is,

$$y = Cx + Du$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & -\frac{(M+m)g}{Ml} & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ -\frac{1}{Ml} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

3.2 LINEAR QUADRATIC REGULATOR (LQR) CONTROL SCHEME

LQR is derived by a set of linear differential equation and the cost is described by a quadratic functional. The main objective of LQR controller is to find the performance of dynamic system at the minimum cost. Performance of the system focuses on the sway angle of the rode and the Power Spectral Density (PSD) of the sway angle response[3]. LQR is a design technique that provides practical feedback gains for most system [5].

3.2.1 BLOCK DIAGRAM OF LQR

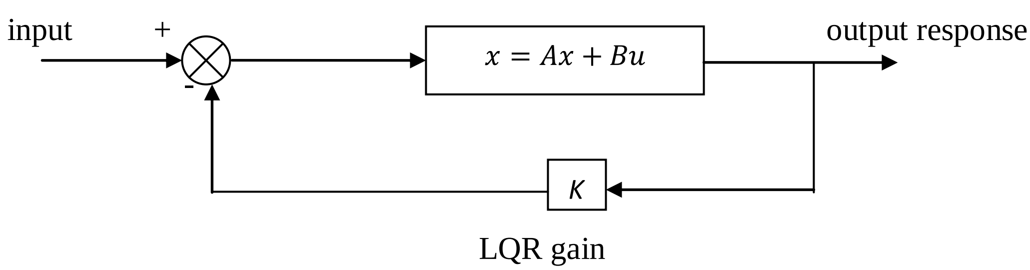

Figure 3.4: Basic system of LQR system

A more common approach in the control of manipulator systems involves the utilization linear quadratic regulator (LQR) design [4]. In order to design the LQR controller, a linear state space model of the gantry crane system was obtained by linearising the equations of motion of the system. For a linear time-invariant (LTI) systems

$$\dot{x} = Ax + Bu$$

The technique involves choosing a control law $u = -Kx$ which stabilizes the origin while minimizing the quadratic cost function

$$J = \int_0^\infty x(t)^T Qx(t) + u(t)^T Ru(t)dt$$

Where;

$$Q = Q^T \geq 0 \text{ and } R = R^T > 0$$

The term “linear-quadratic” refers to the linear system dynamics and the quadratic cost function.

The matrices Q and R are called the state and control penalty matrices, respectively. If the components of Q are chosen large relative to those of R, then deviations of R from zero will be penalized heavily relative to deviations of u from zero. On the other hand, if the components of R are large relative to those of Q then control effort will be more costly and the state will not converge to zero as quickly [1].

Due to Kalman, the control law which minimizes J always takes the form $u = -Kx$. The optimal regulator for a LTI system with respect to quadratic cost function above is always a linear control law. Thus, the closed-loop system takes the form

$$\dot{x} = (A - BK)x$$

and the cost function J take the form

$$J = \int_0^\infty x(t)^T Qx(t) + (-Kx(t))^T R(-Kx(t))dt$$

$$J = \int_0^\infty x(t)^T (Q + K^T RK)x(t)dt$$

3.2.2 LQR SIMULATION (MATLAB)

Figure 3.5 shows block diagram in MATLAB for gantry crane system with LQR controller. The 2D crane block system is developed from few subsystems. Figure 3.6 shows the subsystem for 2D crane system.

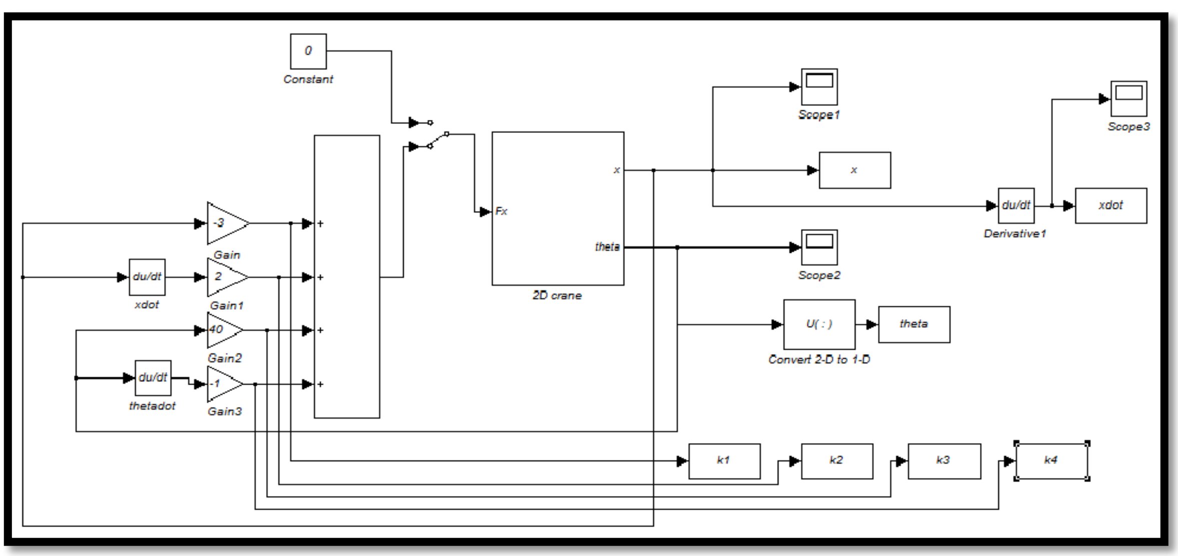

Figure 3.5: LQR system

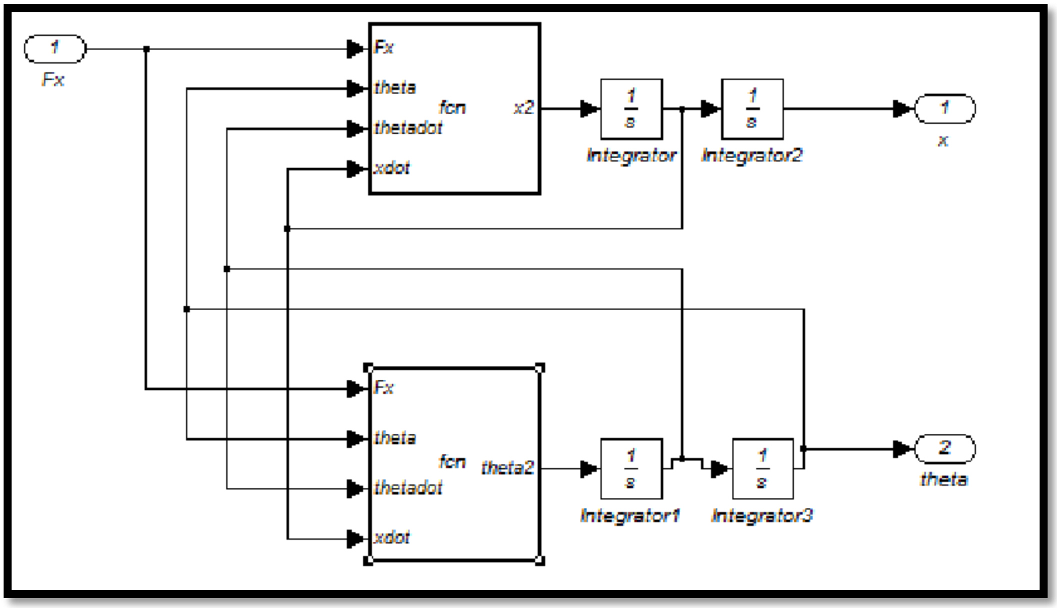

Figure 3.6: 2D crane subsystems

3.2.3 SETTING PARAMETERS, GAINS, EQUATIONS AND GETTING RESULTS

M=2.49; %kg

m=1; %kg

L=1; %m

g=9.81; %m/s^2

k=11.398; %Ns/m

So, the equations (1) and (2) for gantry crane system in MATLAB are shown below:

x2 = (-(Fx-k*xdot)*cos(theta)-g*(M+m)*sin(theta) - m*L*thetadot^2*sin(theta)*cos(theta))/(cos(theta)*(M*(cos(theta))^2-M-m))-g*sin(theta)/cos(theta) ---(1)

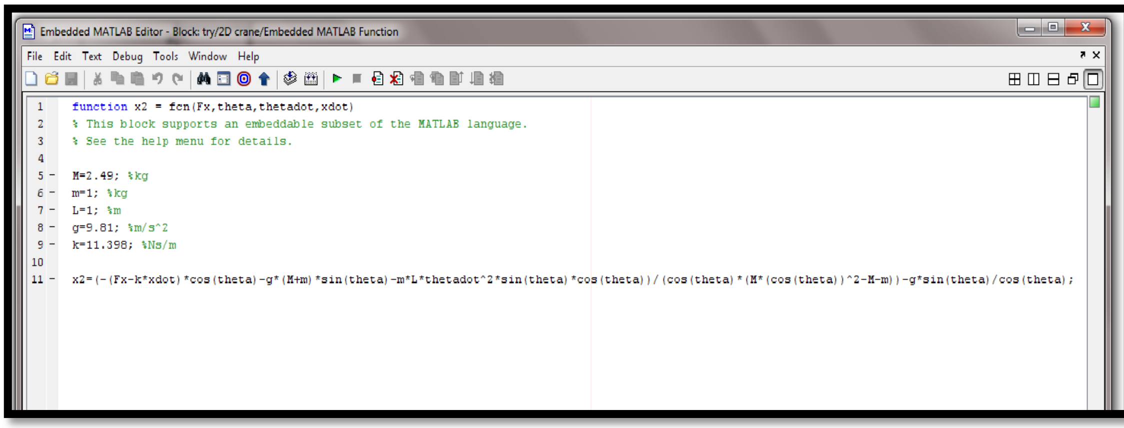

Figure 3.7: Parameters and equation for x

theta2 = ((Fx-k*xdot)*cos(theta) + g*(M+m)*sin(theta) + m*L*thetadot^2*sin(theta)*cos(theta))/(m*L*(cos(theta))^2-(M+m)*L); ---(2)

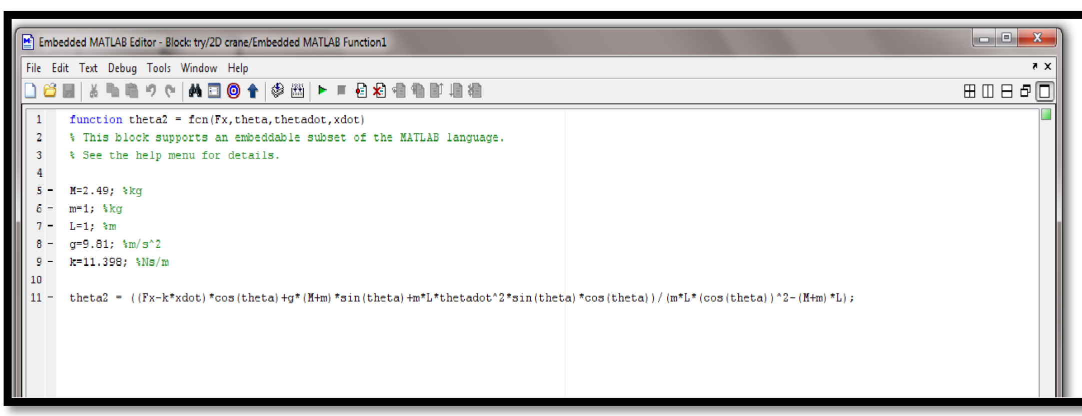

Figure 3.8: Parameters and equation for theta

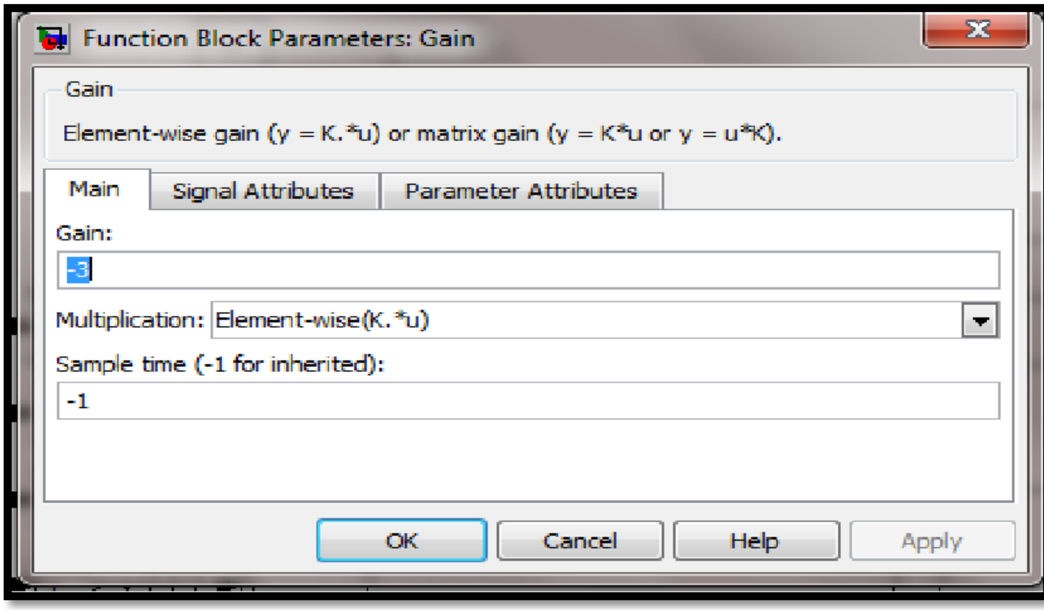

Figure 3.9: LQR gain

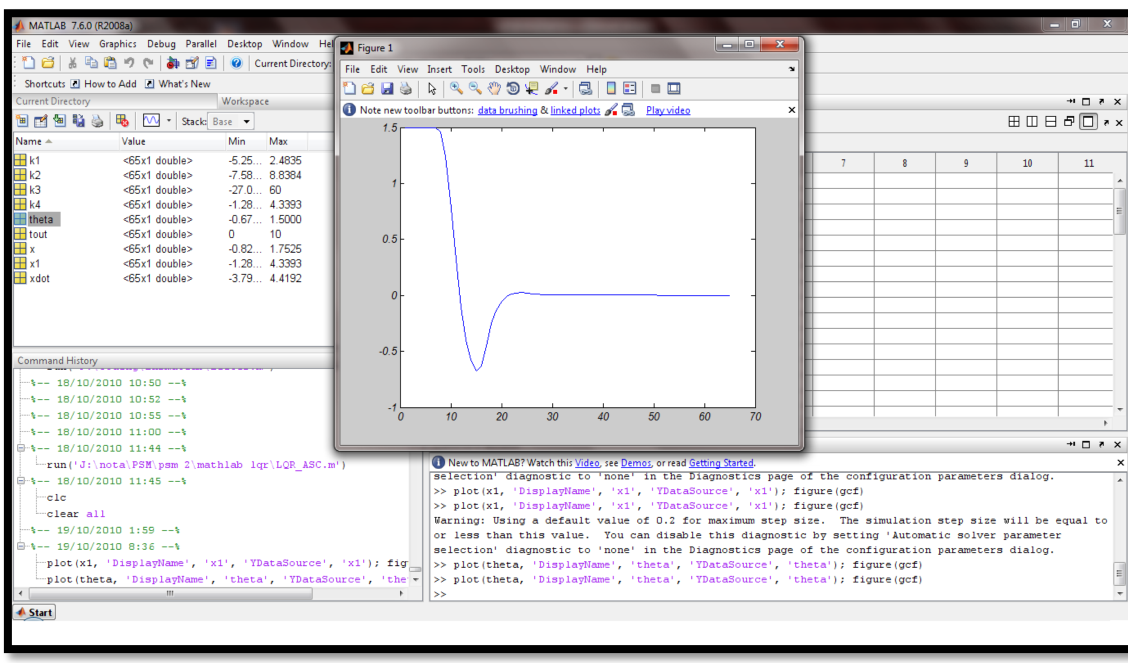

Figure 3.10: Result

During the MATLAB simulation work, we will use three different values for each parameter, length of rope, L, mass of the load, m and starting point of angle of the rope, θ.

All the values will give different performances to the system. We will find the right tuning gain for all the parameters values to get the most efficient performances. Comparison between all the results, parameters and tuning gain will be discussed in next chapter, result and discussion chapter. However, when one of the parameters is changed, the other parameters will remain constant. All the parameters values are shown in table 3.1.

Table 3.1: Parameters for the system

Parameters	**m (kg)**	**L (m)**	**θ (rad)**
m	1.0	1.0	1.5
	2.0	1.0	1.5
	3.0	1.0	1.5
L	1.0	0.5	1.5
	1.0	1.0	1.5
	1.0	1.5	1.5
θ	1.0	1.0	0.5
	1.0	1.0	1.0
	1.0	1.0	1.5

CHAPTER 4

RESULT AND DISCUSSION

4.1 RESULT OF SYSTEM RESPONSES WITH VARIOUS PARAMETERS AND GAINS

In this chapter, gantry crane system responses are presented. It consists of rope sway angle, cart position, cart velocity, input gain to the system and Power Spectral Density (PSD) for both uncontrolled and controlled system responses.

For the gantry crane system performances with LQR controller, the system behaviour for several manipulated parameters, (rope length (L), load weight (m) and initial theta (θ_i) are shown and compared. The injected gain values for all cases are also presented in this chapter along with the rest of the system responses stated above.

Table 4.1: Default parameters

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	starting angle, θ (rad)
Value	2.49	1.0	1.0	9.81	1.5

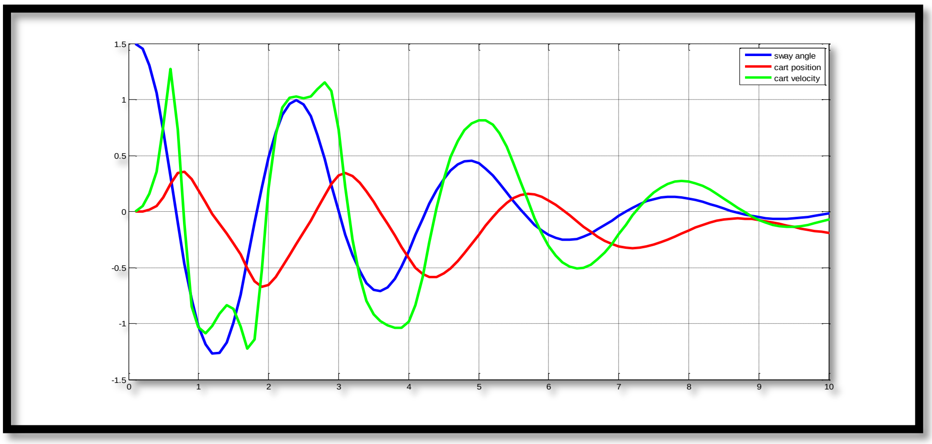

Figure 4.1: Without controller

Figure 4.1 shows the responses of pendulum sway angle, theta, cart's position, x and cart's speed, xdot without a controller while table 4.1 shows the parameters that applied to the system. Blue color indicates the response of theta. Green indicates x response, while red shows the response of xdot. All the responses are oscillating in unbalance condition due to the non-existence of system's controller. Responses show the rise times and settling times are in slow responses. The system gives bad performances due to the delay of the response to achieve zero steady state during settling time. It shows that, pendulum and cart are moving randomly until they stop automatically by themselves. This will increase the settling time and rise time of the response. Therefore, this response proved that the system is totally undesired because of the poor performance.

Table 4.2: Default parameters

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	initial angle, θ_i (rad)
Value	2.49	1.0	1.0	9.81	1.5

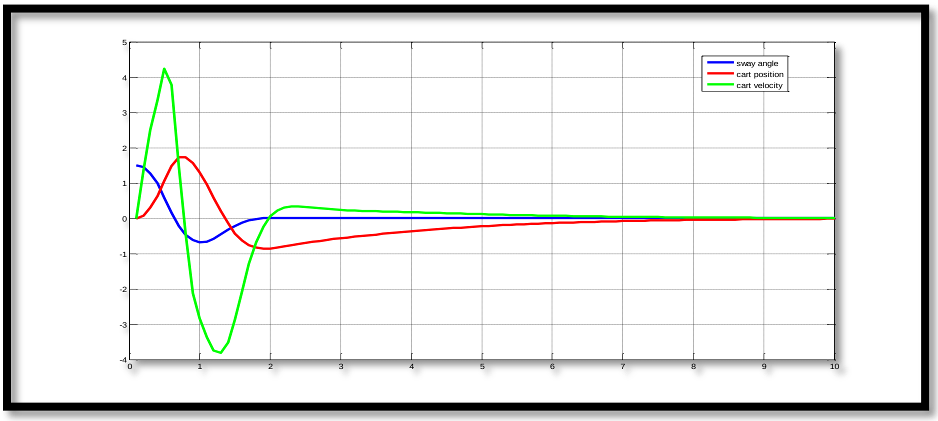

Figure 4.2: Response with default parameter

Figure 4.2 shows responses of system with default parameters applied to the system. Table 4.2 shows the default parameters that are applied to the system. A controller is applied in order to reduce the error from the system. Blue response indicates the sway angle of the pendulum. Green for the position of cart while red indicates the speed of the cart. If compared to Figure 4.2, it is shown that there are major differences between both responses in terms of the rise time and settling time. Sway angle response achieves zero steady state after 1.78 seconds, while the cart stop moving when time=7.88 seconds. The same case occures for the speed of cart. When cart stops moving (x=0), the speed is also achieving to its final steady state (xdot=0). As shown in the Figure 4.2, all responses achieve their steady state, rise time and settling time faster than the system without a controller (refer Figure 4.1).

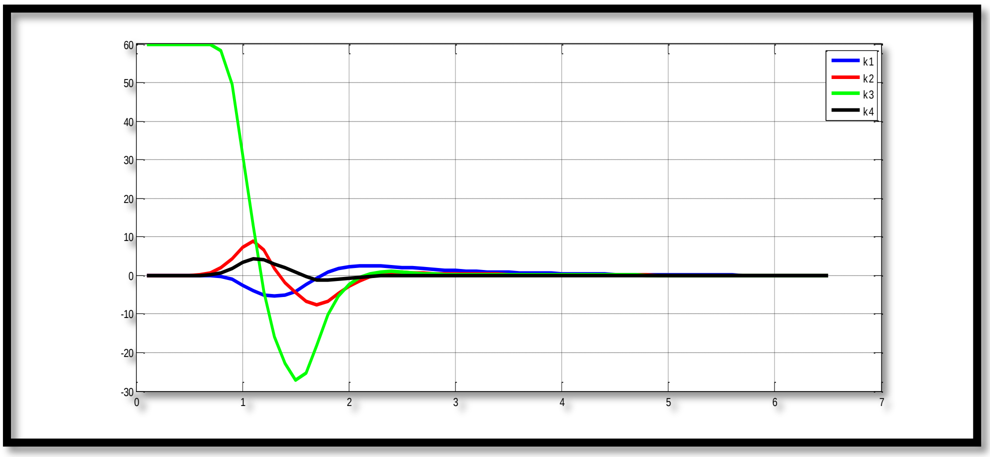

Figure 4.3: Gain= [-3 2 40 -1] of response with default parameters

Figure above shows the responses of gain= [-3 2 40 -1] that are given to the system in order to make the controller increase the efficiency. With a good gain selection, the controller will be able to calculate the error and reduce it to the steady state.

Table 4.3: Parameters with L=0.5 m

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	initial angle, θ_i (rad)
Value	2.49	1.0	0.5	9.81	1.5

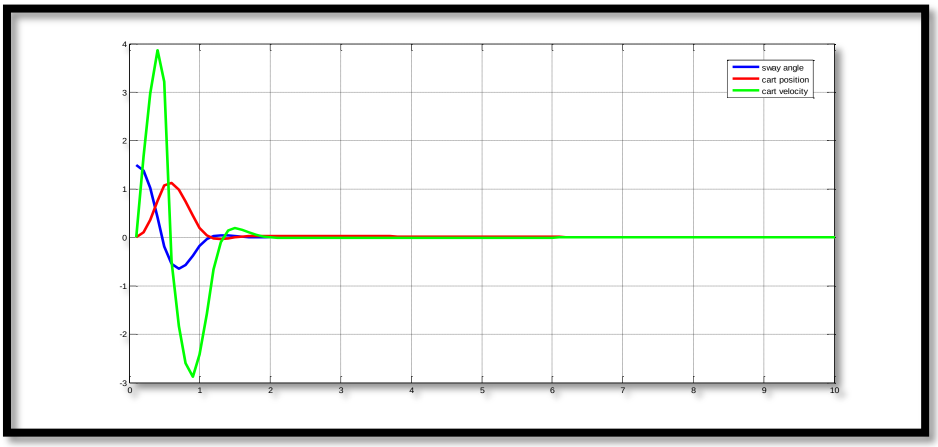

Figure 4.4: Response with L=0.5m

Figure 4.4 shows the response of sway angle, cart's movement and speed with the parameters from table 4.3. Sway angle achieve its steady state at settling time=1.46 seconds. Even the sway angle response is starting at 1.5 rad during the starting transition and reducing to the -0.5 rad at 0.75 seconds, it returns to its steady state after the error is reduced by the controller. When sway angle is 0 rad, the cart stops moving at time=3.42 seconds. The controller manages to reduce the settling time and rise time for the sway angle response by the movement of the cart. When the sway angle is increasing, speed of cart also is increasing and it achieves velocity=0 at time=1.75 seconds.

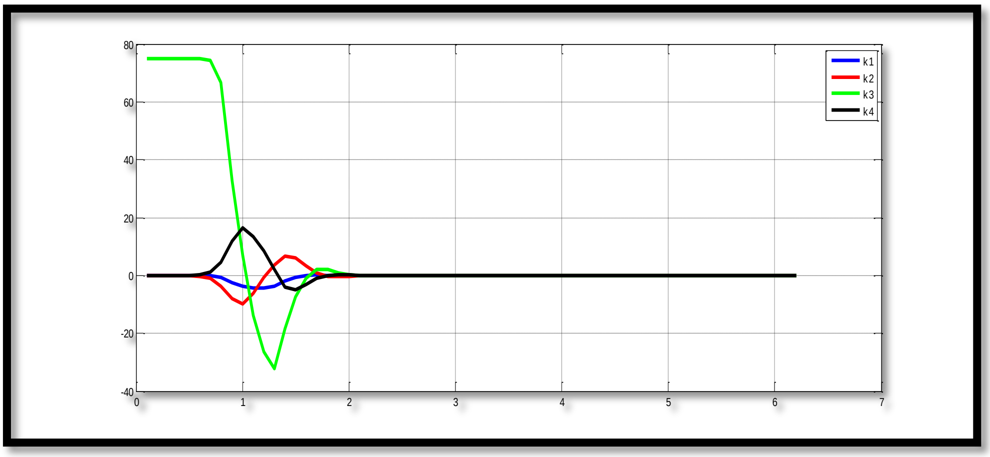

Figure 4.5: Gain= [-4 -2.5 50 -2.5] of response with L=0.5m

Figure 4.5 above shows the responses of gain= [-4 -2.5 50 -2.5] that are given to the system for L=0.5m in order to make the controller increase the efficiency. With a good gain selection, the controller will be able to calculate the error and reduce it to the steady state.

Table 4.4: Parameters with L=1.5m

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	initial angle, θ_i (rad)
Value	2.49	1.0	1.5	9.81	1.5

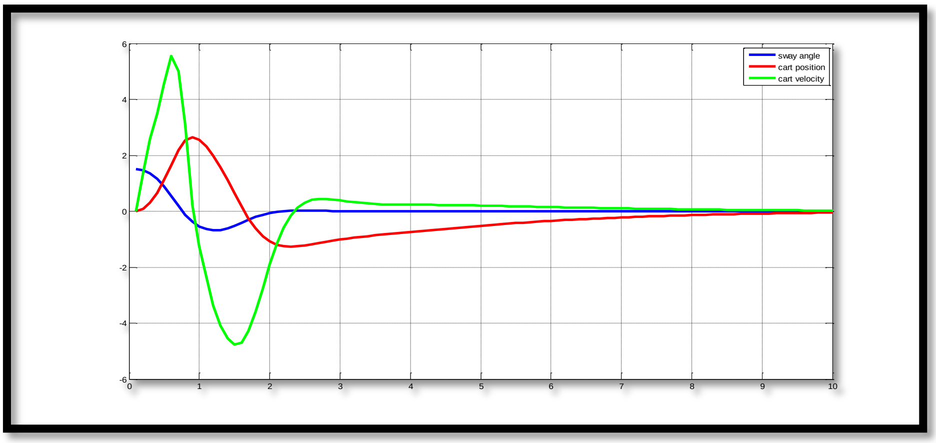

Figure 4.6: Response with L=1.5m

Table 4.4 shows the parameters that used in the system with L= 1.5m. Figure 4.6 shows the responses of sway angle, position and speed of the cart. It is shown that the increasing of length will affect the rise time and settling time of the sway angle. Initially, the sway angle response is starting at 1.5 rad and drop to -0.5 rad, but the response returns to its steady state at 0 rad during settling time=2.09 seconds. The settling time for cart position is at 8.61 seconds while the settling time for cart speed is at 6.30 seconds. It is shown that the cart is moving according to the sway angle of pendulum in order to the reduce the swaying motion. The speed of the cart is directly proportional with the distance of the cart. Therefore, the speed response will achieve its zero steady state when the cart stop moving.

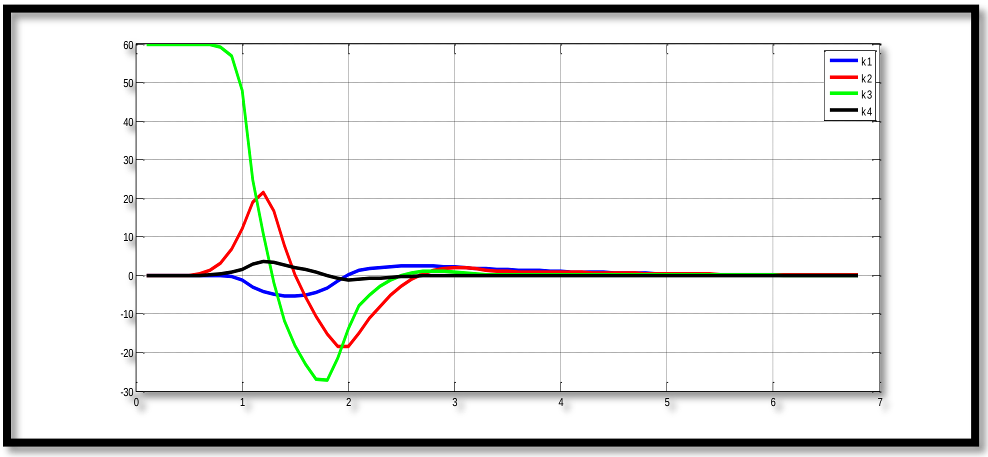

Figure 4.7: Gain= [-4 -2.5 50 -2.5] of response with L=1.5m

Figure 4.7 above shows the responses of gain= [-4 -2.5 50 -2.5] that are given to the system with L=1.5m in order to make the controller increase the efficiency. With a goo gain selection, the controller will be able to calculate the error and reduce it to the steady state.

Table 4.5: Parameters with m=2 kg

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	initial angle, θ_i (rad)
Value	2.49	2.0	1.0	9.81	1.5

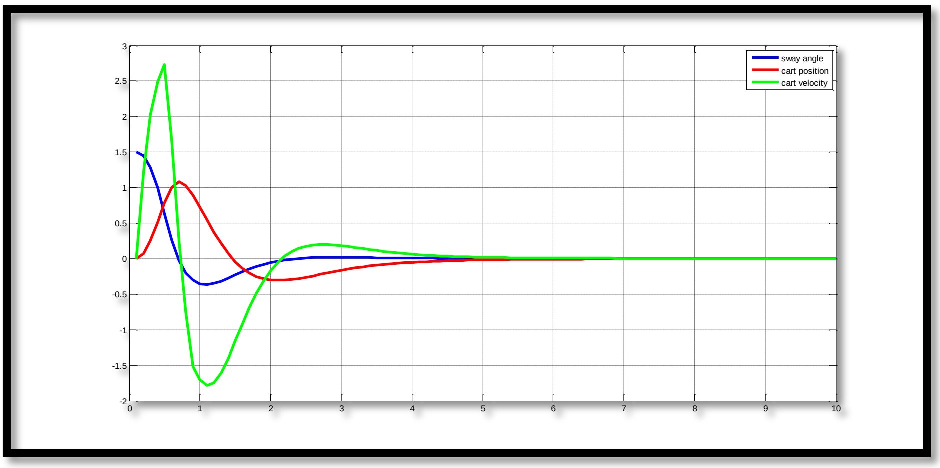

Figure 4.8: Response with m=2kg

Figure 4.8 shows the response of sway angle, cart's movement and velocity with the parameters from table 4.5. Sway angle achieve its steady state at settling time=2.13 seconds. Even the sway angle response is starting at 1.5 rad during the starting transition and reducing to the -0.3 rad at 1.15 seconds, it returns to its steady state after the error is reduced by the controller. When sway angle is 0 rad, the cart stop from moving at time=4.88 seconds. The controller manages to reduce the settling time and rise time for the sway angle response by the movement of the cart. When the sway angle is increasing, speed of cart also is increasing and it achieves velocity=0 at time=4.09 seconds.

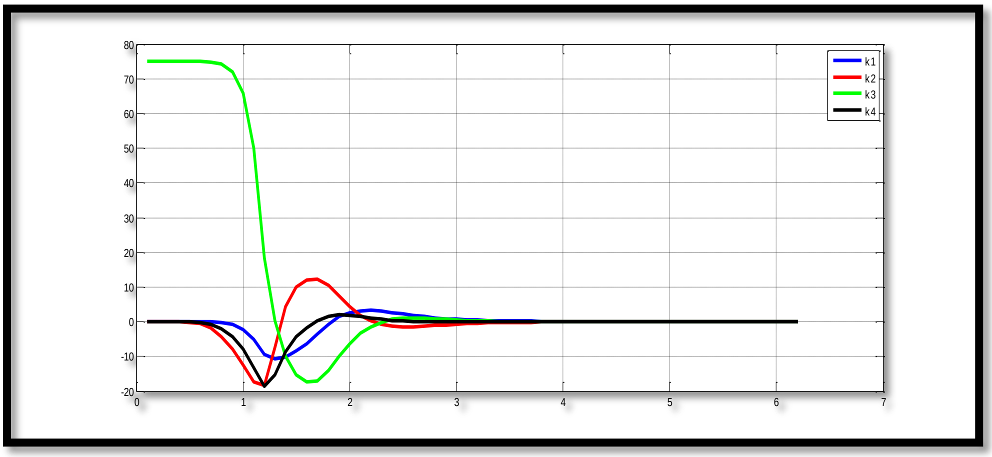

Figure 4.9: Gain= [-10 -7 50 5] of response with m=2kg

Figure 4.9 above shows the response of gain= [-10 -7 50 5] that are given to the system with m=2kg in order to make the controller increase the efficiency. With a good gain selection, the controller will be able to calculate the error and reduce it to the steady state.

Table 4.6: Parameters with m=3 kg

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	Initial angle, θ_i (rad)
Value	2.49	3.0	1.0	9.81	1.5

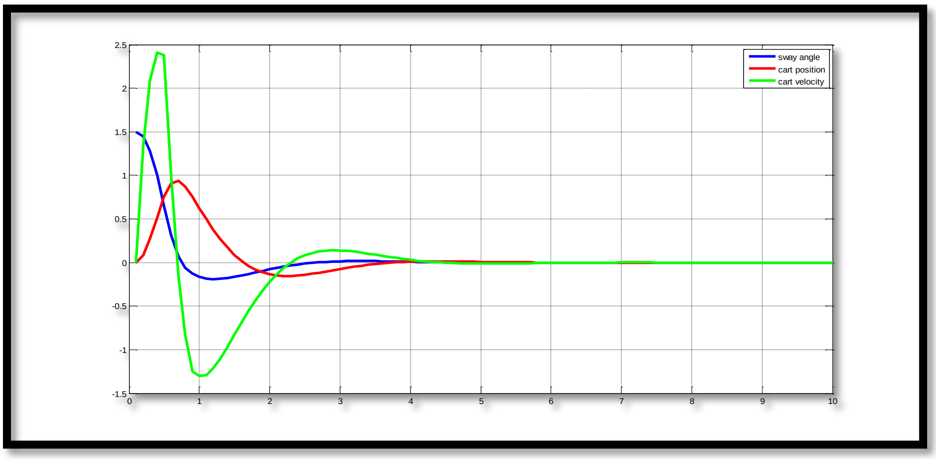

Figure 4.10: Response with m=3 kg

Table 4.6 shows the parameters that used in the system with m=3 kg. Figure 4.10 shows the responses of sway angle, position and speed of the cart. It is shown that the increasing of payload mass will affect the rise time and settling time of the sway angle. Initially, the sway angle response is starting at 1.5 rad and drop to -0.1 rad, but the response return to its steady state at 0 rad during settling time=2.33 seconds. The settling time for cart position is at 3.49 seconds while the settling time for cart speed is at the 3.83 seconds. It is shown that the cart is moving according to the sway angle of pendulum in order to the reduce the swaying motion. The speed of the cart is directly proportional with the distance of the cart. Therefore, the speed of response will achieve to zero when the cart is stop from moving.

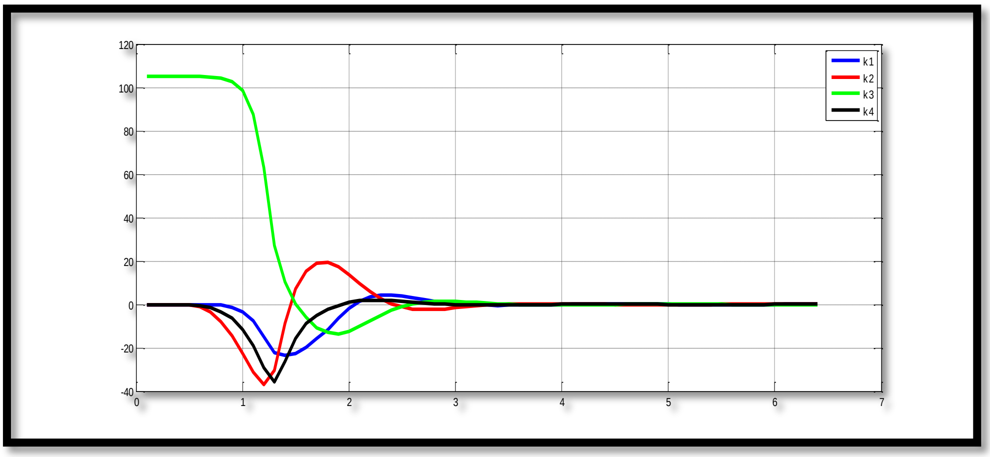

Figure 4.11: Gain= [-25 -15 70 10] of response with m=3kg

Figure 4.11 above shows the response of gain= [-25 -15 70 10] that are given to the system with m=3kg in order to make the controller increase the efficiency. With a good gain, the controller will be able to calculate the error and reduce it to the steady state.

Table 4.7: Parameters with θ_i =0.5 rad

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	Initial angle, θ_i (rad)
Value	2.49	1.0	1.0	9.81	0.5

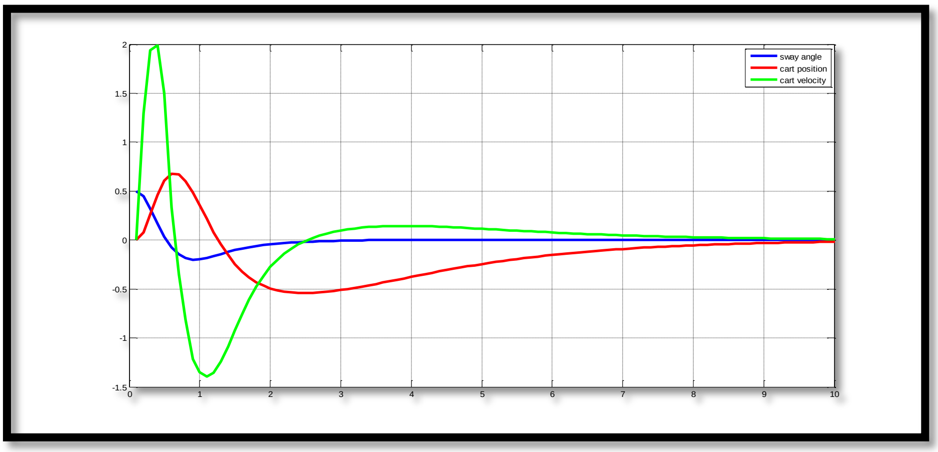

Figure 4.12: Response with θ_i =0.5 rad

Figure 4.12 shows the response of sway angle, cart's movement and speed with the parameters from Table 4.7. Sway angle achieve its steady state at settling time=2.85 seconds. Even the sway angle response is starting at 0.5 rad during the starting transition and reducing to the -0.2 rad at 1.15 seconds, it returns to its steady state after the error is reduced by the controller. When sway angle is 0 rad, the cart stop moving at time=9.01 seconds. The controller succeed to reduce the settling time and rise time for the sway angle response by the movement of the cart. When the sway angle is increasing, speed of cart also is increasing and it acieves velocity=0 at time=6.97 seconds.

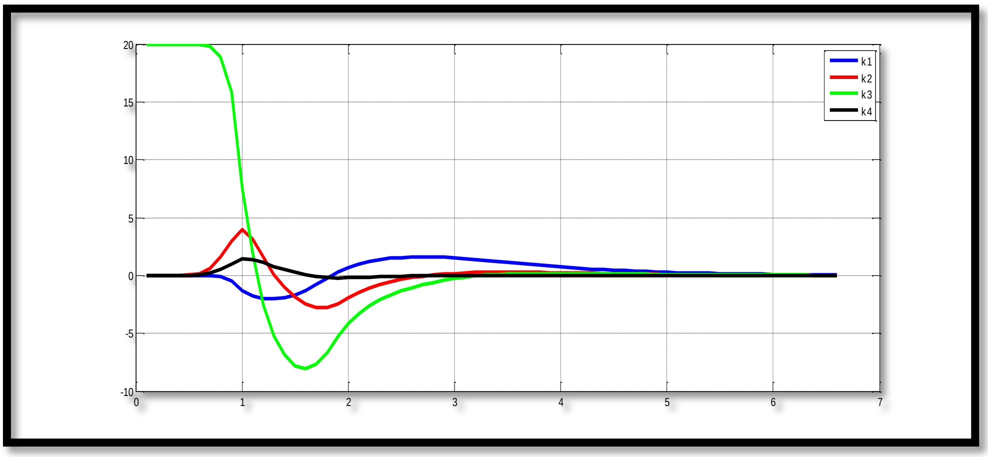

Figure 4.13: Gain= [-3 2 40 -1] of response with θ_i =0.5 rad

Figure 4.13 above shows the response of gain= [-3 2 40 -1] that are given to the system with initial theta at 0.5 rad in order to make the controller increase the efficiency. With a good gain selection, the controller will be able to calculate the error and reduce it to the steady state.

Table 4.8: Parameters with θ_i =1.0 rad

Parameter	M (kg)	m (kg)	L (m)	G (m/s^2)	Initial angle, θ_i (rad)
Value	2.49	1.0	1.0	9.81	1.0

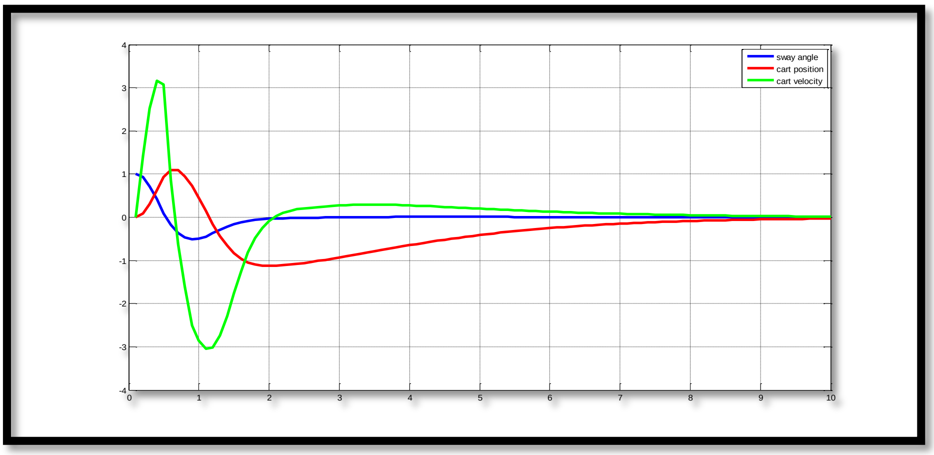

Figure 4.14: Response with θ_i =1.0 rad

Table 4.8 shows the parameters that used in the system with θ_i =1.0 rad. Figure 4.14 shows the responses of sway angle, position and speed of the cart. It is shown that the increasing of initial angle will affect the rise time and settling time of the sway angle. Initially, the sway angle response is starting at 1.0 rad and drop to -0.2 rad, but the response return to its steady state at 0 rad at settling time=2.34 seconds. The settling time for cart position is at 9.02 seconds while the settling time for cart speed is at 7.06 seconds. It is shown that the cart is moving according to the sway angle of pendulum in order to the reduce the swaying motion. The speed of the cart is directly proportional with the distance of the cart. Therefore, the speed response will achieve to zero when the cart is stop from moving.

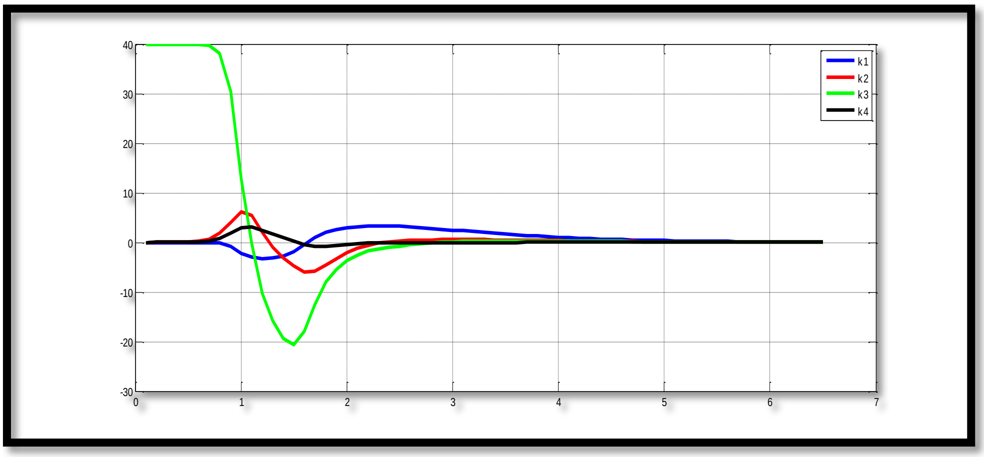

Figure 4.15: Gain= [-3 2 40 -1] of response with θ_i =1.0 rad

Figure 4.14 above shows the response of gain= [-3 2 40 -1] that are given to the system initial theta at 1.0 rad in order to make the controller increase the efficiency. With a good gain selection, the controller will be able to calculate the error and reduce it to the steady state.

4.2 COMPARISON OF RESPONSES

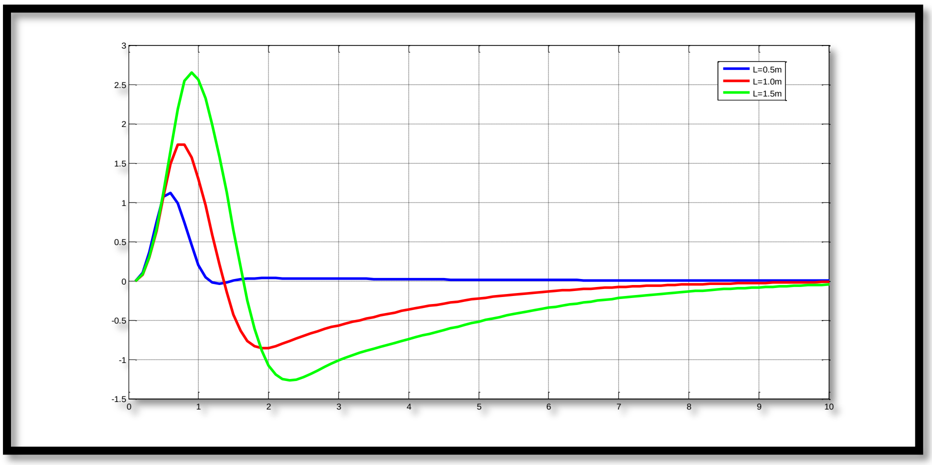

Figure 4.16: Cart position for L=0.5, 1.0 and 1.5 m

Figure 4.16 shows the responses of cart positions for L=0.5, 1.0 and 1.5 m due to the effect of controlling the sway angle of the rope. Cart will move according to the rope's movement in order to reduce the swaying motion. At the final state, the cart will return to its initial position (x=0). Analysis of the responses will be discussed in Figure 4.17 in histogram chart.

Table 4.9: Tr and Ts of cart position for L=0.5, 1.0 and 1.5 m

L (m)	**Rise time Tr(s)**	**Settling time Ts(s)**
0.5	0.0171	3.42159
1.0	0.0333	7.87562
1.5	0.0866	8.60986

Table 4.9 shows the values of the position's responses for the rise time, Tr and settling time, Ts that taken from the output responses.

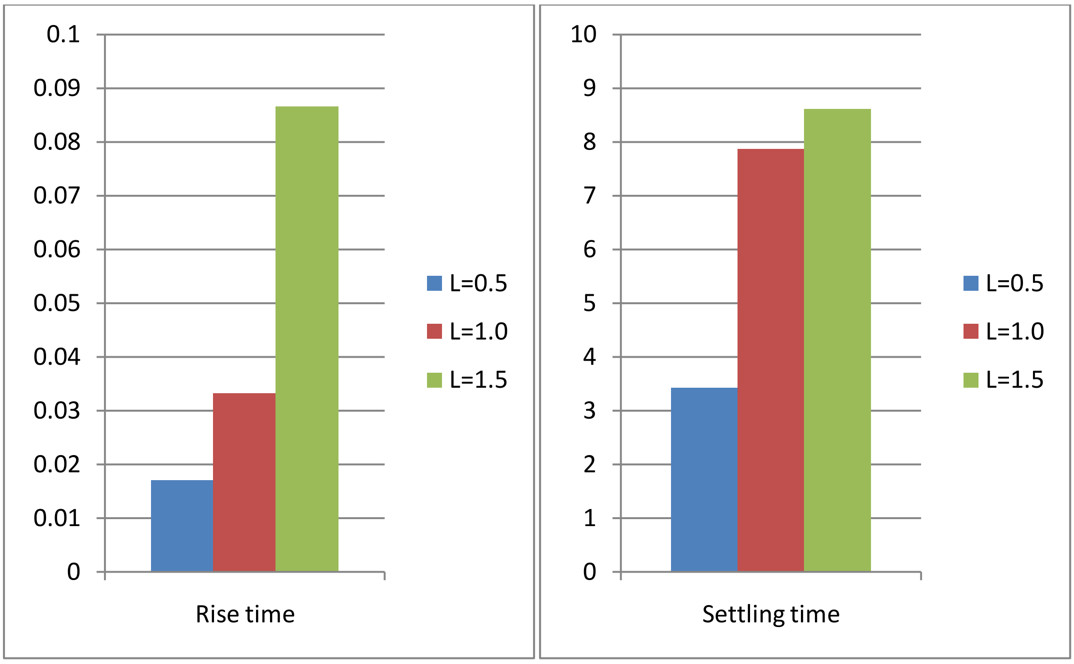

Figure 4.17: Response of specification of cart position for L=0.5, 1.0 and 1.5 m

Figure 4.17 shows the response of rise time and settling time of the position's response in histogram view. The Figure shows that all the rise times and settling times are increasing simultaneously with the increasing of the length of rope, L. The rise time for L=0.5m is at 0.0171s and it increasing to 0.0866 when the length is changed to 1.5m. Same manner is occured for the settling time. The settling time starts at 3.42159s when L=0.5m and increased to 8.60986s when L=1.5m. It is concluded that the length of rope will increase the time response of the cart's movement.

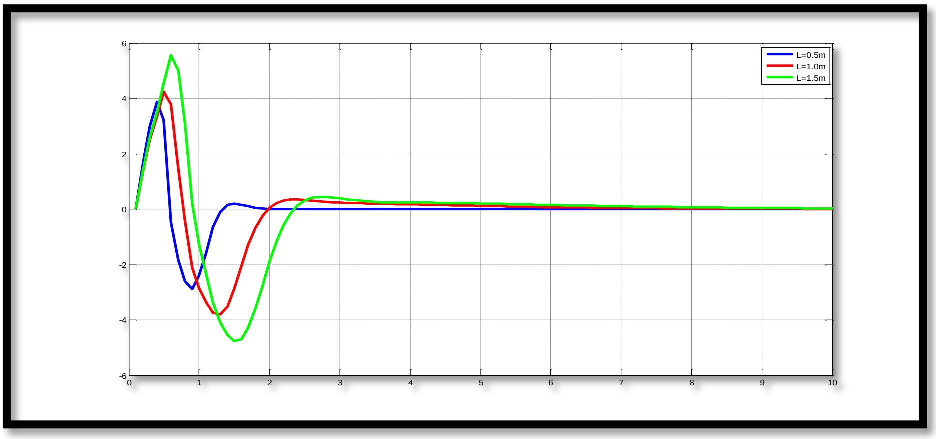

Figure 4.18: Cart speed for L=0.5, 1.0 and 1.5 m

Figure 4.18 shows the responses of cart speed when L is changed from 0.5m to 1.0m and 1.5m. It is shown that the responses increased intially in order to stop the sway angle of the rope. The responses achieve steady state when the sway angle and position of the cart is zero. The reaction of the speed responses are depending on the responses of position and sway angle of the system.

Table 4.10: Tr and Ts of cart speed for L=0.5, 1.0 and 1.5 m

L (m)	**Rise time Tr(s)**	**Settling time Ts(s)**
0.5	0.00017367	1.75047
1	0.0048	5.48745
1.5	0.0144	6.30328

Table 4.10 shows the rise time and settling time for the responses in Figure 4.18. All the rise time and settling time of the cart speed are including for L=0.5m until L=1.5m. The analysis and comparison between all the rise time and settling time are discussed in Figure 4.19 below.

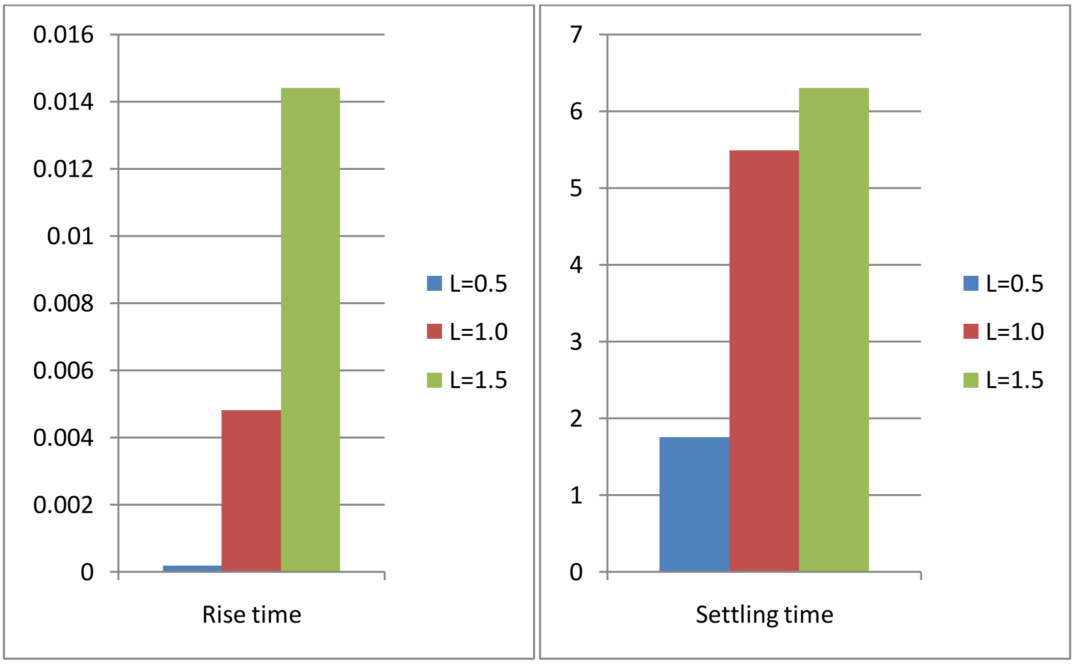

Figure 4.19: Response of specification of cart speed for L=0.5, 1.0 and 1.5 m

As shown in Figure 4.19, the histogram chart are analyzing and comparing the rise time and settling time for responses of cart speed for L=0.5m until 1.5m. From the Figure, it is shown that the rise time are increasing simultaneously when length of the rope is increased. As we can see from the chart, rise time is equal to 0.00017367s and increased to 0.0048s and 0.0144s when L=0.5m, 1.0m and 1.5m respectively.

Same manner with the settling time, the changed of the length of rope will also affect the value of settling time. Settling time will increased when length of the rope is increased. L=0.5m, 1.0m and 1.5m will make the settling time to achieve 1.75047s, 5.48745s and 6.30328s respectively.

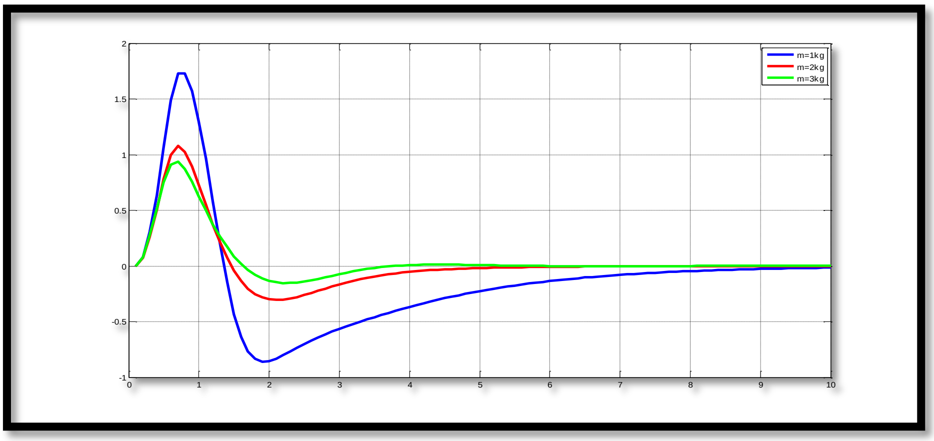

Figure 4.20: Cart position for m=1, 2, 3 kg

All the position responses for mass of payload at m=1 kg, 2kg and 3 kg is shown in Figure 4.20. Green indicates response for m=3 kg, red reprsesent response for m=2kg and blue for default parameter, m=1kg. As shown from the Figure, eventhough the responses have different settling time and rise time, all the responses are achieve their steady state at 0 cm.

Table 4.11: Tr and Ts of cart position for m=1, 2, 3 kg

m (kg)	**Rise time Tr(s)**	**Settling time Ts(s)**
1	0.0333	7.87562
2	0.00013	4.87995
3	0.00022915	3.49037

Table 4.11 shows the parameters that been considered in this response whis is the mass of payload m=1kg, 2kg and 3kg. Values of rise time and settling time are also shown in the table for all three different parameters.

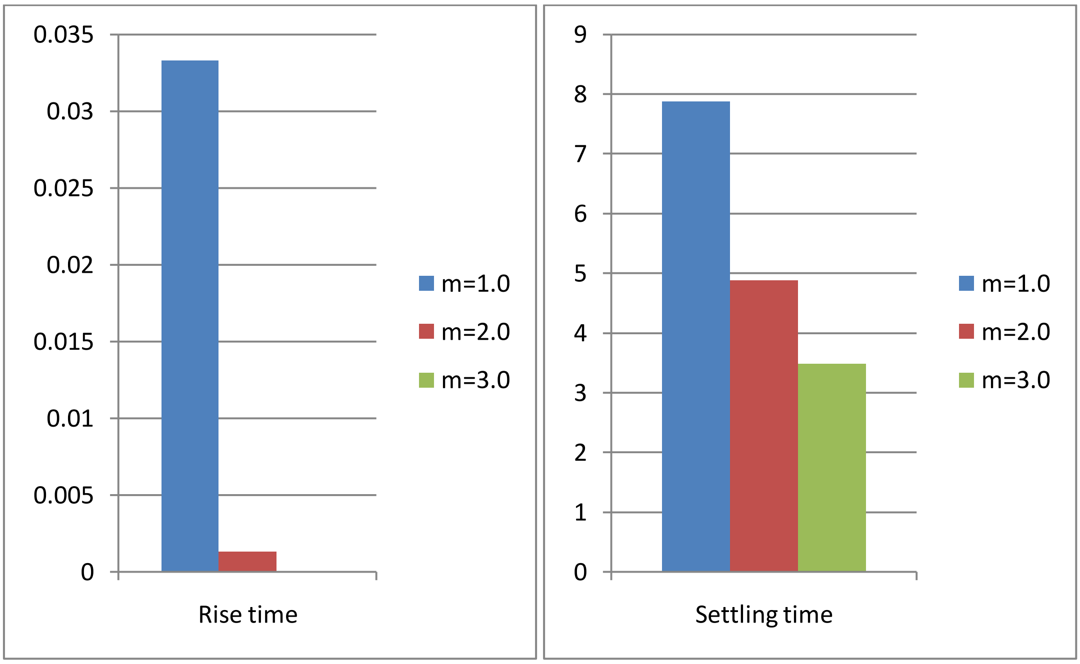

Figure 4.21: Response of specification of cart position for m=1, 2, 3 kg

Figure 4.21 shows the response of settling time and rise time that been represented in histogram chart. The values are extracted from the values at table 4.11. It is shown that all the rise time and settling time are decreased with the increasing mass of payload. It proved that the mass of payload will affect the response of system by increasing or decreasing the rise time and settling time.

The rise time is at 0.0333s when m=1kg. However. It decreased 0.0013s when we changed the mass to 2kg. The rise time keep decreasing to 0.0000022915s when we further increased the mass of payload to m=3kg. Settling time for the response also decreased from 7.87562s to 4.87995s and 3.49037s when we changed the mass of payload from m=1kg to 3kg.

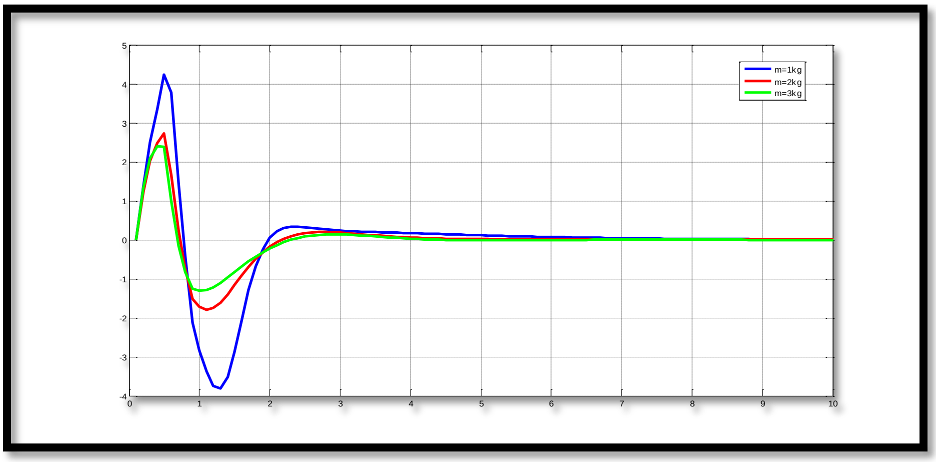

Figure 4.22: Cart speed for m=1, 2, 3 kg

Figure 4.22 shows the responses of cart speed for m=1kg,2kg and 3kg due to the effect of controlling the sway angle of the rope. Cart will move according to the rope's movement and the mass of payload in order to reduce the swaying motion. At the final state, the cart will return to its initial position (x=0). Analysis of the responses will be discussed in Figure 4.23 in histogram chart

Table 4.12: Tr and Ts of cart speed for m=1, 2, 3 kg

m (kg)	**Rise time Tr(s)**	**Settling time Ts(s)**
1	0.0048	5.48745
2	0.00011477	4.09244
3	0.000027151	3.83385

Table 4.12 shows the values of the speed's responses for the rise time, Tr and settling time, Ts that taken from the output responses.

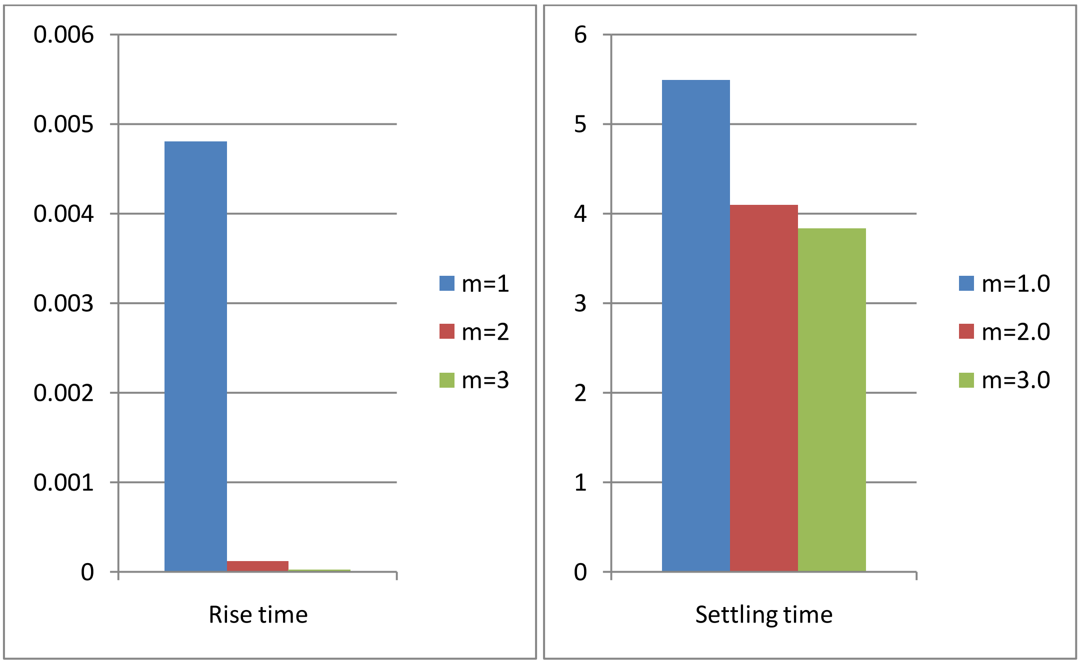

Figure 4.23: Response of specification of cart speed for m=1, 2, 3 kg

Figure 4.23 shows the response of rise time and settling time of the speed's response in histogram view. The Figure shows that all the rise times and settling times are decreasing simultaneously with the increasing of the mass of payload,m. The rise time for m=1kg is at 0.0048s and is decreasing to 0.00011477s when the mass is changed to 2kg and 0.000027151s for m=3kg. Same manner is occured for the settling time. The settling time starts at 5.48745s when m=1kg and decreased to 3.83385s when m=3kg. It is concluded that the mass of payload will affect and decrease the time response of the cart's speed.

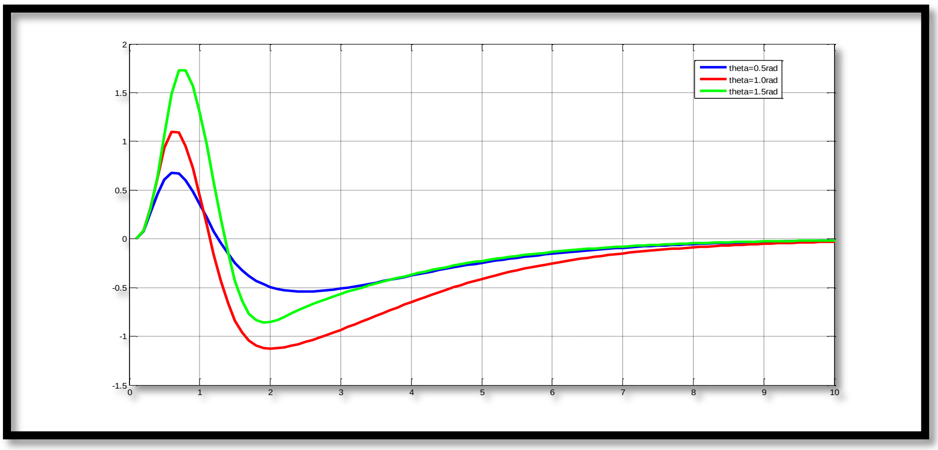

Figure 4.24: Cart position for θ_i =0.5, 1.0, 1.5 rad

Figure 4.24 shows the responses of cart position when theta is changed from 0.5 rad to 1.0 rad and 4.5 rad. It is shown that the responses increased intially in order to stop the sway angle of the rope. The responses achieve steady state when the sway angle and position of the cart is zero. The reaction of the theta responses are depending on the responses of position and final sway angle of the system.

Table 4.13: Tr and Ts of cart position for θ_i =0.5, 1.0, 1.5 rad

theta (rad)	**Rise time Tr(s)**	**Settling time Ts(s)**
0.5	0.1105	9.02129
1.0	0.0739	9.01332
1.5	0.0333	7.87562

Table 4.13 shows the rise time and settling time for the responses in Figure 4.24. All the rise time and settling time of the initial theta including theta=0.5 rad

until 1.5rad. The analysis and comparison between all the rise time and settling time are discussed in Figure 4.25 below.

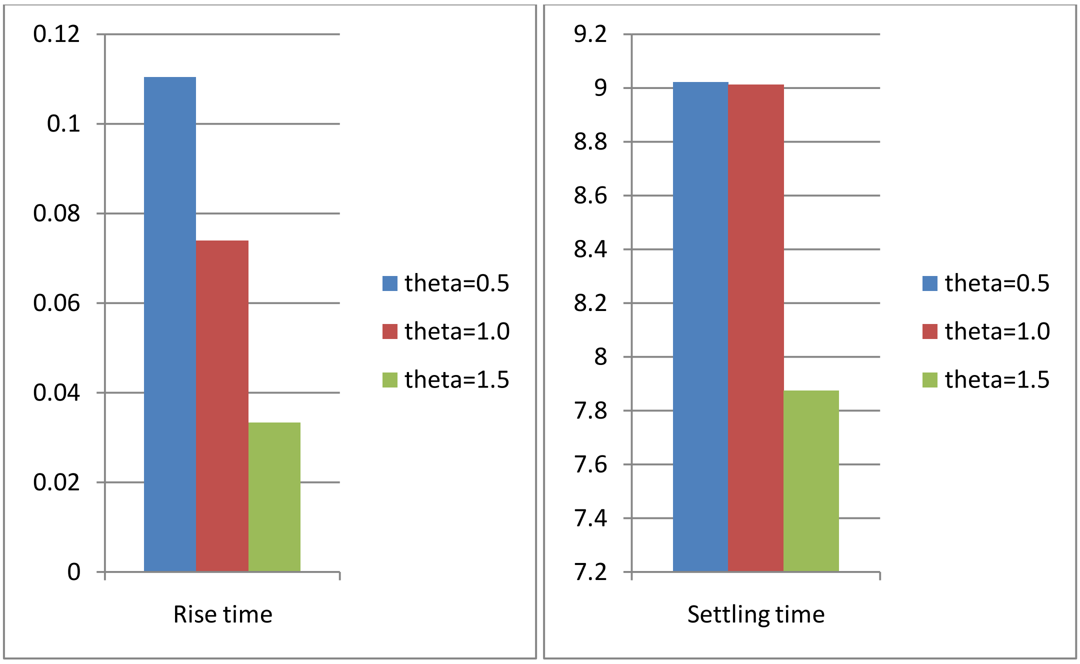

Figure 4.25: Response of specification of cart position for θ_i =0.5, 1.0, 1.5 rad

As shown in Figure 4.25, the histogram chart are analyzing and comparing the rise time and settling time for responses of cart position for theta 0.5 rad until 1.5 rad. From the Figure, it is shown that the rise time are decreasing simultaneously when the initial theta is increased. As we can see from the chart, rise time is equal to 0.1105s and decreased to 0.0739s and 0.0333s when theta=0.5 rad, 1.0 rad and 1.5 rad respectively.

Same manner with the settling time, the changed of the initial theta will also affect the value of settling time. Settling time will decreased when initial theta is increased. Theta= 0.5 rad, 1.0 rad and 1.5 rad will make the settling time to achieve 9.02129s, 9.01332s and 7.87562 respectively.

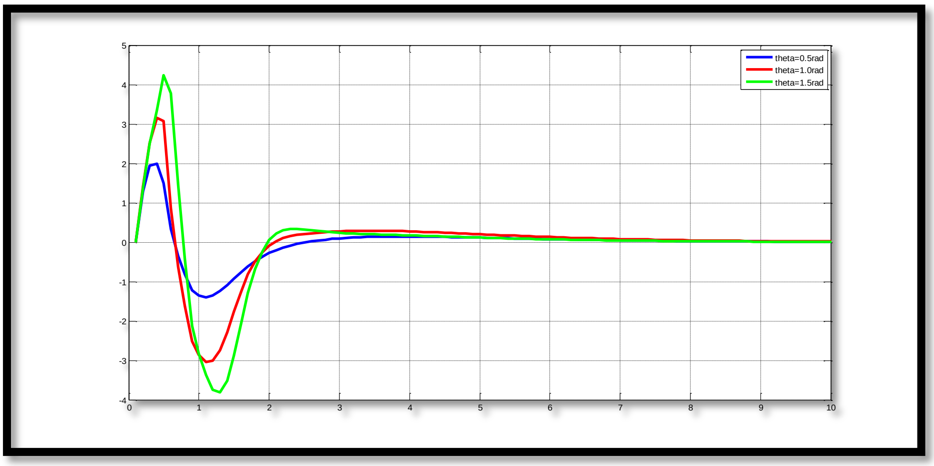

Figure 4.26: Cart speed for θ_i=0.5, 1.0, 1.5 rad

All the speed responses for initial theta at 0.5 rad, 1.0 rad and 1.5 rad are shown in Figure 4.26. Green indicates response for theta=1.5 rad, red reprsesent response for theta=1.0 rad and blue for theta=0.5 rad. As shown from the Figure, eventhough the responses have different settling time and rise time, all the responses are achieve their steady state at 0 cm/s.

Table 4.14: Tr and Ts of cart speed for θ_i=0.5, 1.0, 1.5 rad

Theta (rad)	**Rise time Tr(s)**	**Settling time Ts(s)**
0.5	0.0089	7.05752
1.0	0.0060	6.97717
1.5	0.0048	5.48745

Table 4.14 shows the parameters that been considered in this response which the initial theta is at 0.5rad, 1.0rad and 1.5rad. Values of rise time and settling time are also shown in the table for all three different parameters.

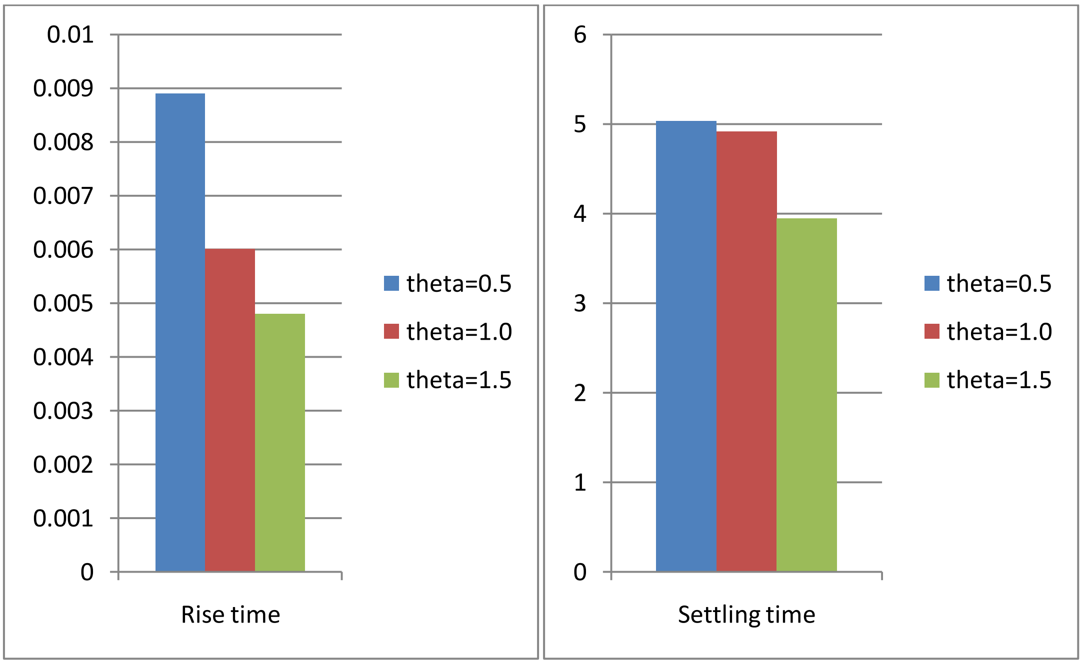

Figure 4.27: Response of specification of cart speed for θ_i =0.5, 1.0, 1.5 rad

Figure 4.27 shows the response of settling time and rise time that been represented in histogram chart. The values are extracted from the values at table 4.14. It is shown that all the rise time and settling time are decreased respectively with the increasing of initial theta. It is proved that the initial theta will affect the response of system by increasing or decreasing the rise time and settling time.

The rise time is at 0.0089s when theta=0.5 rad. However. It decreased to 0.0060s when we changed the initial theta to 1.0 rad. The rise time keep decreasing to 0.0048s when we further increased the theta to 1.5 rad. However, the settling time for the response is decreased from 7.05752s to 6.97717s and 5.48745s when we changed the initial theta from 0.5 rad to 1.0 rad and 1.5 rad.

Table 4.15: Rise time comparison for all position data

Parameter	Rise Time	Parameter	Rise Time
L=0.5	0.0171	theta=0.5	0.1105
L=1.0	0.0333	theta=1.0	0.0739
L=1.5	0.0866	theta=1.5	0.0333
m=1.0	0.0333		
m=2.0	0.0013		
m=3.0	0.0000022915		

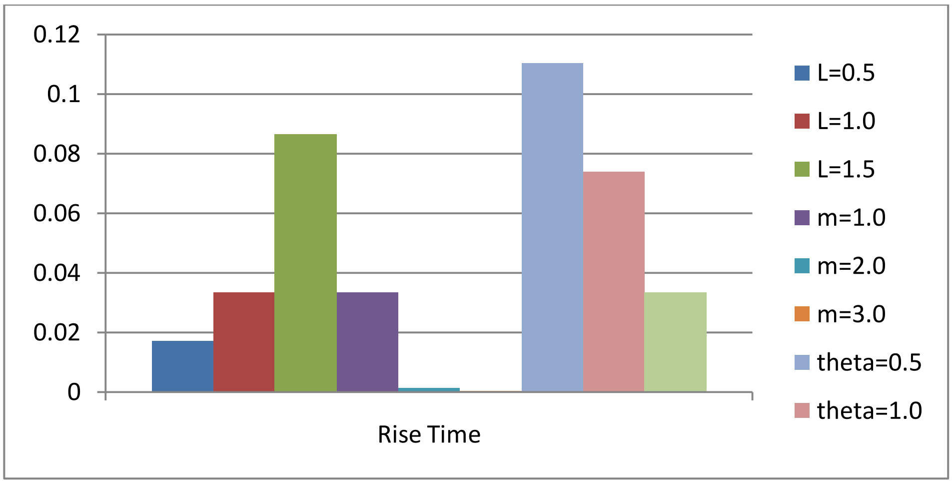

Figure 4.28 : Rise time comparison for all position data

Table 4.15 shows all the values of rise time for all position data that we get from the project. The data are been represented in chart in Figure 4.28. The Figure shows the comparison between all the rise time for position response with various parameters for L, m and theta. It is shown that the rise time are increasing and decreasing sequencely according to the changed parameters. It is shown that the rise time is decreasing simultaneously with the increasing of the length of the rope. However, the rise time is decreasing when the mass of payload is increased. For the changed theta, it is shown that, there is big amount of difference when we increase the theta value. The rise time is increase rapidly from theta=0.5 to theta=1.5 rad.

Table 4.16: Settling time comparison for all position data

Parameter	Settling Time	Parameter	Settling Time
L=0.5	3.42159	theta=0.5	9.02129
L=1.0	7.87562	theta=1.0	9.01332
L=1.5	8.60986	theta=1.5	7.87562
m=1.0	7.87562		
m=2.0	4.87995		
m=3.0	3.49037		

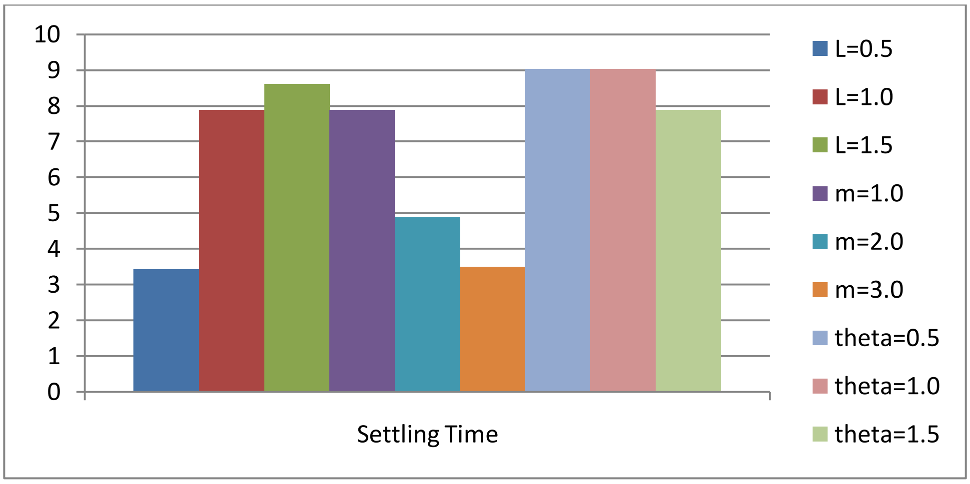

Figure 4.29: Settling time comparison for all position data

Figure 4.29 shows the comparison of settling time for all position data for all parameters in histogtram representation. Table 4.16 shows the value of all the settling time and the parameters. As shown from the figure, there are differences between the changed parameters for all settling time value. When L is increased, it is shown that the settling time also increased. However, the settling time is decreasing simultaneously with the increasing of mass of payload. Same manner with the theta, the settling time is slightly decreased when we increase the value of initial theta.

Table 4.17: Rise time comparison for all speed data

Parameter	Rise Time	Parameter	Rise Time
L=0.5	0.00017967	theta=0.5	0.0089
L=1.0	0.0048	theta=1.0	0.0060
L=1.5	0.0144	theta=1.5	0.0048
m=1.0	0.0048		
m=2.0	0.00011477		
m=3.0	0.000027151		

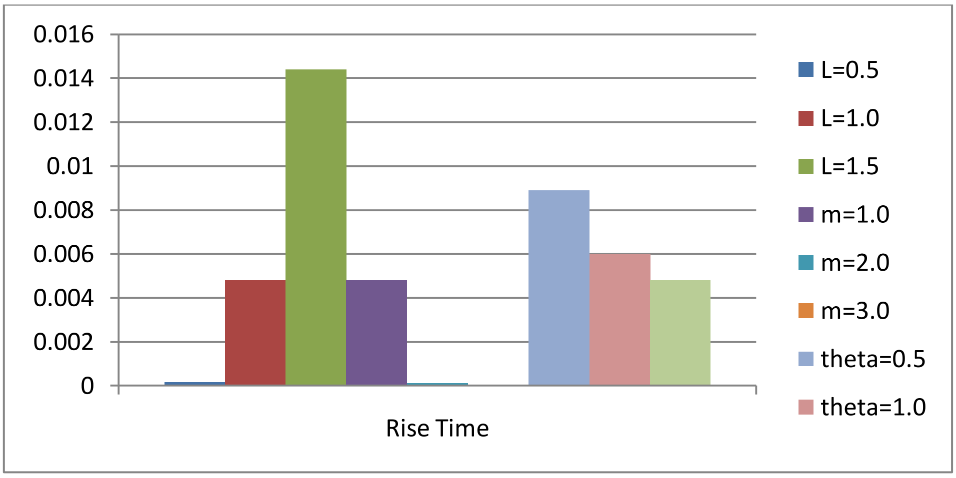

Figure 4.30: Rise time comparison for all speed data

Table 4.17 shows all the values of rise time for all speed data that we get from the project. The data are been represented in chart in Figure 4.30 in order to see the differences and comparison. The Figure shows the comparison between all the rise time for position response with various parameters for L, m and theta. It is shown that the rise time are increasing and decreasing sequencely according to the changed parameters. It is shown that the rise time is sligthly increased simultaneously with the increasing of the length of the rope. However, the rise time is decreasing when the mass of payload is increased. For the changed theta, it is shown that, there is only small amount of difference when we increase the theta value.

Table 4.18: Settling time comparison for all speed data

Parameter	Settling Time	Parameter	Settling Time
L=0.5	1.75047	theta=0.5	7.05752
L=1.0	5.48745	theta=1.0	6.97717
L=1.5	6.30328	theta=1.5	5.48745
m=1.0	5.48745		
m=2.0	4.09244		
m=3.0	3.83385		

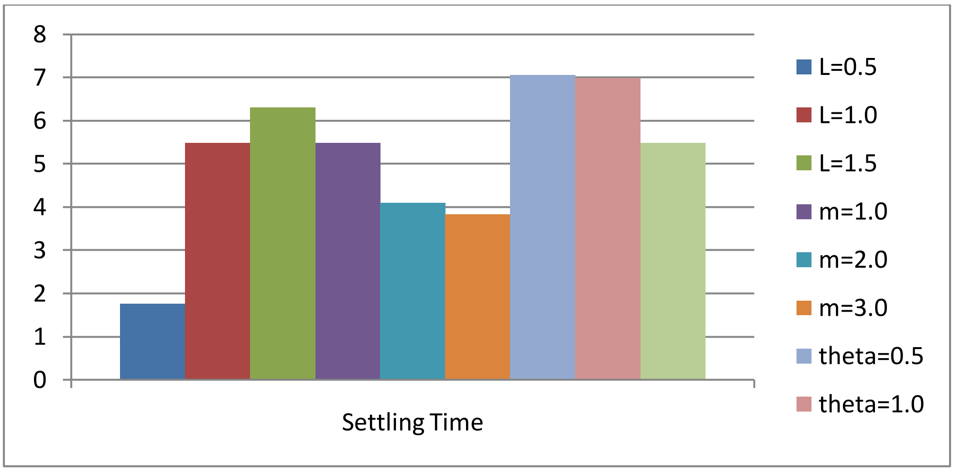

Figure 4.31: Settling time comparison for all speed data

Figure 4.31 shows the comparison of settling time for all speed data for all parameters in histogtram representation. Table 4.18 shows the value of all the settling time and the parameters. As shown from the Figure, there are differences between the changed parameters for all settling time value. When L is increased, it is shown that the settling time also increased. However, the settling time is decreasing simultaneously with the increasing of mass of payload. Same manner with the theta, the settling time is slightly decreased when we increase the value of initial theta.

4.2.1 POWER SPECTRAL DENSITY

Three modes of reduction level are considered in order to analyze the performances of the system to the PSD of the sway angle. Three data are taken from response without controller and with a controller. Difference of sway reduction is discussed in the histograms. Figure 4.32 shows two PSD response with table 4.19 as the parameters applied to the system.

Table 4.19: Default parameters

Parameter	**M (kg)**	**M (kg)**	**L (m)**	**g (m/s^2)**	**Initial angle, θ_i (rad)**
Value	2.49	1.0	1.0	9.81	1.5

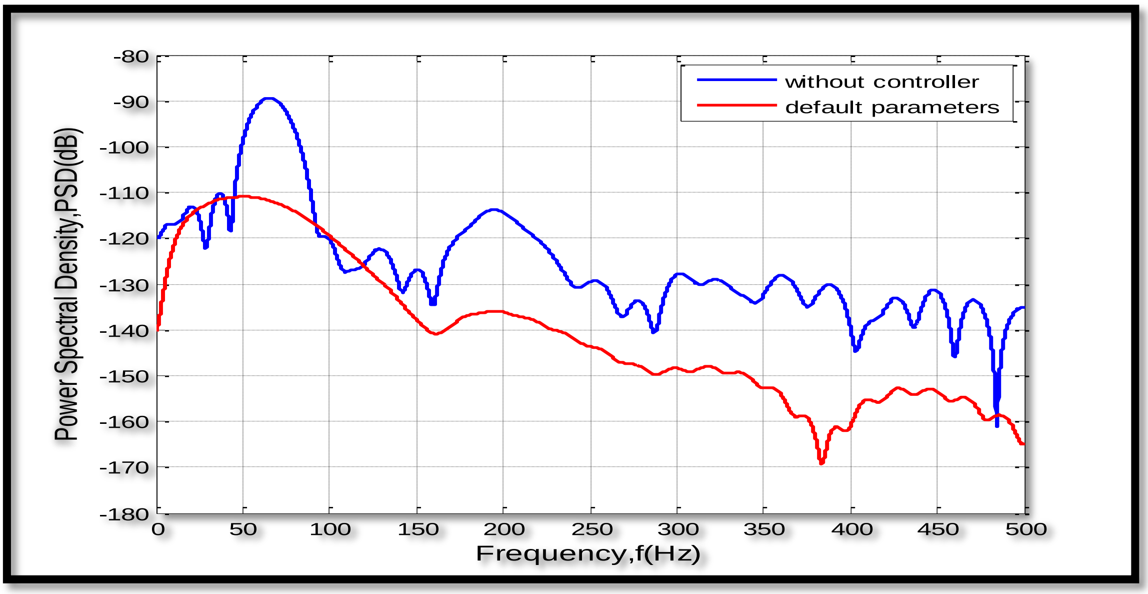

Figure 4.32: PSD response with default parameters

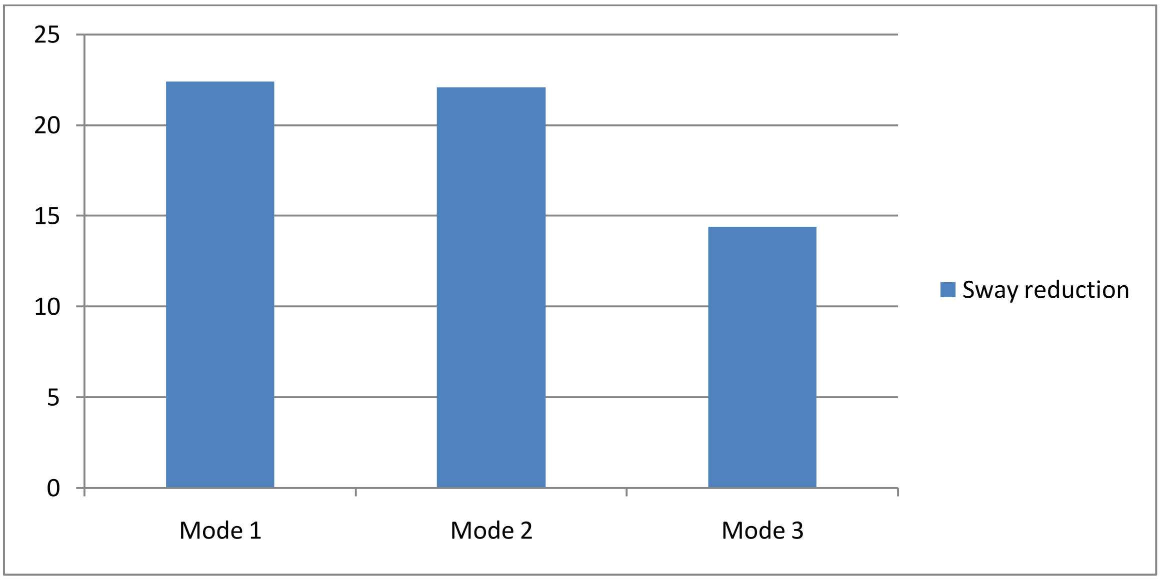

Figure 4.33: Attenuation level of sway different for default parameters

Figure 4.33 shows a histogram that describes and analyzes the attenuation level of sway different for default parameters between a controlled and uncontrolled system. Three data are taken at frequency 65.19 Hz, 195.3 Hz and 252 Hz with the sway reduction levels are 22.4 dB, 22.1 dB and 14.4 dB respectively.

It is shown that sway reduction level at mode 1 and mode 2 are almost the same. It is because there is still large error during mode 1 and 2 (65.19 Hz and 195.3 Hz). Therefore, the controller calculates the large error and reduces it. During mode 3, there is still an error but the error is already reduced to certain value during previous modes. Therefore, it is shown that, the attenuation level of sway reduction is decreased at mode 3 compared to mode 1 and 2.

The controller reaction is in best performance regarding to the reduced error as shown in the Figure.

Table 4.20: Parameters with L=0.5 m

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	Initial angle, θ_i (rad)
Value	2.49	1.0	0.5	9.81	1.5

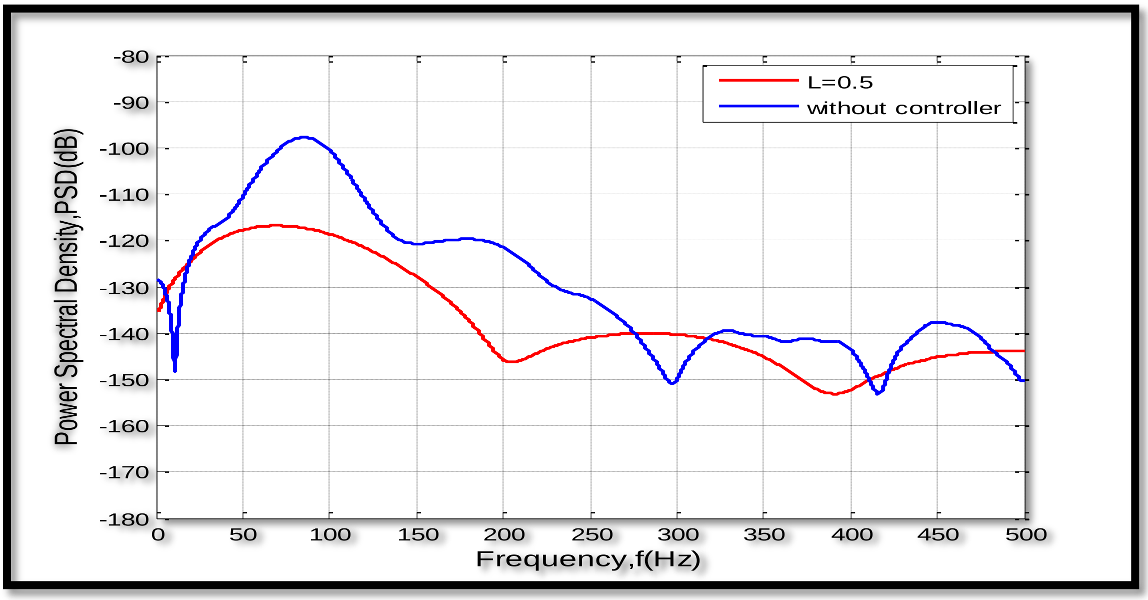

Figure 4.34: PSD response with L=0.5 m

Table 4.20 shows the parameters applied in this system with length of rope are changed to 0.5m. Figure 4.34 shows the responses of the system with two different responses, uncontrolled and controlled system. Three different data from three different frequencies are taken in order to analyze the performances of the controller to the sway angle response. All the data taken are represented in histograms graph at Figure 4.35.

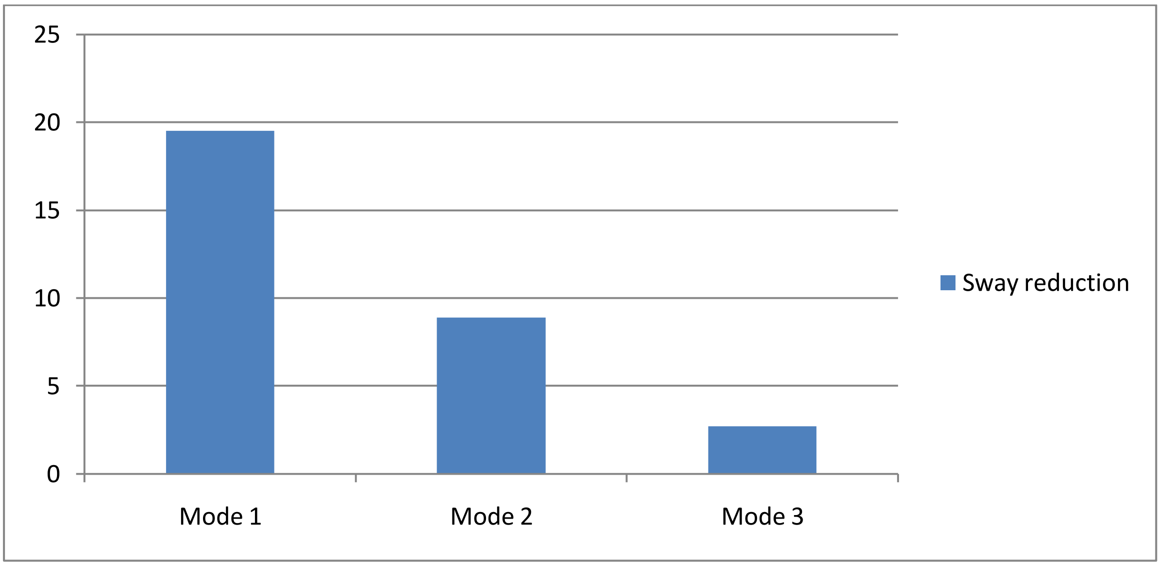

Figure 4.35: Attenuation level of sway different for L=0.5 m

Figure 4.35 shows the attenuation level of sway reduction for L=0.5m between controlled and uncontrolled system. Three modes are been considered in order to analyze the PSD response starting at 84.72 Hz, 248 Hz and 329 Hz. The level of sway reduction data from the system are 19.53dB, 8.9dB and 2.7dB.

As shown in the above Figure, a large error due from the uncontrolled system is reduced during mode 1. The level of sway reduction is decreased from mode 1 to mode 3. It proved that the error is already reduced during mode 1. But there is still some error from the system. Therefore, the controller is still reducing the error during mode 2. Mode 3 sway reduction is only small value. It is because there is only small error left during mode 3. Therefore, the sway reduction level is decrease from mode 2.

As a conclusion, the controller gives good performance in order to analyze the system and reduces the error occurred from the system.

Table 4.21: Parameters with L=1.5 m

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	Initial angle, θ_i (rad)
Value	2.49	1.0	1.5	9.81	1.5

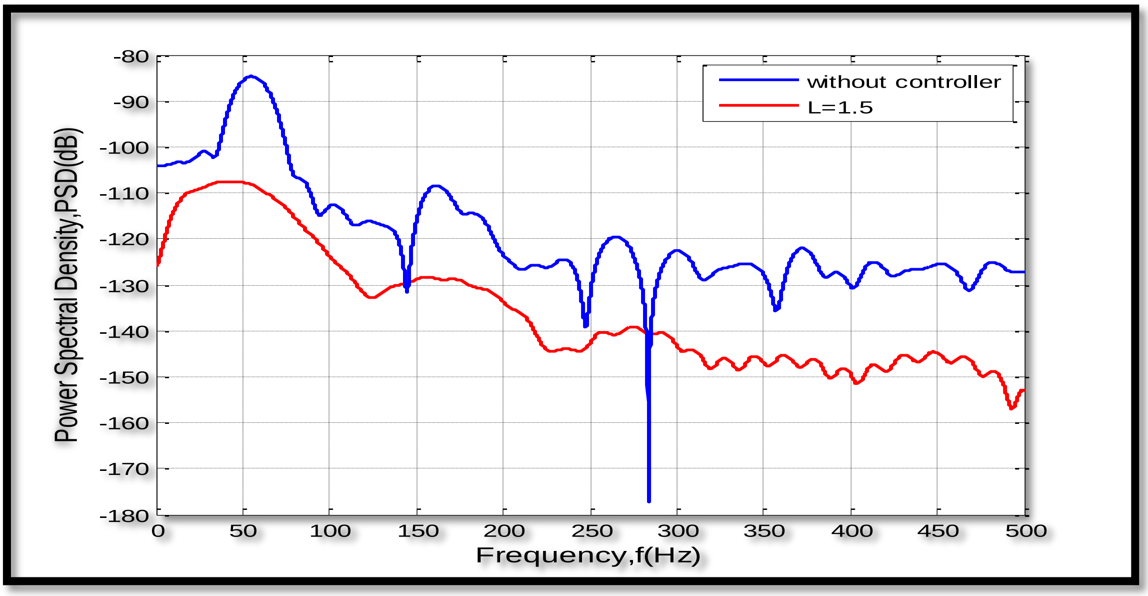

Figure 4.36: PSD response with L=1.5 m

Three modes of reduction level are considered in order to analyze the performances of the system to the PSD of the sway angle. Three different data are taken from response without a controller and with a controller. Difference of sway reduction is discussed in the histograms at Figure 4.37. Figure 4.36 shows two PSD response with table 4.21 as the parameters applied to the system. The length of rope, L is changed from 1m to 1.5m.

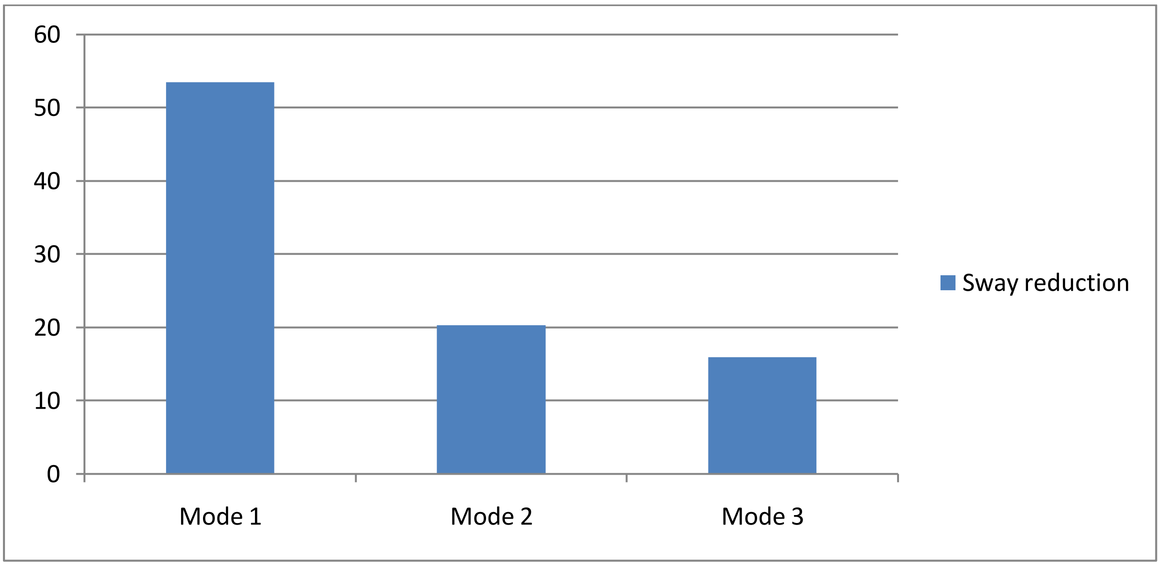

Figure 4.37: Attenuation level of sway different for L=1.5 m

The effect of controller to the system and sway angle is shown in Figure 4.36 in Power Spectral Density (PSD) form. The response is shown in comparative with uncontrolled system response. Details about the reduction level between controlled system and uncontrolled system are shown in Figure 4.37.

The reduction level is reduced from mode 1 to mode 2 and to mode 3. The larger length of pendulum will affect the system error. The system having a larger theta and larger time than L=0.5 and 1.0. So, the controller has enough time react to the system error. The error is reduced during mode 1. So, the error will reduced during mode 2 and mode 3. During mode 3, we can see that the reduction decrease. The error from uncontrolled system is already reduced during mode 1 and mode 2, but the system is still having an error during mode 3, so the controller reducing the left error to achieve theta=0.

Table 4.22: Parameters with m=2 kg

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	Initial angle, θ_i (rad)
Value	2.49	2.0	1.0	9.81	1.5

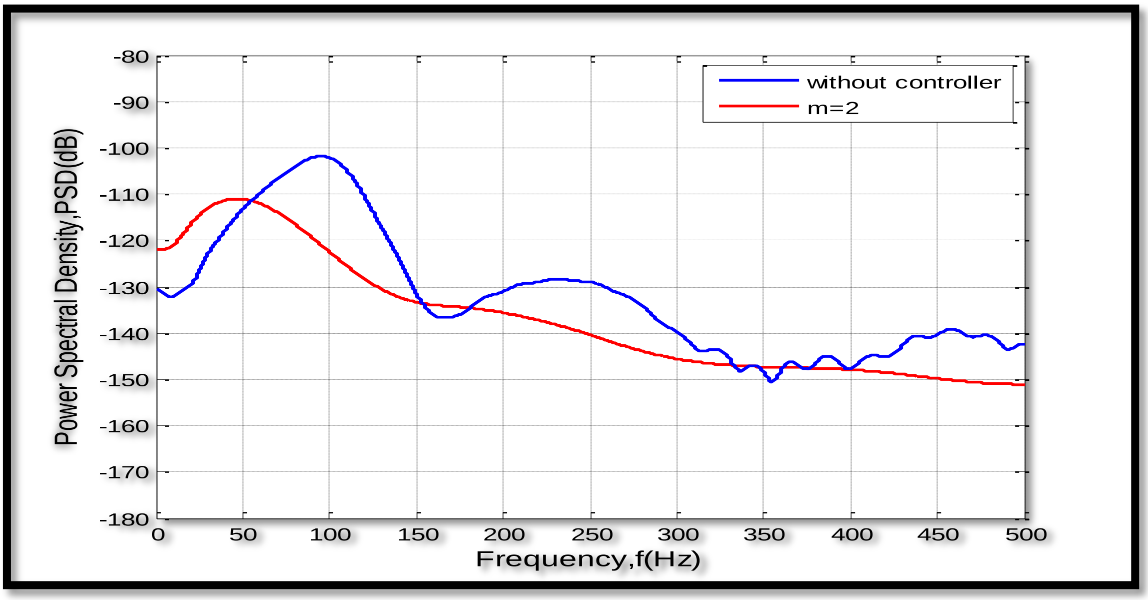

Figure 4.38: PSD response with m=2 kg

Table 4.22 shows the parameters applied in this system with mass of payload is changed to 2kg. Figure 4.38 shows the responses of the system with two different responses, uncontrolled and controlled system. Three different data from three different frequencies are taken in order to analyze the performances of the controller to the sway angle response. All the taken data are represented in histograms graph at Figure 4.39.

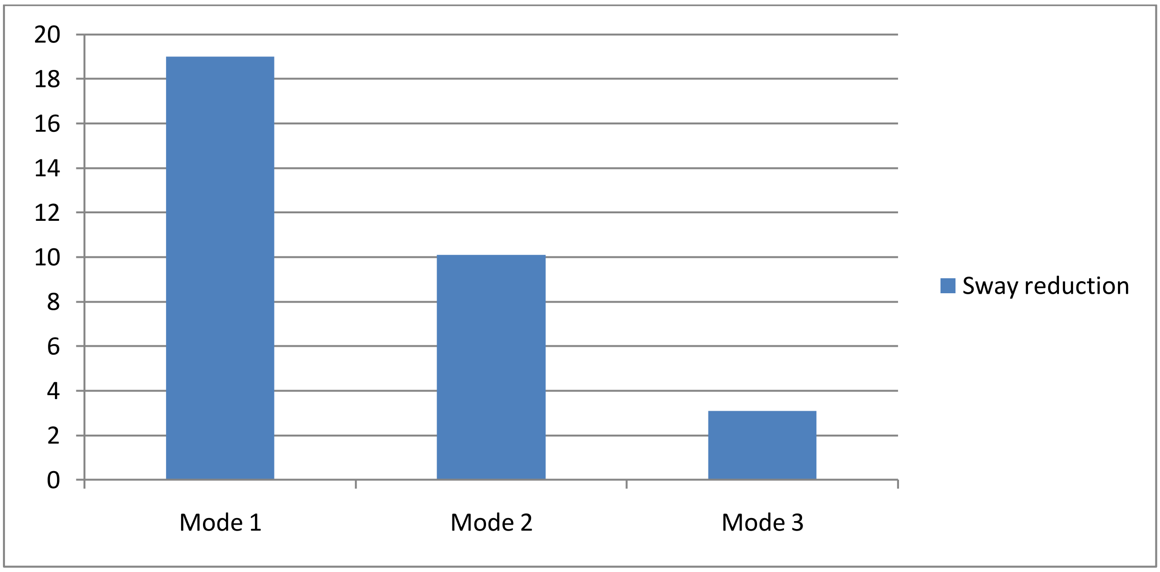

Figure 4.39: Attenuation level of sway different for m=2 kg

Figure 4.39 shows the attenuation level of sway reduction for m=2kg between controlled and uncontrolled system. Three modes are been considered in order to analyze the PSD response starting at 94.85 Hz, 233 Hz and 325.1 Hz. The level of sway reduction data from the system are 19dB, 10.1dB and 3.1dB.

As shown above, a large error due from the uncontrolled system is reduced during mode 1. The level of sway reduction is decreased from mode 1 to mode 3. It proved that the error is already reduced during mode 1. But there is still some error from the system. Therefore, the controller is still reducing the error during mode 2. Mode 3 sway reduction is only small value. It is because there is only small error left during mode 3. Therefore, the sway reduction level is decrease from mode 2.

As a conclusion, the controller gives good performance in order to analyze the system and reduces the error occurred from the system.

Table 4.23: Parameters with m=3 kg

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	Initial angle, θ_i (rad)
Value	2.49	3.0	1.0	9.81	1.5

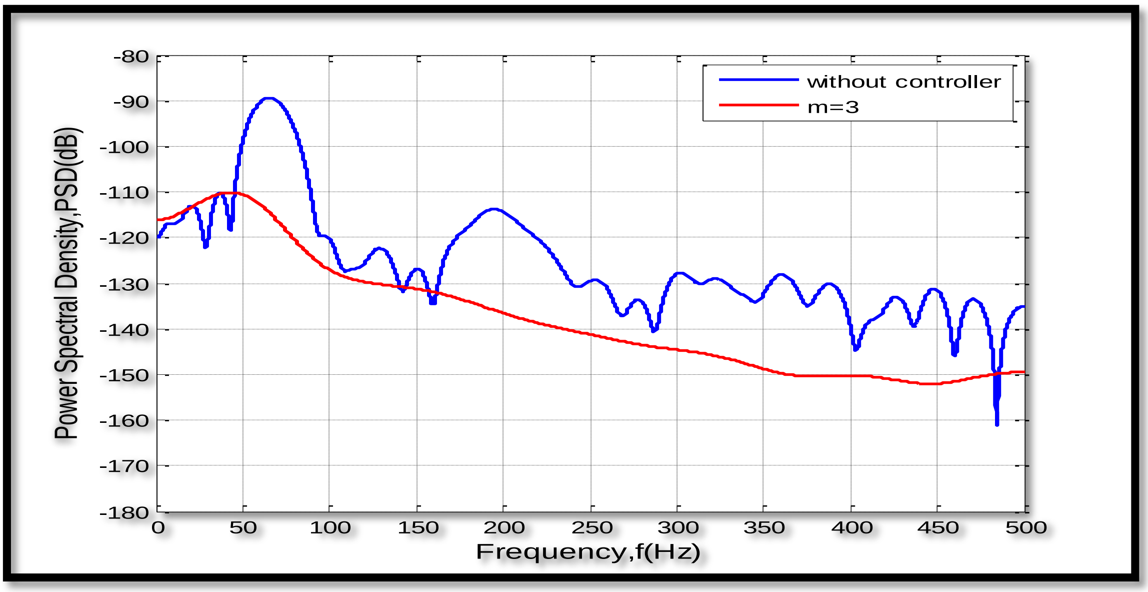

Figure 4.40: PSD response with m=3 kg

Three modes of reduction level are considered in order to analyze the performances of the system to the PSD of the sway angle. Three different data are taken from response without a controller and with a controller. Difference of sway reduction is discussed in the histograms at Figure 4.41. Figure 4.40 shows two PSD response with table 4.23 as the parameters applied to the system. The mass of payload, m is changed from 1kg to 3kg.

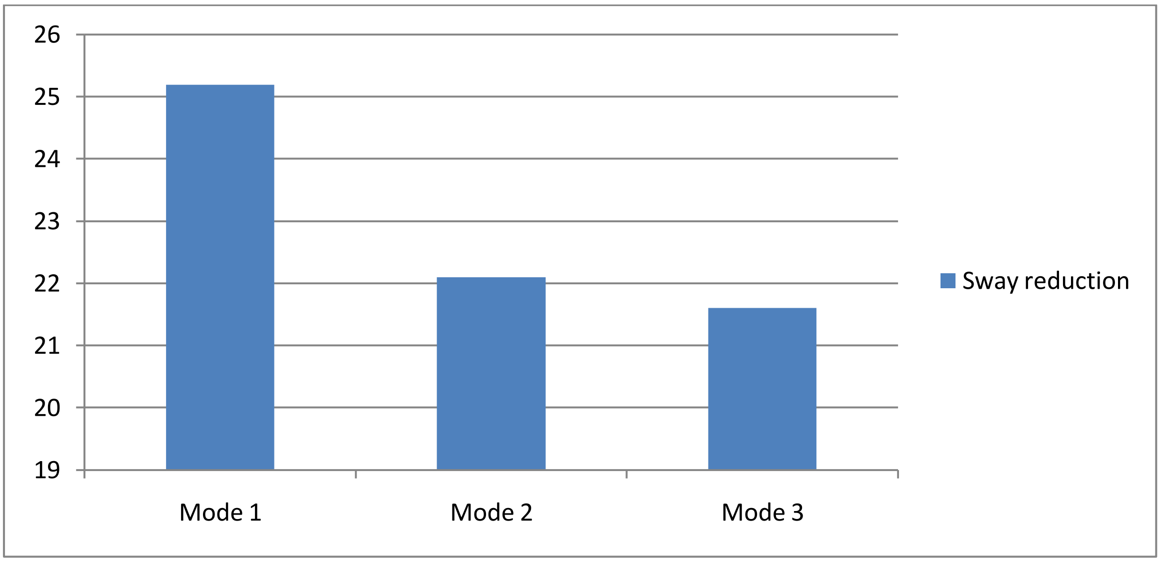

Figure 4.41: Attenuation level of sway different for m=3 kg

The effect of controller to the system and sway angle is shown in Figure 4.36 in Power Spectral Density (PSD) form. The response is shown in comparative with uncontrolled system response. Details about the reduction level between controlled system and uncontrolled system are shown in Figure 4.41.

The reduction level is reduced from mode 1 to mode 2 and to mode 3. The larger mass of payload will affect the system error. The system is having a larger theta and larger time than m=1.0 and 2.0 kg. So, the controller has enough time react to the system error. From mode 1, it is shown that larger level of reduction is achieved because the controller has larger time to react with the error from the system. The error is reduced during mode 1, during mode 2 and mode 3. During mode 3, we can see that the reduction level decrease. The error from uncontrolled system is already reduced during mode 1 and mode 2, but the system is still having an error during mode 3, so the controller reducing the lesser error to achieve theta=0.

Table 4.24: Parameters with θ_i =0.5 rad

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	Initial angle, θ_i (rad)
Value	2.49	1.0	1.0	9.81	0.5

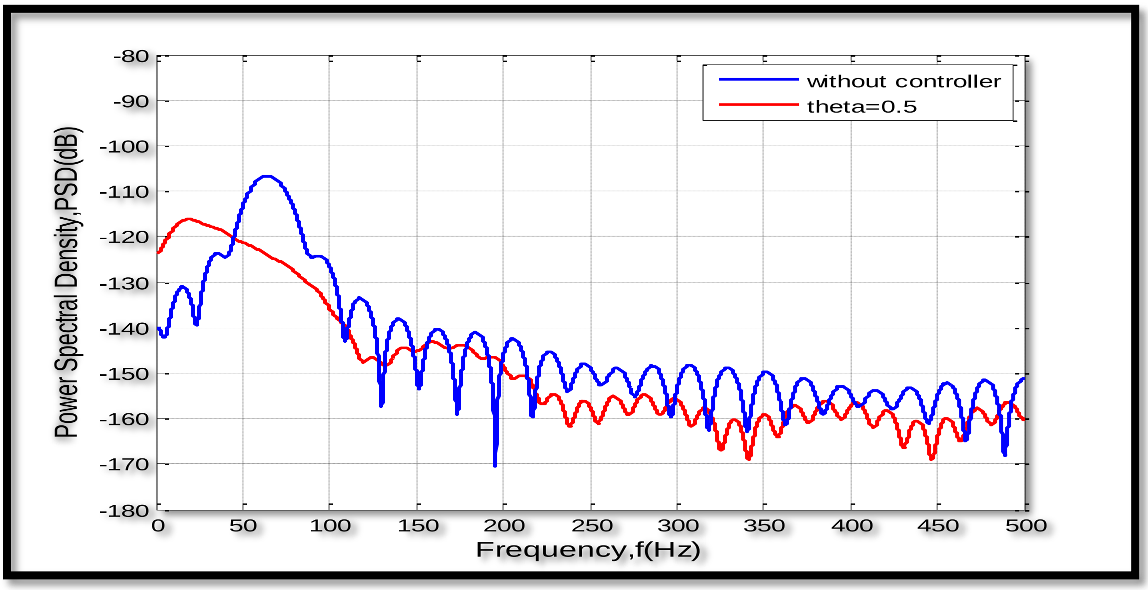

Figure 4.42: PSD response with θ_i =0.5 rad

Table 4.24 shows the parameters applied in this system with initial angle is changed to 0.5 rad. Figure 4.42 shows the responses of the system with two different responses, uncontrolled and controlled system. Three different data from three different frequencies are taken in order to analyze the performances of the controller to the sway angle response. All the taken data are represented in histograms graph at Figure 4.43.

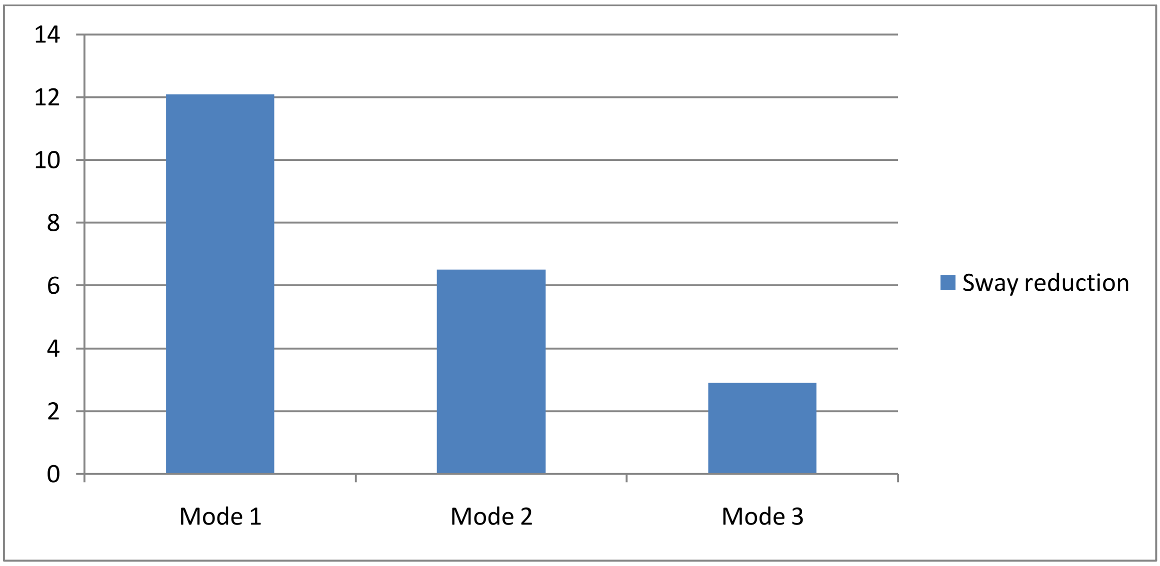

Figure 4.43: Attenuation level of sway different for θ_i =0.5 rad

Figure 4.43 shows a histogram that describes and analyzes the attenuation level of sway different for default parameters between a controlled and uncontrolled system. Three data are taken at frequency 63.35 Hz, 140.7 Hz and 161.7 Hz with the sway reduction levels are 12.1 dB, 6.5dB and 2.9 dB respectively.

It is shown that sway reduction level at mode 1 and mode 2 are almost the same. It is because there is still large error during mode 1 and 2 (63.35 Hz and 140.7 Hz). Therefore, the controller calculates the large error and reduces it. During mode 3, there is still an error but the error is already reduced to certain value during previous modes. Therefore, it is shown that, the attenuation level of sway reduction is decreased at mode 3 compared to mode 1 and 2.

The controller reaction is in best performance regarding to the reduced error as shown in the Figure.

Table 4.25: Parameters with θ_i =1.0 rad

Parameter	M (kg)	m (kg)	L (m)	g (m/s^2)	Initial angle, θ_i (rad)
Value	2.49	1.0	1.0	9.81	1.0

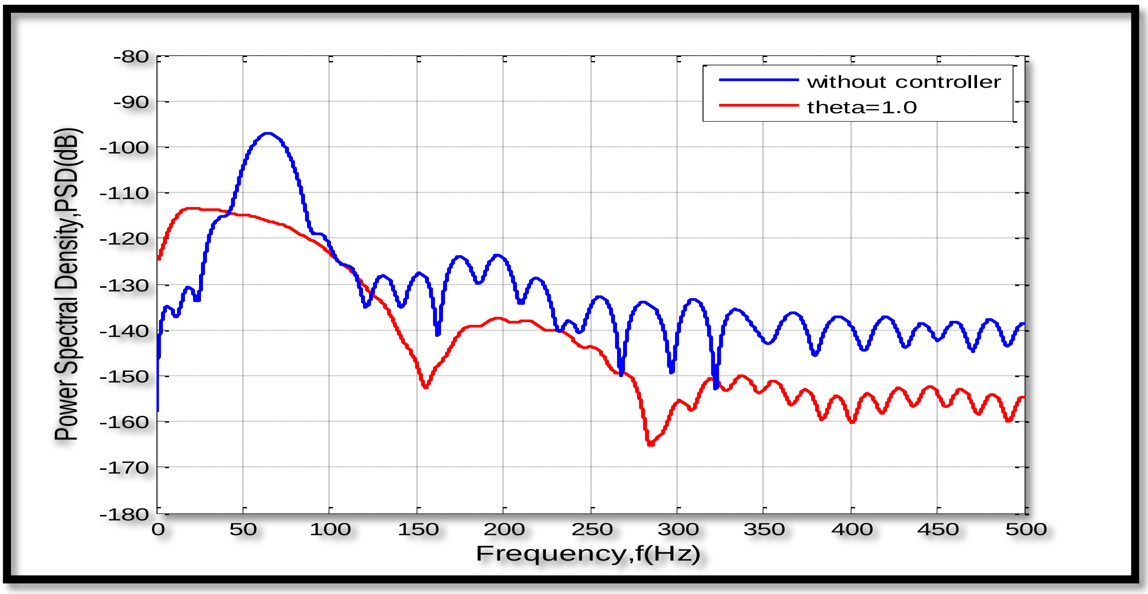

Figure 4.44: PSD response with θ_i =1.0 rad

Three modes of reduction level are considered in order to analyze the performances of the system to the PSD of the sway angle. Three different data are taken from response without a controller and with a controller. Difference of sway reduction is discussed in the histograms at Figure 4.45. Figure 4.44 shows two PSD response with table 4.25 as the parameters applied to the system. The initial angle is changed to 1.0 rad.

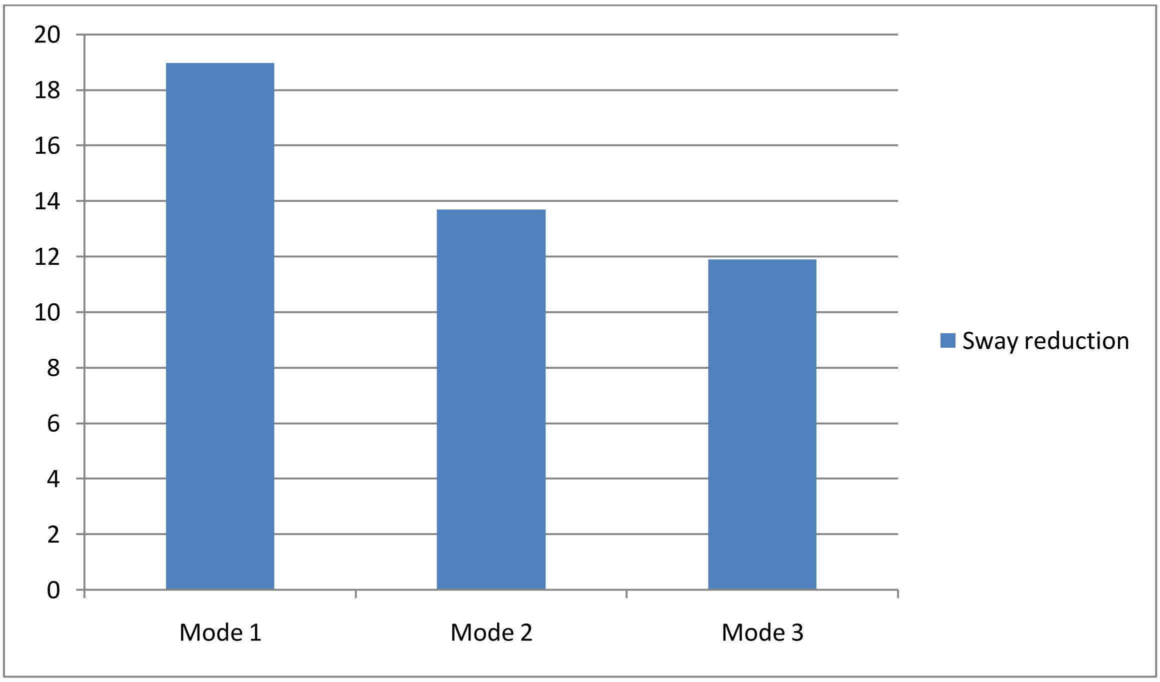

Figure 4.45: Attenuation level of sway different for θ_i =1.0 rad

Figure 4.45 shows the attenuation level of sway reduction for m=2kg between controlled and uncontrolled system. Three modes are been considered in order to analyze the PSD response starting at 63.96 Hz, 197.4 Hz and 255.4 Hz. The level of sway reduction data from the system are 18.97dB, 13.7dB and 11.9dB.

As shown in the above Figure, a large error due from the uncontrolled system is reduced during mode 1. The level of sway reduction is decreased from mode 1 to mode 3. It proved that the error is already reduced during mode 1. But there is still some error from the system. Therefore, the controller is still reducing the error during mode 2. Mode 3 sway reduction is only small value. It is because there is only small error left during mode 3. Therefore, the sway reduction level is decrease from mode 2.

As a conclusion, the controller gives good performance in order to analyze the system and reduces the error occurred from the system.

CHAPTER 5

CONCLUSION

5.1 CONCLUSION

From the simulation results shown in previous chapter, it is shown that a gantry crane controller using Linear Quadratic Regulator (LQR) controller is successfully designed in order to fulfill the main objective of the project, which is to reduce the sway angle of the rope. As can be seen in chapter 4, LQR controller manages to control the system and reduces the sway angle even after several types of parameters of crane system are applied. LQR controller react with the given gains and the systems, so the controller will able to control all the necessary parts that related to achieve zero offset (error=0).

The responses for different parameters are shown in chapter 4. All the analysis shows that, different parameters will lead to different system performances such as, the rise time, settling time and sway angle. Even though, all the responses will return to its steady state due to the existence of LQR controller. LQR controller manages to reduce the error although the parameters are changed.

As a conclusion, LQR controller is a good controller when it is applied to a gantry crane system. It gives best performances in order to reduce the sway angle error from the system.

5.2 SYSTEM LIMITATION

As we know, LQR controller has four gains to be considered. These gains will affect the LQR performances. In order to get the best LQR performances, we need to tune the gain perfectly. During the tuning method, there is no any efficient method except trial and error method as same as PID tuning method. So, the LQR tuning method will take much longer times because its need four gains to be tuned at the same time. The tuning time will possibly gives problems to the user in order to find the best possible gains for the system under consideration.

5.3 FUTURE WORK RECOMMENDATION

There are some upgrades that can be done in order to study the robustness of LQR controller for a gantry crane system. We can apply the LQR system combined with other controllers such as PID controller or delay feedback system. The combination of these controllers can provide different system performance to decrease the sway angle.

Moreover, we can also control the position of the crane and the speed of the crane. This method can be applied in real life at industrial park. The industry will need a controller that can control all these specifications, desired sway angle, speed and position of the cart.

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